

Math 4315 - PDEs Home Work 4

Due: Fri. Oct. 24, 2014

1. Transform the following PDEs to standard form. In the case of hyperbolic, transform to both regular and modified forms.

- (i) $u_{xx} + 2u_{xy} + u_{yy} = 0,$
- (ii) $y^2 u_{xx} + 2xy u_{xy} + x^2 u_{yy} - 2xu_x = 0,$
- (iii) $2u_{xx} - 3u_{xy} + u_{yy} = u_x + u_y,$
- (iv) $x^2 u_{xx} - 3xy u_{xy} + 2y^2 u_{yy} = 0,$
- (v) $4u_{xx} + 4u_{xy} + 5u_{yy} = 1,$
- (iv) $u_{xx} + (1 + y^2)^2 u_{yy} = 0.$

Bonus

The PDE

$$x^2 u_{xx} - 4xy u_{xy} + 4y^2 u_{yy} + xu_x = 0$$

is parabolic. Introducing new coordinates

$$r = x^2 y, \quad s = y,$$

reduces the PDE to

$$u_{ss} - \frac{r}{s^2} u_r = 0.$$

In fact, any choice of

$$r = f(x^2 y), \quad s = g(x, y),$$

will transform the original PDE to one that is in parabolic standard form. Can the choice of f and g be made such that we can transform to

$$u_{ss} = u_r?$$