Determination of Elastic Moduli in Brass, Aluminum, Plastic, and Wood bars using Resonance

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Abstract

This experiment investigated how to excite the Flexural and Torsional modes in Brass, Aluminum, Wood, and Plastic rods. The setup consists of a two channel dynamic signal analyzer, a power amp, an operational amplifier and high pass filter circuit. Two coils of copper wire act as transducers on each end of the rod. The rod is suspended with the transducer coils in strong magnetic fields provided by neodymium magnets. The signal analyzer sends a swept sine wave into the power amplifier which drives one transducer coil. The rod's response is measured with the op-amp circuit connected to the second transducer coil which is connected to the signal analyzer. The resonance peaks were found using the two channel dynamic signal generator. With these peaks, the Flexural and Torsional modes can be determined, thereby allowing one to calculate the Elastic Moduli.

Experimental Setup

This experiment looked at the mechanical resonance of brass, aluminum, plastic, and wood bars. This was achieved by measuring and massing one of each type of bar. The physical values for each bar are given in Table 1. Copper coils were used, one epoxied on each end of the bars totaling two for each bar. They were created by taking six meters of 34 gauge copper wire and 28 gauge copper wire and wrapping it around a corrugated screwdriver handle that had a diameter of 1.234 inches for 50 turns. About fourteen cm of the same copper wire was used to lace around the coil to keep it wound tight. By using six meters and fifty turns, two half meters of the wire were left for connections. Two vertical stands were used to hold horseshoe magnets, each with four neodymium magnets on the pole faces. One horseshoe magnet was placed on each stand via clamps. A free-free boundary condition on the bars was obtained by supporting the bars with rubber bands on each end. A bar was then placed such that it was in the middle of the horseshoe magnets. One coil was connected to an op-amp high pass filter circuit while the other was connected to a power amp. The power amp was connected to a two channel dynamic signal analyze. The signal analyzer sent a swept sine wave into the power amp driving the coil. The response is measured by the op-amp circuit and read by the signal analyzer.

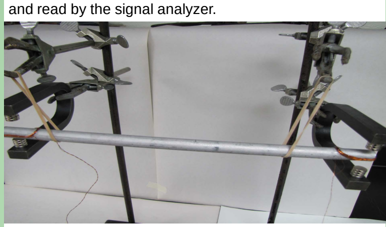
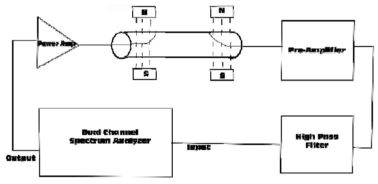


Figure 1: In this figure, the aluminum bar was used as an example to show how the apparatus was set such that a free-free boundary was used.



Bar Type	Length (cm)	Diameter (cm)	Mass (g)	Density (g/cm³)
Brass	30.2	1.273	327.300	8.49 0.03
Aluminum	33.8	1.428	148.360	2.74 0.06
Plastic	30.5	1.288	54.720	1.255 0.004
Wood	30.3	1.214	14.600	0.4171 0.0016

Table 1: This table shows the length, diameter, mass and density of each of the four rods used. The uncertainty in the length was ± 0.1 cm. Diameter uncertainty was ± 0.002 cm. Mass uncertainty was ± 0.005 g/cm^3

Theory

Torsional Modes

In order to detect the modes that is governed by the shear modulus and the Young's modulus separately, the rods need to be fashioned so that the wavelengths of the waves and the length of the bar are much greater than the diameter of the bar. By doing this the waves will propagate independently. To excite the torsional modes of the bars, they need to be oriented like in Figure 2. With this orientation, a force is created on the bar as given by the Lorentz force law for stationary wire given by:

$$F = iB \otimes l \qquad (1)$$

These forces produce torques on the bar forcing it to move back and forth within the magnetic field. As a consequence, an e.m.f is produced because the area of the coil is changing in the magnetic field. By imposing a free-free boundary condition on the bars, harmonic modes are obtained and their frequencies are:

$$f_n^T = \frac{nc_T}{2L} \qquad (2)$$

where n takes integer values one, two, three, etc. The phase speed in equation 2 relates the shear modulus and the density of the bar. By using the phase speed one can obtain the shear modulus:

$$G = 4\rho L^2 \left(\frac{f_n^T}{n}\right)^2 \qquad (3)$$

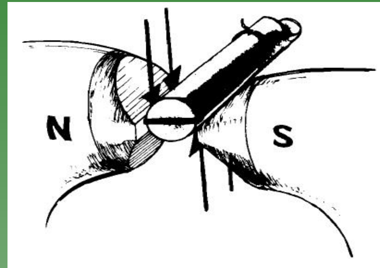


Figure 2: A schematic of how the bars should be oriented in the magnetic field in order to attain torsional vibration. Figure taken from Penn State Lab Manual.

Flexural Modes

To excite the flexural modes, the bars need to be oriented like in Figure 3. By setting it up in this manner, forces act upon the rod but in unequal magnitudes. This is because while one part of the coil feels a strong magnetic force, the rest of the coil does not because it lies in the fringe field. This leads to modes that are not harmonic, and their frequencies are found by:

$$f_n^F = \frac{\pi n^2 c_L \kappa}{8L^2} \qquad (4)$$

where n=3.0112,4.9994,7,9… and κ is the radius of gyration. By using the phase speed one can find the Young's modulus as:

$$E = \frac{1024\rho L^4}{\pi^2 d^2}\left(\frac{f_n^F}{n^2}\right)^2 \qquad (5)$$

where n takes on the same values as mentioned above.

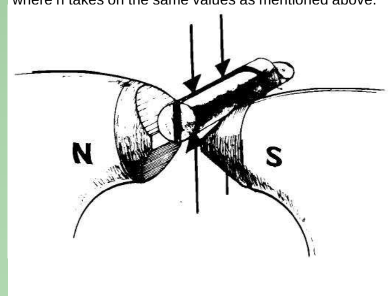


Figure 3: A schematic of how the bars should be oriented in the magnetic field in order to attain flexural vibrations. Figure taken from Penn State lab manual.

Data

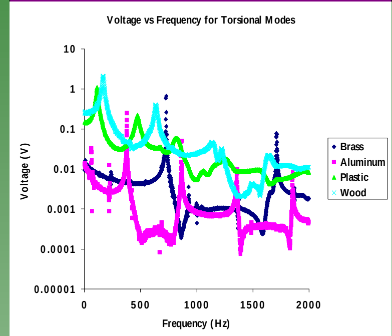


Figure 4: This figure shows the response of the four different bars to torsional excitation. The uncertainty in the frequency was found to be ± 5 Hz.

Bar Type	f (Hz)	n	f/n (Hz)
Brass	3800	1	3800
	7680	2	3840
		Average=	3820 ± 20
Aluminum	4600	1	4600
	9250	2	4630
	14000	3	4670
		Average=	4633 ± 20
Plastic	1490	1	1490
	3070	2	1540
	4590	3	1530
		Average=	1520 ± 20
Wood	1730	1	1730
	3780	2	1890
		Average=	1810 ± 20

Table 2: These values represent the resonant peak values for the frequency, mode number, and frequency per mode number. The average values were used to calculate G using equation (3).

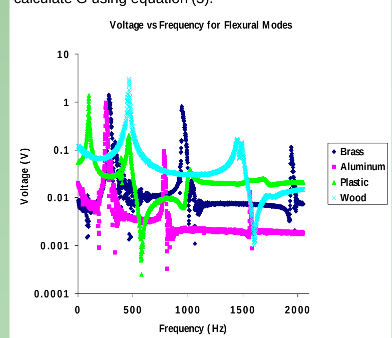


Figure 5: To the bottom left is a plot of the resonance response to flexural excitation of the four bars. Table 3 has the corresponding resonant peak values for the frequency, mode number and frequency per mode number squared for the different bars that were used. The uncertainty for the frequency was found to be ± 1 Hz.

Bar Type	f (Hz)	n	f/n² (Hz)
Brass	356	3.0112	39.26
	975	4.9994	39
	1900	7	38.78
		Average=	39.01 0.14
Aluminum	582.2	3.0112	64.2
	1590	4.9994	63.6
	3080	7	62.9
		Average=	63.57 0.14
Plastic	189	3.0112	20.84
	527	4.9994	21.09
		Average=	20.97 0.14
Wood	530	3.0112	58.45
	1440	4.9994	57.61
		Average=	58.03 0.14

Table 3: These values represent the resonant peak values for the frequency, mode number, and frequency per mode number. The average values were used to calculate E using equation (5)

Analysis of Data

In order to find the Shear Modulus and the Young's Modulus with the given data, the frequency per mode number and frequency per mode number squared are needed. Because the bars are assumed to be uniform and there is no dispersion of the wave, the average values of the frequencies per mode number were used to calculate the moduli using equations (3) and (5).

Bar Type	Shear Modulus (GPa)	Accepted Values (GPa)
Brass	45.4 ± 0.6	36 - 41[2]
Aluminum	26.4 ± 0.6	26 - 30[2]
Plastic	1.3 ± 0.04	---
Wood	0.5 ± 0.01	---

Table 4: In this table the calculated Shear moduli for the four rods are given. A range of currently accepted values, if known are also given for comparison.

Bar Type	Young's Modulus (GPa)	Accepted Values (GPa)
Brass	68.8 ± 1.1	96 - 110[2]
Aluminum	72.0 ± 1.8	70 - 79[2]
Plastic	4.30 ± 0.08	2.4 - 4.1[1]
Wood	8.3 ± 0.12	5.5 – 15.7[1]

Table 5: This table has the calculated Young's moduli for the four rods used as well as the range of accepted values for comparison.

Literature Cited

[1]W.D. Callister Jr., *Materials Science and Engineering an Introduction*, 4th Ed. (Wiley and Sons, New York, 1994), pp. 778-779.

[2] Properties of Common Solid Materials, WWW document, (http://www.efunda.com/materials/common_matl/common_matl.cfm?matlphase=solid&matlprop=mechanical).

Acknowledgments

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