Calc 1 — Some Notes

Binomial Theorem

$$(x+h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{3.2}x^{n-3}h^3 + \cdots + nxh^{n-1} + h^n$$

Some limits

$$\lim_{h\to 0}\frac{\sin h}{h} = 1, \quad \lim_{h\to 0}\frac{1-\cos h}{h} = 0, \quad \lim_{h\to 0}\frac{e^h - 1}{h} = 1$$

Some Trig Identities

$$\begin{aligned} \sin(A+B) &= \sin A\ \cos B + \cos A\ \sin B, \\ \cos(A+B) &= \cos A\ \cos B - \sin A\ \sin B \end{aligned}$$

$$\lim_{h\to 0}\frac{\sin h}{h} = 1, \quad \lim_{h\to 0}\frac{1-\cos h}{h} = 0, \quad \lim_{h\to 0}\frac{e^h - 1}{h} = 1$$

Proofs

Sum

$$\begin{aligned} (f+g)' &= \lim_{h\to 0}\frac{(f(x+h)+g(x+h)) - (f(x)+g(x))}{h} \\ &= \lim_{h\to 0}\frac{(f(x+h)-f(x)) + (g(x+h)-g(x))}{h} \\ &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\ &= f' + g' \end{aligned}$$

Difference

$$\begin{aligned} (f-g)' &= \lim_{h\to 0}\frac{(f(x+h)-g(x+h)) - (f(x)-g(x))}{h} \\ &= \lim_{h\to 0}\frac{(f(x+h)-f(x)) - (g(x+h)-g(x))}{h} \\ &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} - \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\ &= f' - g' \end{aligned}$$

Product

$$
\begin{aligned}
(fg)' &= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{(f(x+h-f(x))\,g(x+h) + f(x)\,(g(x+h) - g(x))}{h} \\
&= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \cdot \lim_{h\to 0} g(x+h) + f(x) \cdot \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \\
&= f'g + fg'
\end{aligned}
$$

Quotient

$$
\begin{aligned}
\left(\frac{f}{g}\right)' &= \lim_{h\to 0} \frac{\dfrac{f(x+h)}{g(x+h)} - \dfrac{f(x)}{g(x)}}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x) - g(x+h)f(x)}{g(x+h)g(x)h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - g(x+h)f(x)}{g(x+h)g(x)h} \\
&= \lim_{h\to 0} \frac{(f(x+h) - f(x))\,g(x) - f(x)\,(g(x+h) - g(x))}{g(x+h)g(x)h} \\
&= \lim_{h\to 0} \frac{\dfrac{f(x+h) - f(x)}{h} \cdot g(x) - f(x) \cdot \dfrac{g(x+h) - g(x)}{h}}{g(x+h)g(x)} \\
&= \frac{f'g - fg'}{g^2}
\end{aligned}
$$

Some Standard Formula

Areas		Volumes	
circle	$A = \pi r^2$	cylinder	$V = \pi r^2 h$
rectangle	$A = lw$	box	$V = lwh$
sphere	$A = 4\pi r^2$	sphere	$V = \frac{4}{3}\pi r^3$
triangle	$A = \frac{1}{2}bh$	cone	$V = \frac{1}{3}\pi r^2 h$