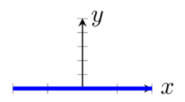
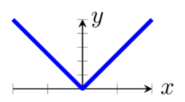
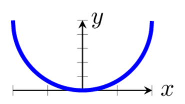
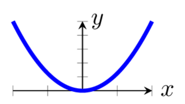
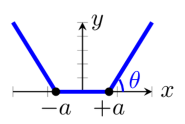
Nadira Amadou Mahamane
NAMADOUMAHAMANE1@cub.uca.edu

Dr. William Slaton
WVSLATON@uca.edu

Dept. of Physics & Astronomy
The Univ. of Central Arkansas

Analysis and Experiment of Air Drag on a Sphere

Model Results

Shape	Function	R_π
	$y(x) = 0$	1
	$y(x) = \|x\|$	0.5
	$y(x) = 1 - \sqrt{1 - x^2}$	0.5
	$y(x) = x^2$	$\approx 0.40236\ldots$
	$y(x) = \begin{cases} 0 & -a < x \le a \\ +mx + b & +a < x \le +1 \\ -mx + b & -a < x \le -1 \end{cases}$	$\approx 0.381966\ldots$

For the conical frustum $\theta = 90° - \alpha$, where $\alpha \approx 31.7°$ and $a = 1 - \tan(\alpha)$. Note that the reduced drag coefficient is less than the parabola! This surprising result was determined by Newton[1].

STL Files

The `.STL` file extension common in 3D printing is an abbreviation of the word **ST**ereo**L**ithography, though sometimes it is also referred to as **Standard Triangle Language** or **Standard Tessellation Language**. The main purpose of the `STL` file format is to encode the surface geometry of a 3D object as a *tessellation* in a particular format as illustrated below. Tessellation is the process of tiling a surface with one or more geometric shapes such that there are no overlaps or gaps.

```
solid simple
facet normal 0 -1 0
   outer loop
      vertex 0 0 0
      vertex 0 0 1
      vertex 1 1 0
   endloop
endfacet
endsolid simple
```

Figure 4a. `STL` code format (example).

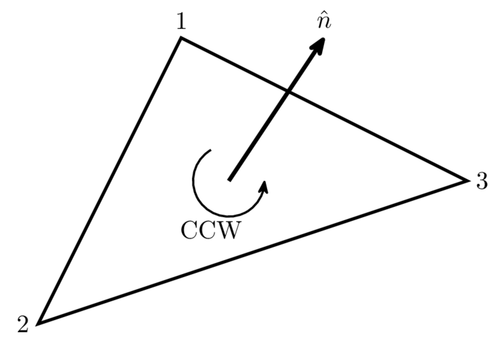


Figure 4b. `STL` triangular facet indicating vertices and normal.

Type of Nose Cone

The images below represents other types of nose cone we were able to investigate in our research. We were able to draw the 3D models and perform the theoretical calculations.

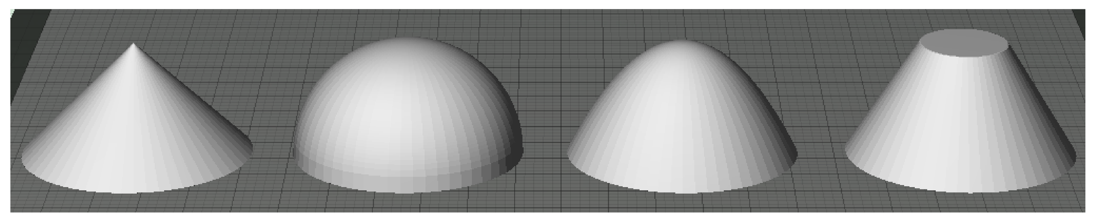


Figure 5: Rendered 3D model of generated `STL` nose cone for (left to right): cone, hemisphere, paraboloid, and truncated frustum.

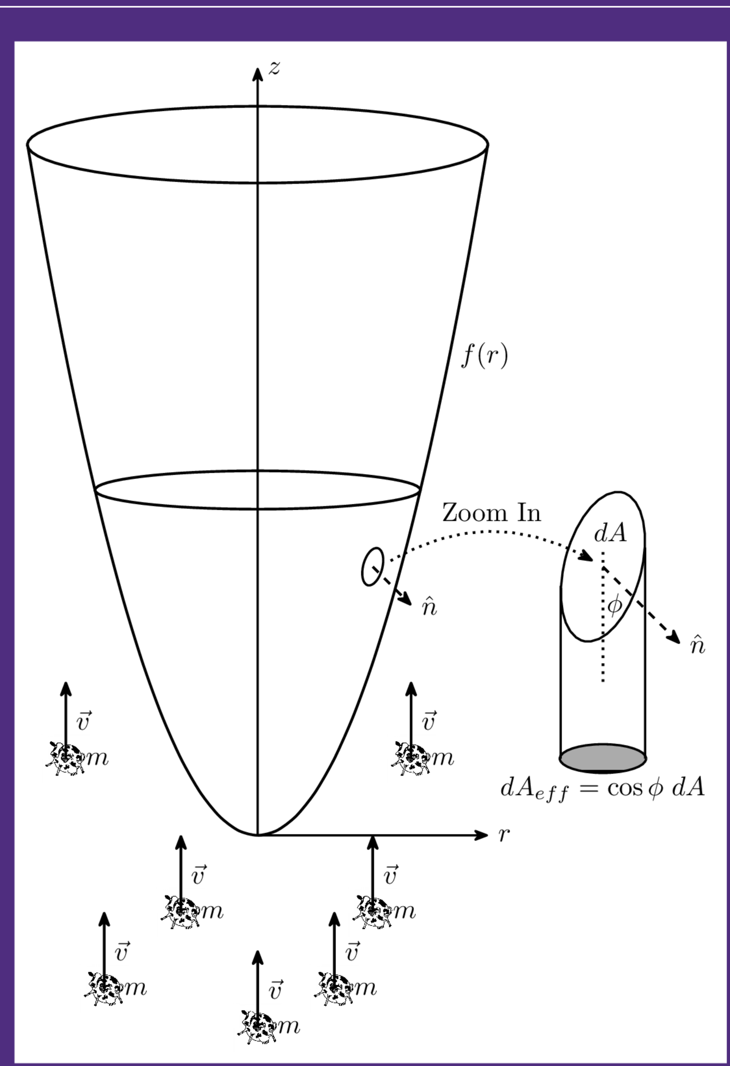


Fig 1: A symmetric object defined by $f(r)$ with "particles" of air, m, with velocity, $\vec{v}$.

Model

Newton's Nose-Cone[1] follows Newton's analytical modeling of air drag as the result of impacts from tiny "particles" of air as illustrated in Fig 1. The impulse-momentum theorem is,

$$\vec{F}\Delta t = \Delta\vec{p},$$

where $\vec{p} = m\vec{v}$ is the momentum. Fig 2 illustrates the vector components of interest. The nose-cone is symmetric about z so dF_r will sum to zero leaving only a component in the z-direction,

$$dF_z = \cos\phi\, dF_\perp.$$

The impact force, $dF_\perp$, is from mass, m, hitting the surface. Thus,

$$dF_\perp \Delta t = mv\cos\phi - (-mv\cos\phi)$$

or for the z-component,

$$dF_z = 2\frac{m}{\Delta t} v\cos^2\phi. \quad (1)$$

$m/\Delta t$ is the rate at which particles are hitting the surface, dA, which is dependent on the density of particles, ρ, in a volume, V. Hence,

$$\frac{m}{\Delta t} = \frac{\rho V}{\Delta t}.$$

The particles in volume, V, that hit the surface in time Δt depends in the velocity, v, and the effective cross-sectional area, dA_{eff} as, $V = v\ \Delta t\ dA_{eff}$. However, the effective cross-sectional area can be written as $dA_{eff} = \cos\phi\ dA$. Thus,

$$\frac{m}{\Delta t} = \rho v \cos\phi\, dA \quad (2)$$

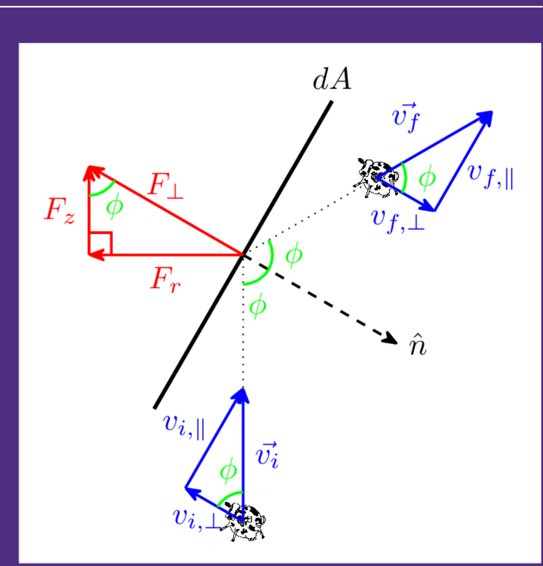


Fig 2: Components of the force and velocities for a single impact.

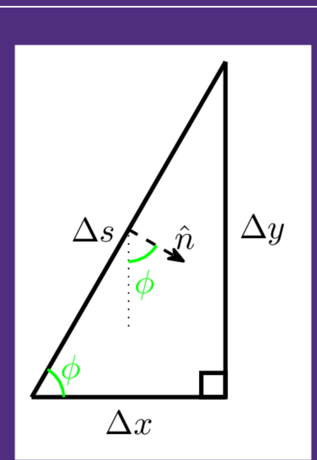


Fig 3: Arc length.

Combining Eq 1 and Eq 2 yield,

$$dF_z = 2\rho v^2 \cos^3\phi\, dA,$$

for the differential element of force on dA due to the collisions. Integrating this over the total surface area of the nose-cone gives the net drag force,

$$F_{drag} = \int dF_z = 2\rho v^2 \underbrace{\iint_A \cos^3\phi\, dA}_{R} \quad (3)$$

The arc length along the surface, Δs, as illustrated in Fig 3, can be written as $\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ while $\cos\phi = \Delta x/\Delta s$. Making use of these in the expression for R yield in polar coordinates yields,

$$R = \iint_A \cos^3\phi\, dA = \iint_A \frac{\cos\phi}{1 + (\Delta y/\Delta x)^2}\, dA = \int_0^{2\pi} d\theta \int_0^{x_0} \frac{x\, dx}{1 + (dy/dx)^2}.$$

The Newton's reduced drag coefficient is then $R_\pi = R/\pi$ or,

$$R_\pi = \int_0^{x_0} \frac{2x\, dx}{1 + (dy/dx)^2}, \quad (4)$$

for any function $y(x)$ that is symmetric about the vertical.

Experiment

Direct measurements of the change in apparent weight of a sphere suspended in a vertical flow field for a range of flow velocities are presented in Fig 5. The change in apparent weight is caused by the air drag on the sphere.

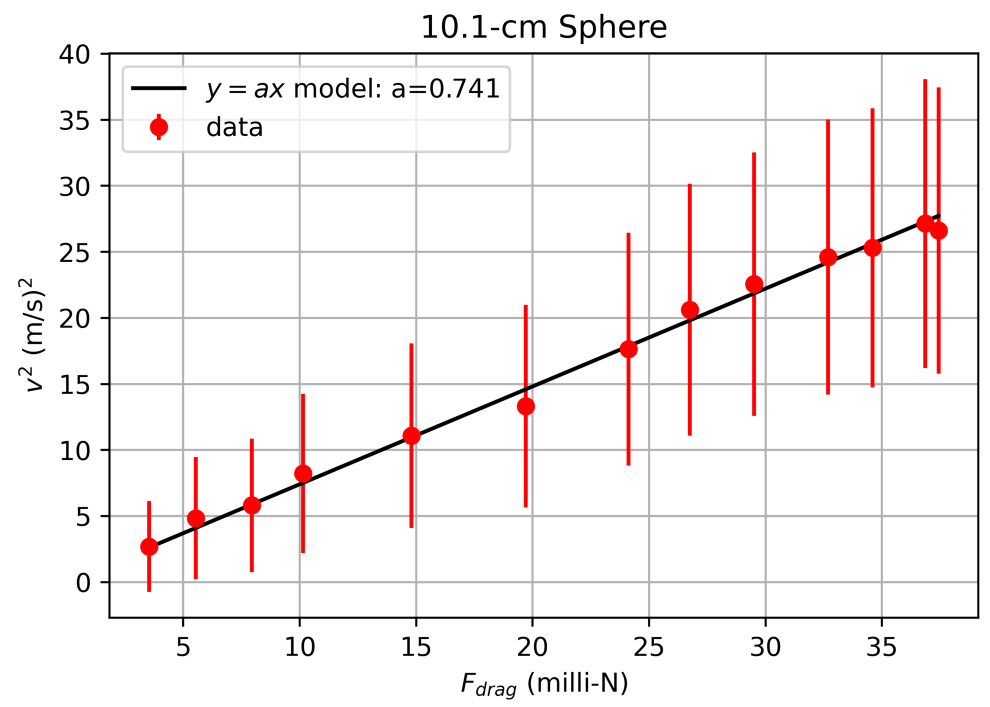


Fig 5: Experimental Data with Analysis [2]

The accepted model[3] for air drag is,

$$F_{drag} = \frac{1}{2} C_D \rho A v^2,$$

where C_D is the dimensionless drag coefficient which depends on the object's shape and flow regime, ρ is the air density, A is the cross-sectional area of the object, and v is the air speed. Plotting v^2 vs F_{drag} results in a linear relationship which can be fitted to the model. Doing so yields the following for the experimental determination of the drag coefficient for a sphere of the range of Reynolds numbers $11 \times 10^3 < Re < 35 \times 10^3$:

$$C_{D,exp} = 0.278 \pm 0.004$$

The accepted value for the drag coefficient for a smooth sphere in this range of Reynolds numbers is $C_D = 0.5$ as illustrated in Fig 6.

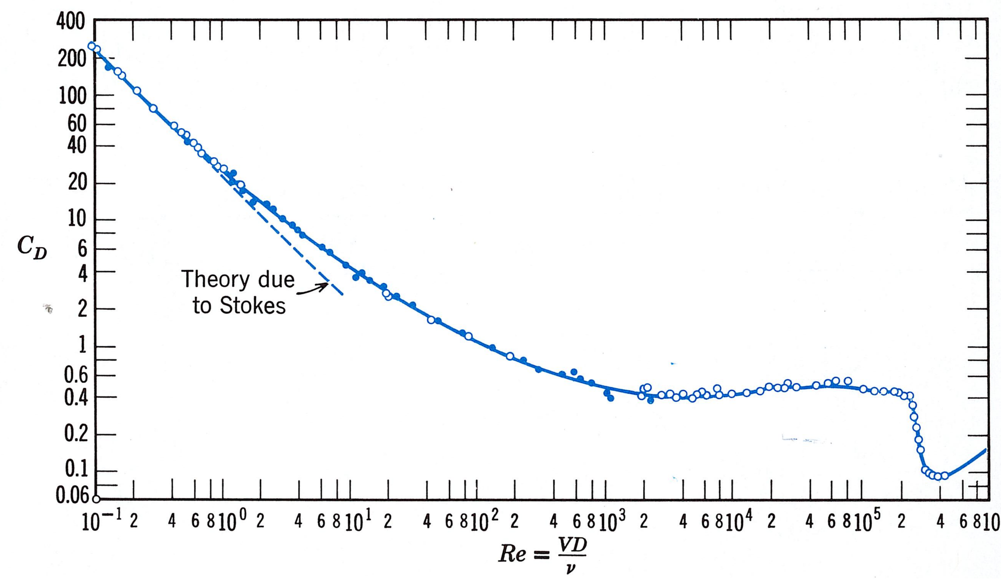


Fig 6: Drag coefficient of a smooth sphere[4], C_D for a smooth sphere vs Reynolds number, $Re = Dv/\nu$ where D is the sphere's diameter, v is the fluid's stream velocity, and ν is the fluid's kinematic viscosity.

Conclusions

To summarize this research was an amazing opportunity to get introduced to numerous subjects such as 3D modeling, programming, and hands-on research experience. Due to the lack of sensitivity of our anemometer we were unable to pursue the research and compare the drag of the different shaped nose-cones. The cross-section of our vertical wind tunnel was also too small to have a uniform flow. Additionally, Newton's method does not account for the real physics of fluid flow around an object.

References

[1] C. Henry Edwards. "Newton's Nose-Cone Problem", *The Mathematical Journal*, Vol 7, Winter 1997, pp. 64-71.
[2] John R. Taylor. *An Introduction to Error Analysis, 2nd Ed.*, University Science Books, 1997.
[3] Randall D. Knight. *Physics for Scientists and Engineers, 4th Ed.*, Pearson, 2017.
[4] Robert W. Fox and Alan T. McDonald. *Introduction to Fluid Mechanics, 4th Ed.*, Wiley & Sons, 1992.