Classifying Improper Interval Graphs

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Interval Graphs

• Definition: A graph whose vertices may be represented as a set of closed intervals: where an edge occurs iff the corresponding intervals intersect
• (Equivalently: the intersection graph of closed intervals)
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• Definition: A graph whose vertices may be represented as a set of closed intervals: where an edge occurs iff the corresponding intervals intersect

• (Equivalently: the intersection graph of closed intervals)
Nonuniqueness (of representation)
Proper Interval Graph

• Definition: An interval graph which has a representation in which no interval contains another.
Proper Interval Graphs

• Characterized in 1969 by Fred Roberts (Right)
• Characterization: an interval graph is proper iff it has no induced $K_{1,3}$

$K_{1,3}$ (aka claw, 3-star)
$q$-Proper Interval Graph

- Definition: An interval graph in which no interval is contained by more than $q$ others.
\(q\)-Proper Interval Graphs

• Characterized in 1999 A. Proskurowski (left) and J.A. Telle (right)

• Characterization: an interval graph is \(q\)-proper iff it has no induced \(T_{q+1}\). (\(T_{q+1}\) is a \(q+1\) clique and three independent vertices, each one of which is adjacent to every vertex in the clique.)
$p$-Improper Interval Graph

- Definition: An interval graph in which no interval contains more than $p$ others.
Examples

1-improper

2-improper
$p$-Improper Interval Graph

• Studied in 2008 by R. Jamison (right) and J. Beyerl.

• Classified 1-improper interval graphs.

• Partially classified balanced improper interval graphs
Instability

Remove one vertex

6-Improper

1-Improper
Forbidden Subgraphs

• Definition: A graph $H$ is said to be forbidden if $H$ cannot be an induced subgraph of $G$

• Such an $H$ is said to be minimal if every proper subgraph of $H$ is not forbidden
Examples of Forbidden Subgraphs

(For the class of 1-improper interval graphs)
Local Components

- Given a basepoint, exterior local components are in the same place in every representation.
A local component is considered balanced if it does not contribute to the impropriety when it is represented as a side component.

The side component necessarily contributes to the impropriety.
MFISGs with balanced side components

- Exterior local components are $P_2$
- Non-exterior local components are $K_n$

These vertices contribute to the impropriety; structure does not matter.

The $K_n$ here is as large as it needs to be to block the $K_n$ on the inside.
MFISGs with an unbalanced side component

- Exterior local components are $P_2$
- Non-exterior local components are $K_n$

These vertices contribute to the imprority; structure does not matter as long as the potential $K_n$ on the side is the same.
Applications

• Counting minimal forbidden interval subgraphs
• Comparing minimal forbidden interval subgraphs and critical graphs
Thank you!
Eratta

- It is relevant to note that after the talk we decided not to call balanced/unbalanced side components as such and to instead create the new terminology unconfined/confined.