Optimizing the Angular Acceleration of a Current Carrying Wire in Dipole Magnetic Field

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ABSTRACT

When a current carrying wire is placed in a dipole magnetic field, there will be electromagnetic force acting on it. Fixing the rotation axis, the torque should change with the change of the shape of the wire. To produce the extreme angular acceleration, moment of the inertia of the wire must be considered as well. This research project calculated the torque for a randomly shaped wire, and proved the torque is constant when two ends are fixed using the method of Calculus of Variation. The extreme moment of inertia is also approached in this project. The test of theoretical outcomes is preformed with building the system using Solidworks, and 3D printer. The main materials that are used are copper wire and copper disk, cylindrical magnetic, and batteries. The final result will give the shapes with different length limits wire that will produce the maximum and minimum angular accelerations.

INTRODUCTION

- Angular acceleration is the rate of change of angular velocity. For two-dimensional rotational motion, Newton's second law can be adapted to describe the relation between torque and angular acceleration: $\alpha = \frac{\tau}{I}$, where τ is the total torque exerted on the body, and I is the mass moment of inertia of the body.
- The torque is given by the equation, $\vec{\tau} = \vec{r} \times \vec{F}$, where r is the distance between the point where the force is acting. The total force on the current carrying wire in a magnetic field is given by the equation,
  Dipole magnetic field, B,
  $$B_{dip}(r) = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$
- The moment of inertia, otherwise known as the angular mass or rotational inertia, of a rigid body determines the torque needed for a desired angular acceleration about a rotational axis. It depends on the body's mass distribution and the axis chosen, with larger moments requiring more torque to change the body's rotation. It is an extensive (additive) property: the moment of inertia of a composite system is the sum of the moments of inertia of its component subsystems (all taken about the same axis). One of its definitions is the second moment of mass with respect to distance from an axis r, $I = \int r^2 dm$, integrating over the entire mass.

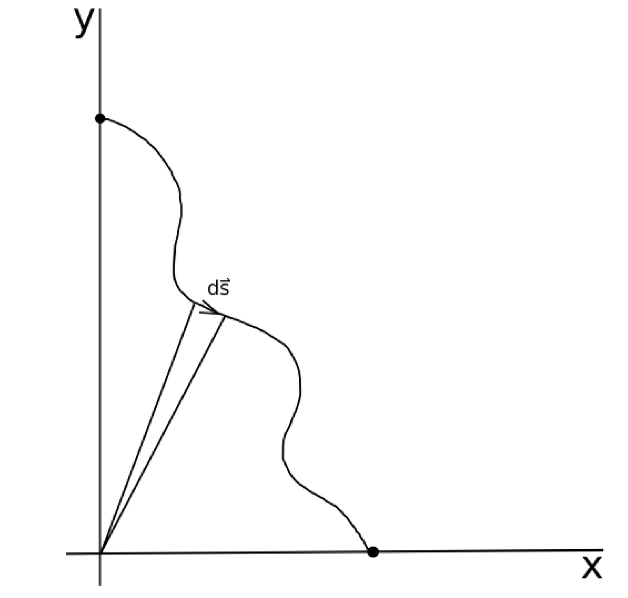


Figure 1 Randomly shaped Current carrying wire

MATERIALS AND METHODS

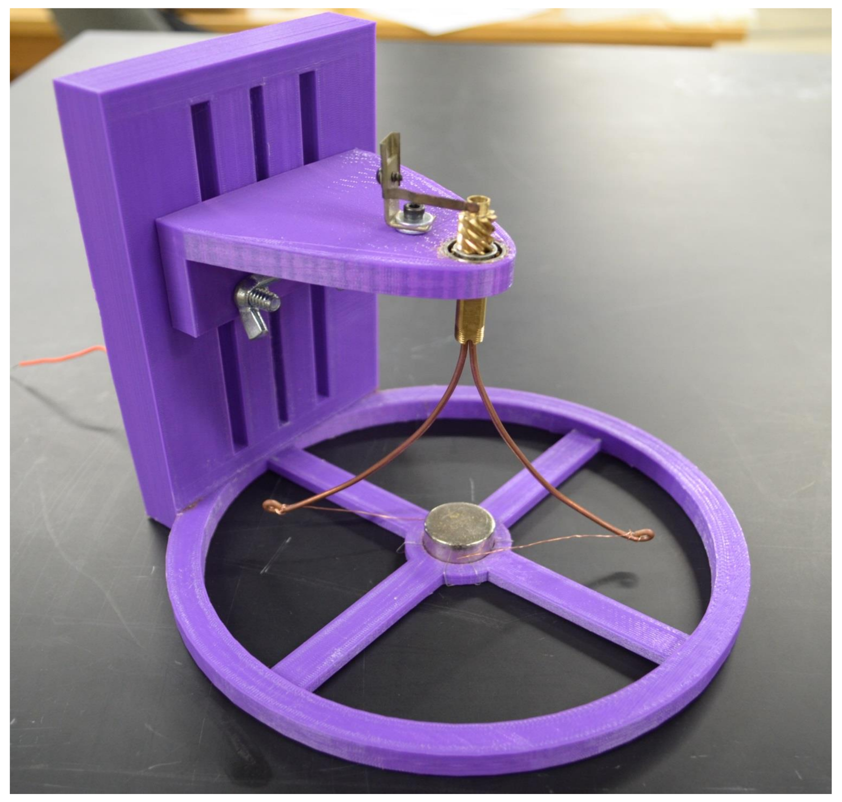
Using the equations mentioned in the interduction, for a randomly shaped wire, the torque equation is given by,

$$\vec{\tau} = \int_{s_i}^{s_f} \vec{r} \times (I_0 d\vec{s} \times \vec{B}) = \frac{\mu_0 m I_0}{4\pi}\left(\frac{sin^2\theta_1}{r_1} - \frac{sin^2\theta_2}{r_2}\right)\hat{y} \qquad (1)$$

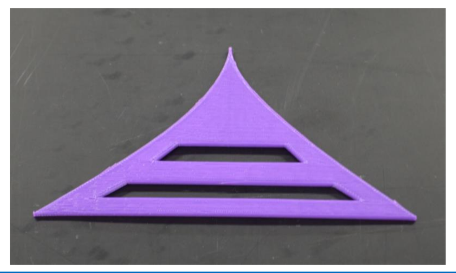
and the moment of inertia is given by,

$$I = \rho\int_{x_1}^{x_2} x^2 \sqrt{1 + (\frac{dy}{dx})^2}\, dx \qquad (2)$$

Where ρ is linear density of the wire. Equation (1) indicates that the torque only depends on two end points of the wire. To extremize the angular acceleration, the method of Variational Calculus is applied to Equation 2.
The program Soildworks is used to design the device shown in the Figure 2(a), and the device was printed using 3-D printer. A model (Figure 2(b)) for bending wire to the "extreme" shape is designed using Solidworks as well as outputting XYZ points of the curve to the program using Python language.



(a)



(b)

Figure 2 (a) device to run the test, (b) model to bend the wire

RESULTS

Applying the Euler's Equation to Equation (2) to obtain the differential equation,

$$y'^2 = \frac{C^2}{(x^2+\lambda)^2 - C^2} \qquad (3)$$

Four cases of solution are obtained by solving this differential equation taking C = 1/b,
Case 1,

$$y = \frac{1}{\sqrt{b(b\lambda+1)}}\int_0^\theta \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}}, \quad x = \sqrt{\frac{b\lambda+1}{b}}\cot\theta$$

where

$$k = \sqrt{\frac{2}{b\lambda+1}}\ , 1 < b\lambda, 0 < b$$

Case 2,

$$y = \frac{1}{\sqrt{2b}}\int_0^\theta \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}}\ , \quad x = \sqrt{\frac{1-b\lambda}{b}}\sec\theta$$

where

$$k = \sqrt{\frac{2}{b\lambda+1}}\ , -1 \le b\lambda \le 1, 0 < b$$

Case 3

$$y = \frac{1}{\sqrt{b(1-b\lambda)}}\int_0^\theta \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}}\ , \quad x = \frac{\sqrt{(1-b\lambda)/b}}{\sin\theta}$$

where

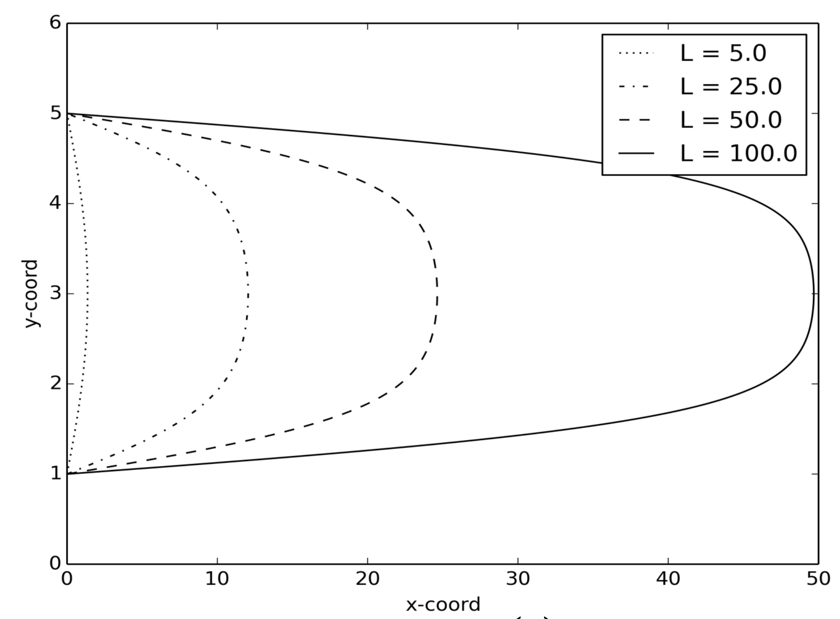
$$k = \sqrt{\frac{b\lambda-1}{b\lambda+1}}\ , b\lambda < -1\ , b > 0$$

Case 4

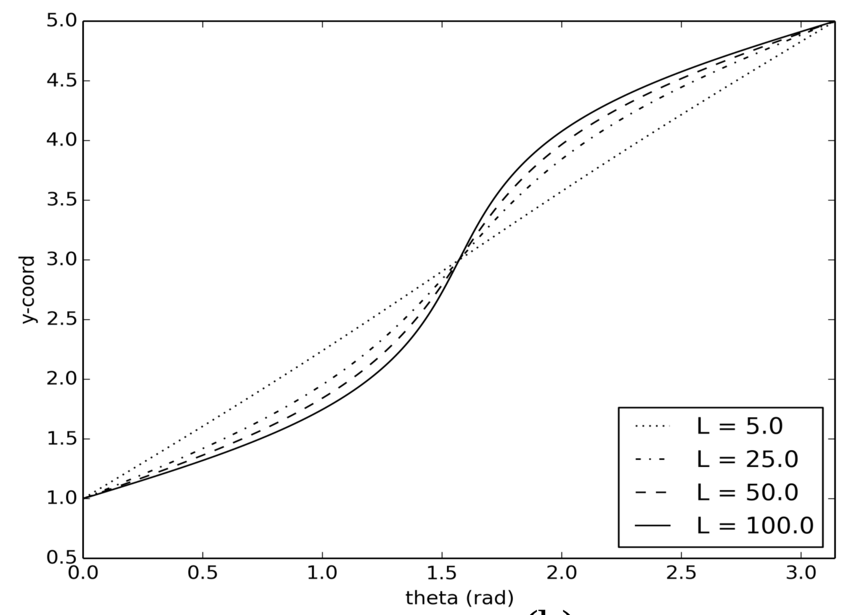
$$y = \frac{1}{\sqrt{b(-1-b\lambda)}}\int_0^\theta \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}}\ , \quad x = \sqrt{(1-b\lambda)/b}\,\sin\theta$$

where

$$k = \sqrt{\frac{b\lambda+1}{b\lambda-1}}\ , 1 < b\lambda\ , b < 0$$



(a)



(b)

Figure 3 (a) Case 4 with different length constraint, (b) y_vs_θ

CONCLUSION

Standard deviation calculations show that while there is no significant difference in the data obtained from the testing of the Case 4. There was less variation using the automated device.
This device then has the potential to reduce errors that may be subjective to the experimenter. For example, different experimenters may approach a test subject at different rates and speeds which may inevitably affect the response. The device maintains the same rate of approach ensuring that the data from different experiments can be compared. This consistent approach will result in more consistent data from experiments.

FUTURE WORK

The solutions of the differential equation (3) consist Elliptical Integral of the First Kind. In the Case 1, 2 and 3, x-value is infinite either when $\theta = 0, or\ \theta = \pi/2$. For two given end points and length, a proper way of choosing θ_1 and θ_2 is significantly important to obtain the curve functions. Due to the time constraint, the curve functions of Case 1, 2, and 3 that are the cases for maximum angular acceleration(minimum moment of inertia) were not solved. The solution of Case 4 gives the minimum angular acceleration(maximum moment of inertia). The following work will focus on finding a method of choosing θ_1 and θ_2, so that a theoretical function for maximum angular acceleration can be obtain and tested.

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