Math 1592 - Calculus II
Sample Test 2

1. Do the following converge (explain)?

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^4+1}, \quad \sum_{n=1}^{\infty} \frac{1}{n^3+1}, \quad \sum_{n=1}^{\infty} \left(\frac{1}{2}+\frac{1}{n}\right)^n, \quad \sum_{n=1}^{\infty} \frac{e^n}{n!}, \quad \sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}, \quad \sum_{n=1}^{\infty} \frac{1}{\ln(n+1)},$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}, \quad \sum_{n=1}^{\infty} \frac{n-1}{n+1}, \quad \sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}, \quad \sum_{n=2}^{\infty} \frac{1}{\ln^2(n)}, \quad \sum_{n=3}^{\infty} \frac{1}{n\ln n}, \quad \sum_{n=1}^{\infty} \frac{1}{2^n+1}.$$

2. Determine whether the following series converge absolutely, conditionally or diverge

$$\sum_{n=1}^{\infty} \frac{(-1)^n(n-1)}{n+1}, \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n(n+1)}}, \quad \sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!},$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n+3^n}, \quad \sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}, \quad \sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1},$$

3. Calculate the n^{th} degree Taylor polynomial with remainder for the following. Expand about the point c

(i) $f(x) = e^x, \ c = 1, \quad n = 2$ (ii) $f(x) = \sin x, \ c = \dfrac{\pi}{2}, \quad n = 4$

(iii) $f(x) = \ln(x+1), \ c = 0, \quad n = 3$ (iv) $f(x) = \dfrac{1}{2-x}, \ c = -2, \quad n = 3.$

4. Determine the interval of convergence of the following

$$\sum_{n=1}^{\infty} \frac{2^n x^n}{\sqrt{n+1}}, \quad \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}, \quad \sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^2},$$

5. Given the following power series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 \cdots$$

find a power series for the following:

(i) $\dfrac{1}{(1-x)^2}$, (ii) $x\tan^{-1} x$, $(iii)\dfrac{3}{4-x}$ centered at $x = 1$

and determine the interval in which they converge.