

Math 4315 - PDEs Sample Test 3

1. Determine the Fourier series for

(i)

$$f(x) = \begin{cases} 1 & \text{if } -2 \leq x < 0 \\ x+1 & \text{if } 0 \leq x \leq 2 \end{cases}$$

(ii)

$$f(x) = \begin{cases} -x & \text{if } -1 \leq x < 0 \\ x^2 & \text{if } 0 \leq x \leq 1 \end{cases}$$

2. Solve

$$u_t = u_{xx}, \quad 0 < x < L,$$

subject to the initial condition and boundary conditions

$$(i) \quad u(x, 0) = 5x - x^2, \quad u(0, t) = 0, \quad u(4, t) = 4$$

$$(ii) \quad u(x, 0) = \begin{cases} x^2 + 1 & \text{if } 0 < x < 1, \\ 2(x-2)^2 & \text{if } 1 < x < 2. \end{cases}, \quad u(0, t) = 1, \quad u(2, t) = 0$$

3. Solve Laplace's equation

$$u_{xx} + u_{yy} = 0, \quad 0 < x < L, \quad 0 < y < L,$$

subject to the boundary conditions

$$(i) \quad u(x, 0) = 0, \quad u(0, y) = 0, \quad u(x, 1) = x^2, \quad u(1, y) = 0,$$

$$(ii) \quad u(x, 0) = 0, \quad u(0, y) = 0, \quad u(x, 2) = 0, \quad u(2, y) = 2y - y^2.$$