Math 1591 - Sample Problems for the Final

1. Limits

$$\lim_{x\to 1}\frac{x^2-1}{x-1},\quad \lim_{x\to 2}\frac{\sqrt{x}-4}{x-2},\quad \lim_{x\to 4}\frac{\sqrt{x}-2}{x-4},\quad \lim_{x\to\infty}\frac{x^2-1}{x^2+1},\quad \lim_{x\to -1}\frac{x^2-x-2}{x^2+3x+2},$$

$$\lim_{x\to -2}\frac{x^2+4x+4}{x^2-x-6},\quad \lim_{x\to 4}\frac{x^2-3x-4}{x^2-16},\quad \lim_{x\to 2}\frac{x^2-3x+2}{x^4-16},\quad \lim_{x\to 9}\frac{3-\sqrt{x}}{9+8x-x^2},$$

$$\lim_{x\to 2^-}\frac{10}{x^2-4},\quad \lim_{x\to\infty}\frac{2x^2-3}{-x^2+4x},\quad \lim_{x\to\infty}\frac{2x^2-3}{1+4x},\quad \lim_{x\to 0}\frac{\sin 2x}{x},\quad \lim_{x\to 0}\frac{\sin 4x}{x},$$

$$\lim_{x\to 0}\frac{\tan 3x}{5x},\quad \lim_{x\to 0}\frac{\sin 3x}{\cos 5x},\quad \lim_{x\to 0}\frac{\tan x}{3x},\quad \lim_{\theta\to 0}\frac{\cos 2\theta-1}{\sin 2\theta},\quad \lim_{\theta\to 0}\frac{\tan^2(2\theta)}{\theta\sin\theta}$$

2. Differentiation

$$\sqrt{x^2+1},\quad \frac{x}{x+1},\quad x\sin x,\quad 4(x^3+1)(2\cos x-3),\quad \frac{\sqrt{x}-1}{\sqrt{x}+1},\quad \tan^4 x,$$

$$(1-\sqrt{x})(1-x),\quad x(x+1)(x^2+1),\quad \sqrt{x}\tan x-\tfrac{2}{3}x\sqrt{x},\quad \frac{\sqrt{x}}{\sqrt{x}+1},$$

$$(3x+1)^{5/2}(2x-1)^{3/2},\quad \frac{\cos 2x-1}{\sin 2x},\quad \left(\frac{x}{x+1}\right)\cos x,\quad \tan^3(3x-1),\quad \sqrt{1+\sin x},$$

$$\cos^2 x+\sin(x^2+1),\quad x^2\cos x,\quad \cos^3(2x),\quad \frac{x^3}{\sin x},\quad (x^2-3x+1)^5$$

$$x\sin^{-1}x,\quad \frac{\tan^{-1}x}{\sqrt{x}},\quad x^3e^x,\quad \frac{\ln x^3}{x},\quad (e^x+1)^5,\quad e^{\sin x},\quad x^x,\quad 2^{x-1}$$

Second derivatives

$$\left(x^2+2\sqrt{x}+3\right)'',\quad \left(\frac{x+1}{x^2+1}\right)'',\quad \left(x\sqrt{1-x}\right)'',$$

Implicit Differentiation, find y'

$$x^2-xy+y^4=0,\quad y^2+xy=x^2+1,\quad xy=\frac{1}{x+y},\quad x\sin x+\cos y=2,$$

$$x\sin y=y\sin x$$

3. Tangents

Find the tangent line to the given curves at the points indicated

$y = x^3 + 2x$ at (1,3) $\quad$ $y = x^3 + x^2 + x$ at (1,3) $\quad$ $y = \dfrac{x}{x+1}$ at (1,1/2)

$x^3 + y^3 = 2xy$ at (1,1) $\quad$ $y = x^4 - 6x$ at (1,-5) $\quad$ $x^3 + 4y^3 - 6xy = 0$ at (2,1)

$y = x^3 + x \sin x$ at $x = \frac{\pi}{2}$ $\quad$ $x^2 y^2 - 2xy + x = 2$ at (2,1)

$y = \dfrac{4}{\sqrt{x}+1}$ at $\left(1, \frac{4}{\sqrt{2}}\right)$ $\quad$ $2y^2(x-1)^2 + xy = x$ at $x = 1$

4. Absolute Minimum and Maximum

Find the absolute minimum and maximum for the given functions on the indicated intervals

$x^3 - 12x + 1$ in $[-3,5]$, $x^4 - 8x^2 + 5$ in $[-3,2]$, $4x^3 - 15x^2 + 12x + 7$ in $[0,3]$

5. Sketching

For the following curves, show critical points, points of inflections, intervals of increasing and decreasing, concavity up or down.

$y = x^3 - 6x^2 + 9x$, $\quad y = x^4 - 8x^2 + 5$, $\quad y = x^4 - 4x^3 + 3$,

6. Integrals

Evaluate the following integrals

$\int \cos 2x \, dx$, $\quad \int_0^3 \left(x^2 - 1\right) dx$, $\quad \int_0^1 \dfrac{x}{\left(x^2-1\right)^2} dx$, $\quad \int_0^1 (x+1)^2 \, dx$, $\quad \int_0^{\pi/4} \sin 2x \, dx$, $\quad$,

$\int x^3 \sqrt{2 + x^4} \, dx$, $\int_0^1 x^2 \sqrt{x+1} \, dx$, $\quad \int 2x + \sec x \tan x \, dx$, $\quad \int_0^1 \dfrac{x \, dx}{\left(x^2+3\right)^3}$, $\int \dfrac{\ln x}{x} dx$

$$\int \frac{x-1}{\sqrt{1-x^2}}\,dx\,,\quad \int \frac{\cos x}{1+\sin^2 x}\,dx\,,\quad \int_1^2 2^{x-1}\,dx\,,\quad \int_0^1 \frac{e^x}{\left(e^{2x}-1\right)^2}\,dx\,,\quad \int_1^3 \frac{dx}{5-2x+x^2}\,,$$

$$\int x^5\sqrt{1+x^2}\,dx\,,\quad \int_0^1 x^2\sqrt{9-x^3}\,dx\,,\quad \int_0^{\pi/4} \sin^2 x\cos x\,dx\,,\quad \int \frac{dx}{\sqrt{2x-x^2}}\,,\quad \int_{\pi/3}^{\pi/4} \sin t\,dt\,,$$

$$\int_0^{\pi/4} \cos^3 x\sin x\,dx\,,\quad \int \sin(2x+3)\,dx\,,\quad \int \sin x - 2\cos x\,dx\,,\quad \int_1^3 \left(\frac{1}{t^2}-\frac{1}{t^4}\right)dt\,.$$

7. Areas

Find the area enclosed by the following curves

$y = x, y = \sin x$ on $[0,\pi]$, $\quad x = y^2,\ y = 2 - x$,

8. Volumes of Revolution

Find the volume of revolution where the region bound by the following curves is revolved about both x and y axes. Use both the method of disc's and shells.

$y = \sqrt{x}, y = 1,\ x = 0\,;\quad y = x,\ y = 2 - x,\ y = 0.$

9. Continuity and Differentiability

Determine whether the following functions are continuous and differentiable on R

$$f(x) = \begin{cases} x & x \geq 0 \\ x^2 & x < 0 \end{cases} \qquad\qquad f(x) = \begin{cases} 1 - x^2 & x \leq 1 \\ x^2 - 4x + 3 & x > 1 \end{cases}$$

10. Min/Max Problems

(i) The sum of two positive numbers is 20. Find the numbers such that (i) their product is a max. (ii) the sum of there squares is a min.

(ii) Find the dimensions of the rectangle of max. area that can be inscribed in the portion of the parabola $12x = y^2$ intercepted by $x = 1$.

(iii) A cylindrical container with an open top holds 1000 cm^3. Find the dimensions of the container such that the surface area is a minimum.

11. Related Rate Problems

(i) Find the rate of change of the distance between the origin and a moving point on the graph $y = sin\, x$ if $\frac{dx}{dt} = 2$ cm/s.

(ii) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft. from the wall?

(iii) A water tank has the shape of an inverted circular cylinder with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m3/min, find the rate at which the water level is rising when the water is 3m deep.

12. Center of Mass

Find the center of mass of the following:

(i) $y = x,\ y = 2 - x,\ y = 0$ (ii) $y = \sqrt{x},\ y = -x,\ x = 4.$