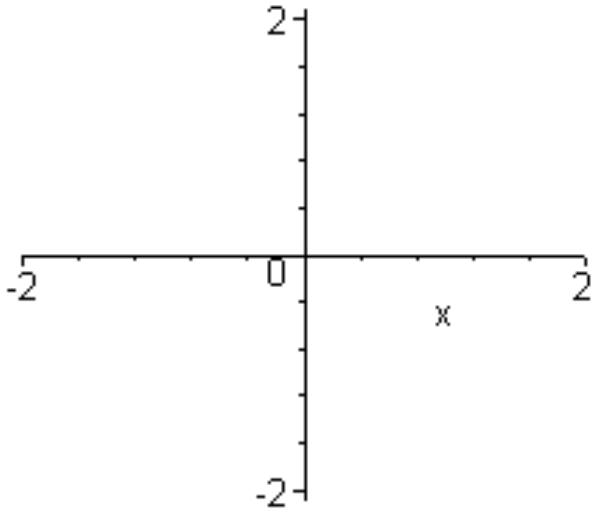
Math 1591 - Calculus 1 Sample Test 1 Name:

1. Calculate $\lim_{x \to 1} \frac{x^3 - x^2}{x - 1}$ using the techniques of graphically, numerically and analytically.

   (i) Graphically. Sketch the function $f(x) = \frac{x^3 - x^2}{x - 1}$ near $x = 1$ and determine the limit.



   (ii) Numerically. Fill the following table where $f(x) = \frac{x^3 - x^2}{x - 1}$ and determine the limit

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$						

   (ii) Analytically. Determine the limit analytically.

2. Calculate the following limits analytically

   (i) $\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2}$ (ii) $\lim_{x \to 0} \frac{\tan 4x}{\tan 2x}$ (iii) $\lim_{x \to \infty} \frac{3x^2 + 4}{x^2 + 2x + 1}$

3. Calculate the first derivatives (either $f'(x)$ or y') of the following (the short way). Do not simplify your answer.

(i) $f(x) = x\sqrt{x} + \dfrac{1}{x\sqrt{x}}$ (ii) $y = \dfrac{4e^x}{x^2+1}$

(iii) $y = x^2 \tan x$ (iv) $f(x) = \sin\left(\sqrt{4x^2+x}\right)$

4. Calculate the derivative of $f(x) = \dfrac{1}{x}$ from the definition (*i.e.* the long way).

5. Find the equation of the tangent line to $y = x^4 - 2x^3 + 2x^2$ at the point (1,1).

6. Consider the function

$$f(x) = \begin{cases} x^2, & x \le 0 \\ x^3, & x > 0 \end{cases}$$

Is $f(x)$ continuous at $x = 0$? Is $f(x)$ differentiable at $x = 0$? What is $f'(0)$ if it exists?

7. Using the δ–ε definition, what is meant by

$$\lim_{x \to 2} 2x - 1 = 3$$

8. Prove the following:

$$\frac{d}{dx}(cf) = c\frac{d}{dx}(f), \qquad \frac{d}{dx}(f \pm g) = \frac{df}{dx} \pm \frac{dg}{dx}$$

$$\frac{d}{dx}(fg) = \frac{df}{dx}g + f\frac{dg}{dx}, \qquad \frac{d}{dx}\left(\frac{f}{g}\right) = \frac{\frac{df}{dx}g - f\frac{dg}{dx}}{g^2}$$