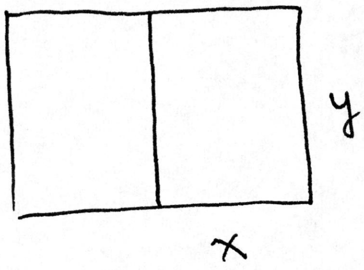
Math 1591 – Calc I   ST 3 – Sol

#1



the area $A = 2xy$

fence $P = 4x + 3y = 240$

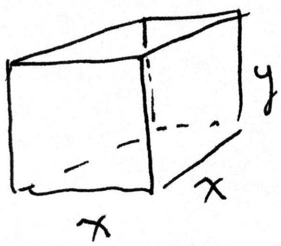
so $y = \dfrac{240 - 4x}{3} \Rightarrow A = 2x\left(\dfrac{240 - 4x}{3}\right)$

$= \dfrac{480x - 8x^2}{3}$

$A' = \dfrac{480 - 16x}{3}$ and $A' = 0$ when $x = \dfrac{480}{16} = 30$

$A'' = -\dfrac{16}{3} < 0$ so a maximum. Dimension's of each pen $30' \times 40'$

#2



$V = x^2 y = 64$ and $A = 2x^2 + 4xy$

Now $y = \dfrac{64}{x^2}$ so $A = 2x^2 + \dfrac{4x \cdot 64}{x^2} = 2x^2 + \dfrac{256}{x}$

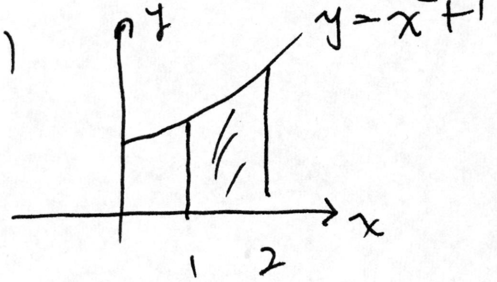
$A' = 4x - \dfrac{256}{x^2}$ $A' = 0$ when $x^3 = \dfrac{256}{4} = 64 \Rightarrow x = 4$

$A'' = 4 + \dfrac{512}{x^3} > 0$ when $x = 4$ so a min.

when $x = 4$, $y = 64/4^2 = 4$ so dimensions $4' \times 4' \times 4'$

#3 Areas

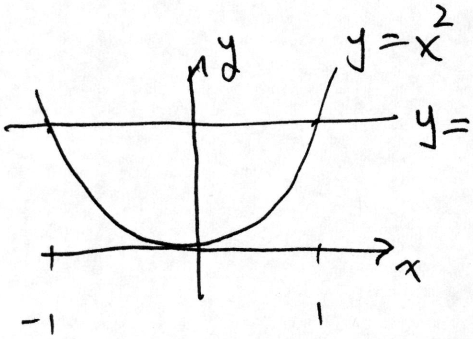
(i)



$$A = \int_1^2 (x^2+1)\,dx = \frac{x^3}{3} + x \Big|_1^2$$

$$= \left(\frac{8}{3}+2\right) - \left(\frac{1}{3}+1\right) = 10/3$$

(ii)

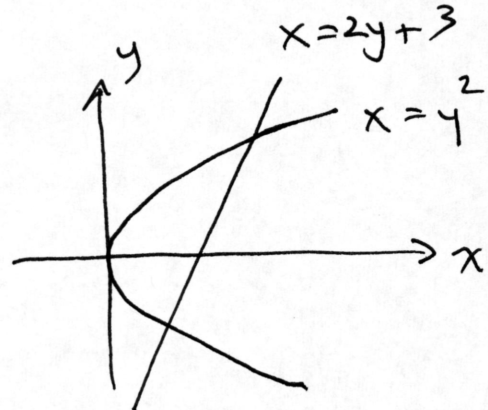


Intersection pt $x^2 = 1 \Rightarrow x = \pm 1$

$$A = \int_{-1}^{1} (1-x^2)\,dx = x - \frac{x^3}{3} \Big|_{-1}^{1} = \left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)$$

$$= 4/3$$

(iii)



Intersection pts

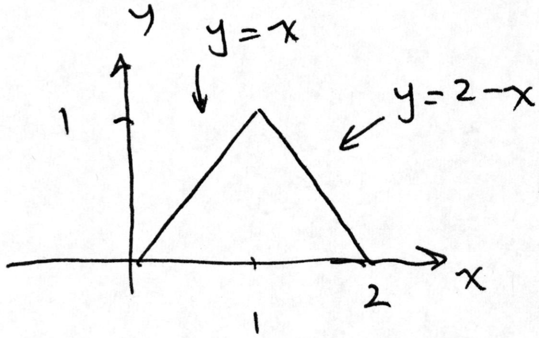
$y^2 = 2y+3$ or $y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0 \quad y = -1, 3$

$$A = \int_{-1}^{3} (2y+3-y^2)\,dy = y^2 + 3y - y^3/3 \Big|_{-1}^{3} = (9+9-9) - \left(1-3+\frac{1}{3}\right)$$

$$= 32/3$$

(iv)



$$A = \int_0^1 (2-y-y)\,dy = 2y - y^2 \Big|_0^1$$

$$= (2-1) - (0-0)$$

$$= 2 - 1 = 1$$

4(i) $-\dfrac{d}{dx}\displaystyle\int_1^x \sin t^2\,dt = -\sin x^2$

(ii) $\dfrac{d}{dx}\displaystyle\int_x^{x^2} \dfrac{dt}{\sqrt{1-t^2}} = -\dfrac{d}{dx}\displaystyle\int_a^{x} \dfrac{dt}{\sqrt{1-t^2}} + \dfrac{d}{dx}\displaystyle\int_0^{x^2} \dfrac{dt}{\sqrt{1-t^2}}$

$$= -\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^4}}\cdot 2x$$

5 (i) $\displaystyle\int \frac{x-1}{\sqrt{x}}\,dx = \int x^{1/2} - x^{-1/2}\,dx = \frac{2x^{3/2}}{3} - 2x^{1/2} + c$

(ii) $\displaystyle\int_0^{1/2} \frac{dx}{4x^2+1}$ let $u = 2x$ $\quad x = 0\ u = 0$

$du = 2dx \quad x = 1/2\ u = 1$

so $\displaystyle\frac{1}{2}\int_0^1 \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}u\Big|_0^1 = \frac{1}{2}\left(\tan^{-1}1 - \tan^{-1}0\right) = \frac{1}{2}\pi/4 = \frac{\pi}{8}$

(iii) $\displaystyle\int x\sqrt{x-1}\,dx$ let $u = x-1$ so $dx = du$ $\quad x = u+1$

$\Rightarrow \displaystyle\int (u+1)\sqrt{u}\,du = \int u^{3/2} + u^{1/2}\,du = \frac{2u^{5/2}}{5} + \frac{2u^{3/2}}{3} + c$

$$= \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + c$$

(iv) $\displaystyle\int_0^{\pi/4} \sec^2 x\,dx = \tan x\Big|_0^{\pi/4} = \tan\frac{\pi}{4} - \tan 0 = 1 - 0 = 1$

(v) $\displaystyle\int_0^4 \frac{x\,dx}{\sqrt{9+x^2}}$ $\quad u = 9+x^2 \quad x=0 \ u=9$

$du = 2x\,dx \quad x=4 \ u=25$

so $\displaystyle\frac{1}{2}\int_9^{25} \frac{du}{\sqrt{u}} = \frac{1}{2}\cdot 2u^{1/2}\Big|_9^{25} = \sqrt{25}-\sqrt{9} = 5-3 = 4$

(vi) $\displaystyle\int_{\pi/12}^{\pi/6} \sin 2x \cos 2x\,dx$ $\quad$ let $u = \sin 2x \quad x=\pi/12 \ u = 1/2$

$du = 2\cos 2x\,dx \quad x=\pi/6 \ u=\sqrt{3}/2$

so $\displaystyle\int_{1/2}^{\sqrt{3}/2} \frac{1}{2}u\,du = \frac{u^2}{4}\Big|_{1/2}^{\sqrt{3}/2} = \frac{1}{4}\left(\frac{3}{4}-\frac{1}{4}\right) = \frac{1}{8}$

(vii) $\displaystyle\int \frac{dx}{4x^2+4x+2} = \int \frac{dx}{(2x+1)^2+1}$ $\quad$ let $u = 2x+1$

$du = 2dx$

so $\displaystyle\frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}u + c = \frac{1}{2}\tan^{-1}(2x+1)+c$

(viii) $\displaystyle\int x(1+\sec^2 x^2)\,dx = \int x\,dx + \int x\sec^2 x^2\,dx$

let $u = x^2$

$du = 2x\,dx$

$\displaystyle\int x\,dx + \frac{1}{2}\int \sec^2 u\,du$

$\displaystyle = \frac{x^2}{2} + \frac{1}{2}\tan u + c = \frac{x^2}{2} + \frac{1}{2}\tan x^2 + c$

(ix) $\displaystyle\int \frac{x+1}{x^2+1}\,dx = \int \frac{x}{x^2+1}\,dx + \int \frac{1}{1+x^2}\,dx$

let $u = x^2+1$ so $du = 2x\,dx$

$$= \frac{1}{2}\int \frac{du}{u} + \int \frac{1}{1+x^2}\,dx = \frac{1}{2}\ln|u| + \tan^{-1}x + c$$

$$= \frac{1}{2}\ln|x^2+1| + \tan^{-1}x + c$$

(x) $\displaystyle\int_e^{e^2} \frac{dx}{x\ln x}$ let $u = \ln x$, $du = \frac{dx}{x}$ $\quad x = e\ u = 1$, $x = e^2\ u = 2$

$$\int_1^2 \frac{du}{u} = \ln|u|\Big|_1^2 = \ln 2$$

(xi) $\displaystyle\int \frac{dx}{1+\sqrt{x}}$ let $u = 1+\sqrt{x}$ so $du = \frac{dx}{2\sqrt{x}}$ $\quad \displaystyle\int \frac{2(u-1)\,du}{u}$

$$= 2\int 1 - \frac{1}{u}\,du = 2(u - \ln|u|) + c$$

$$= 2(1+\sqrt{x} - \ln|1+\sqrt{x}|) + c$$

(xii) $\displaystyle\int \frac{e^{2x}}{1+e^x}\,dx$ let $u = e^x+1$, $du = e^x\,dx$

$$\int \frac{(u-1)\,du}{u} = \int 1 - \frac{1}{u}\,du = u - \ln|u| + c$$

$$= e^x + 1 - \ln|e^x+1| + c$$

don't need the 1
c will absorb it

(xiii) $\displaystyle\int_0^1 x e^{-x^2}\,dx$ let $u=-x^2$ $\quad x=0\ \ u=0$

$du=-2x\,dx \quad x=1\ \ u=-1$

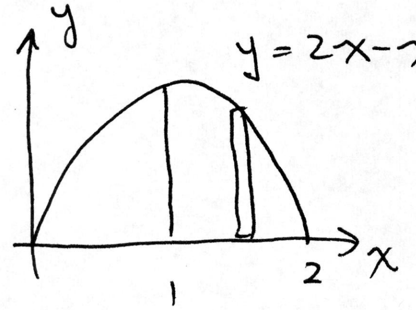
$$-\frac{1}{2}\int_0^{-1} e^u\,du = -\frac{1}{2}e^u\Big|_0^{-1} = -\frac{1}{2}\left(e^{-1}-e^0\right) = \frac{1}{2}\left(1-e^{-1}\right)$$

(xiv) $\displaystyle\int_{-1}^0 \frac{dx}{x^2+2x+2} = \int_{-1}^0 \frac{dx}{(x+1)^2+1}$ let $u=x+1$ $\quad x=0\ u=1$

$du=dx \quad x=-1\ u=0$

$$\int_0^1 \frac{du}{1+u^2} = \tan^{-1}u\Big|_0^1 = \tan^{-1}1-\tan^{-1}0 = \pi/4$$

6 ii)



$\Delta x = \frac{1}{n}, \quad x_i = 1+\frac{i}{n}$

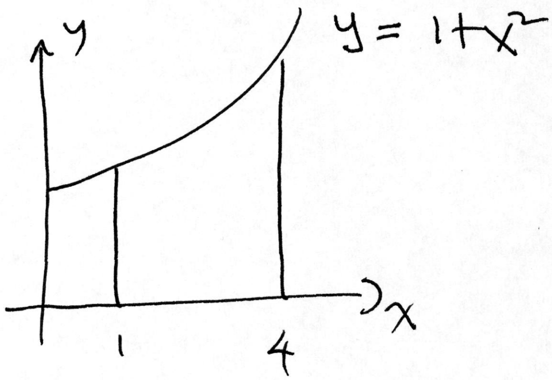
$h_i = 2\left(1+\frac{i}{n}\right)-\left(1+\frac{i}{n}\right)^2$

$$A = \lim_{n\to\infty}\sum_{i=1}^n \left[2\left(1+\frac{i}{n}\right)-\left(1+\frac{i}{n}\right)^2\right]\frac{1}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^n \frac{1}{n}-\frac{i^2}{n^3} = \lim_{n\to\infty} \frac{n}{n} - \frac{n(n+1)(2n+1)}{6n^3}$$

$$= 1-\frac{1}{3} = 2/3$$

6 iii)



$\Delta x = \frac{4-1}{n} = \frac{3}{n} \qquad x_i = 1 + \frac{3i}{n}$

$h_i = 1 + \left(1 + \frac{3i}{n}\right)^2$

$A = \lim_{n \to \infty} \sum_{i=1}^{n} \left[1 + \left(1 + \frac{3i}{n}\right)^2\right] \frac{3}{n}$

$= \lim_{n \to \infty} \sum_{i=1}^{n} \frac{6}{n} + \frac{18i}{n^2} + \frac{27i^2}{n^3}$

$= \lim_{n \to \infty} \frac{6n}{n} + \frac{18}{n^2} \frac{n(n+1)}{2} + \frac{27}{n^3} \frac{n(n+1)(2n+1)}{6}$

$= 6 + \frac{18}{2} + \frac{27 \cdot 2}{6}$

$= 6 + 9 + 9 = 24$