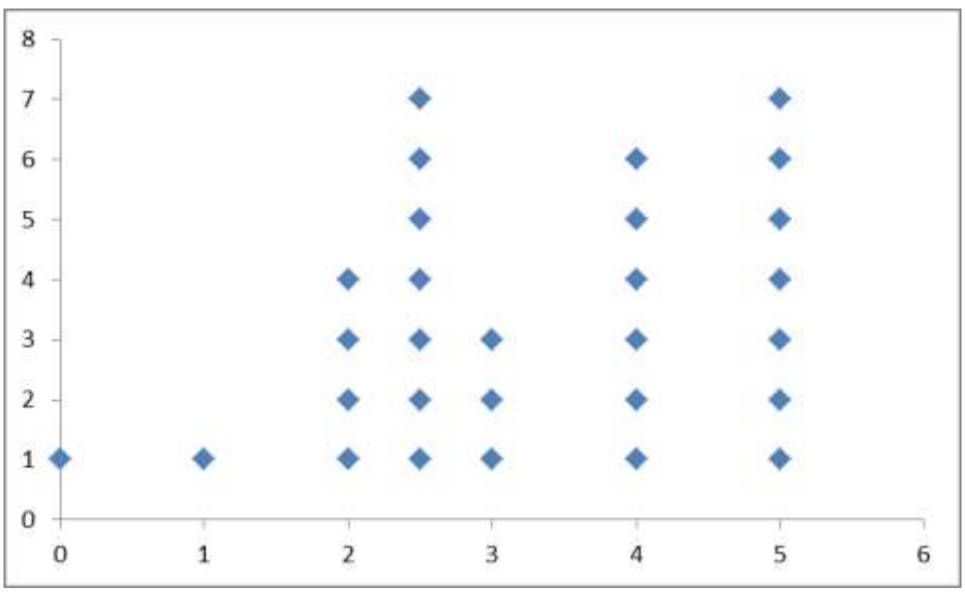
Name__Solutions____ Trigonometry, Test 2

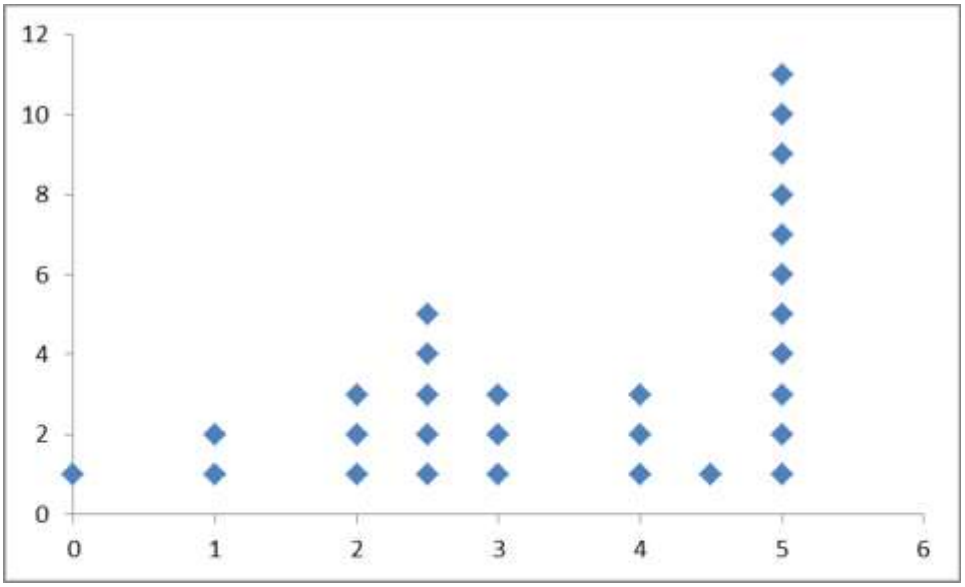
1) Find $\cos\left(\frac{11\pi}{12}\right)$. Show your work. (5 points)

$$\begin{aligned}\cos\left(\frac{11\pi}{12}\right) &= \cos\left(\frac{8\pi}{12}+\frac{3\pi}{12}\right)\\ &= \cos\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)\\ &= \cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)\\ &= \frac{-1}{2}\cdot\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}\\ &= -\frac{1}{2\sqrt{2}}-\frac{\sqrt{3}}{2\sqrt{2}}\\ &= -\frac{1+\sqrt{3}}{2\sqrt{2}}\end{aligned}$$



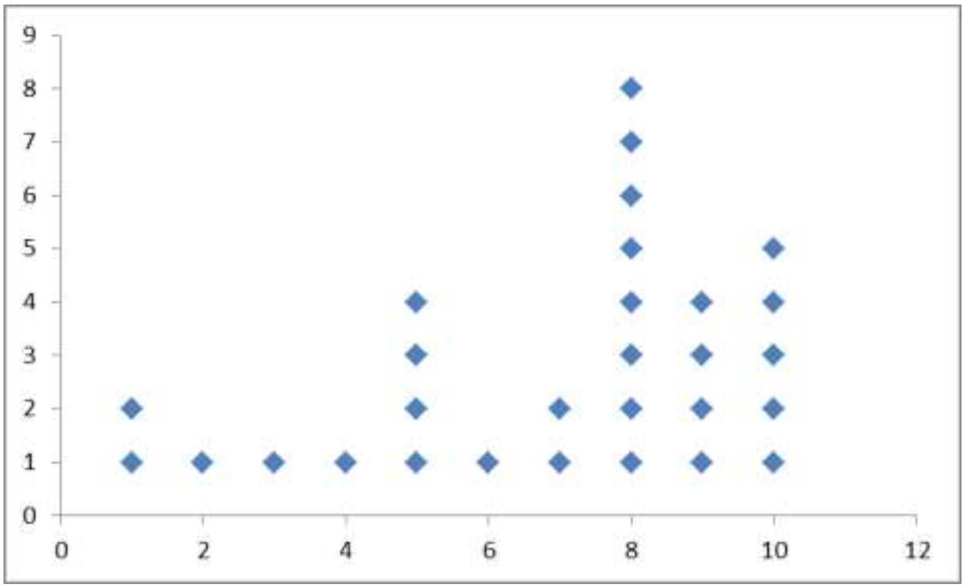
2) Find $\sin\left(\frac{3\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right)$. Show your work. (5 points)

$$\sin\left(\frac{3\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right) = \sin\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) = \sin\left(\frac{2\pi}{2}\right) = \sin(\pi) = 0$$



3) Simplify $(2\sin(x) + \cos(x)) \cdot (\sin(x) + 2\cos(x)) - 4\sin(x)\cos(x)$ as much as possible. Circle your answer. (10 points)

$$(2\sin(x) + \cos(x)) \cdot (\sin(x) + 2\cos(x)) - 4\sin(x)\cos(x)$$
$$= 2\sin^2(x) + \cos(x)\sin(x) + 4\sin(x)\cos(x) + 2\cos^2(x) - 4\sin(x)\cos(x)$$
$$= 2\sin^2(x) + \cos(x)\sin(x) + 2\cos^2(x)$$
$$= 2(\sin^2(x) + \cos^2(x)) + \cos(x)\sin(x)$$
$$= 2 + \cos(x)\sin(x)$$



4) Verify the following identity. (20 points)

$$\frac{\csc(x) - \sin(x)}{\sin(x)} = \cot^2(x)$$

$$\frac{\csc(x) - \sin(x)}{\sin(x)}$$

$$= \frac{\frac{1}{\sin(x)} - \sin(x)}{\sin(x)}$$

$$= \frac{\frac{1}{\sin(x)} - \frac{\sin^2(x)}{\sin(x)}}{\sin(x)}$$

$$= \frac{\frac{1 - \sin^2(x)}{\sin(x)}}{\sin(x)}$$

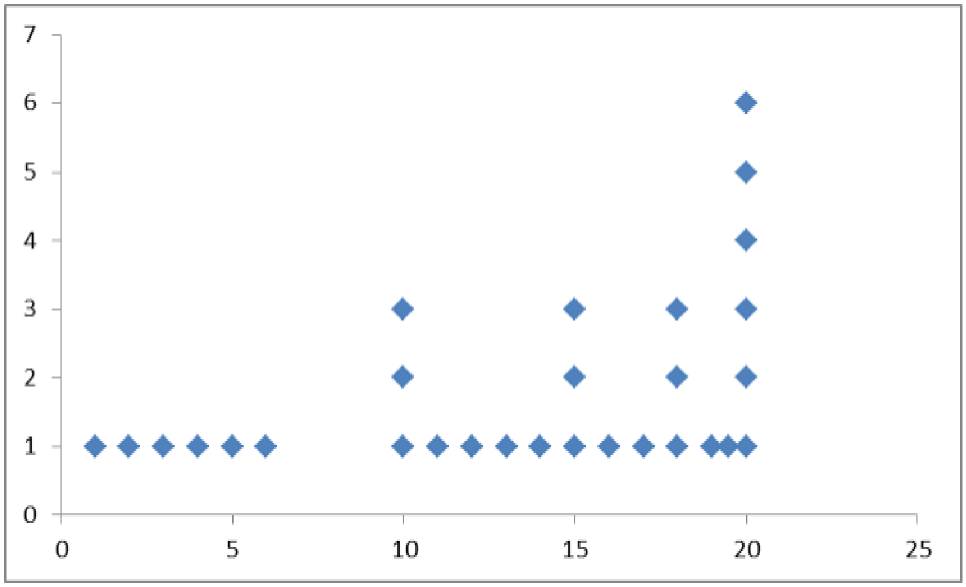
$$= \frac{\frac{\cos^2(x)}{\sin(x)}}{\sin(x)}$$

$$= \frac{\frac{\cos^2(x)}{\sin(x)}}{\frac{\sin(x)}{1}}$$

$$= \frac{\cos^2(x)}{\sin(x)} \cdot \frac{1}{\sin(x)}$$

$$= \frac{\cos^2(x)}{\sin^2(x)}$$

$$= \cot^2(x)$$



5) Write as a single fraction and simplify if possible. Circle your answer. (20 points)

$$\frac{\tan(x)}{\sin(x)\cos(x)} - \frac{\sin^2(x)}{\cos^2(x)}$$

$$\frac{\tan(x)}{\sin(x)\cos(x)} - \frac{\sin^2(x)}{\cos^2(x)}$$

$$= \frac{\tan(x)\cos(x)}{\sin(x)\cos^2(x)} - \frac{\sin^3(x)}{\sin(x)\cos^2(x)}$$

$$= \frac{\tan(x)\cos(x) - \sin^3(x)}{\sin(x)\cos^2(x)}$$

$$= \frac{\frac{\sin(x)}{\cos(x)} \cdot \cos(x) - \sin^3(x)}{\sin(x)\cos^2(x)}$$

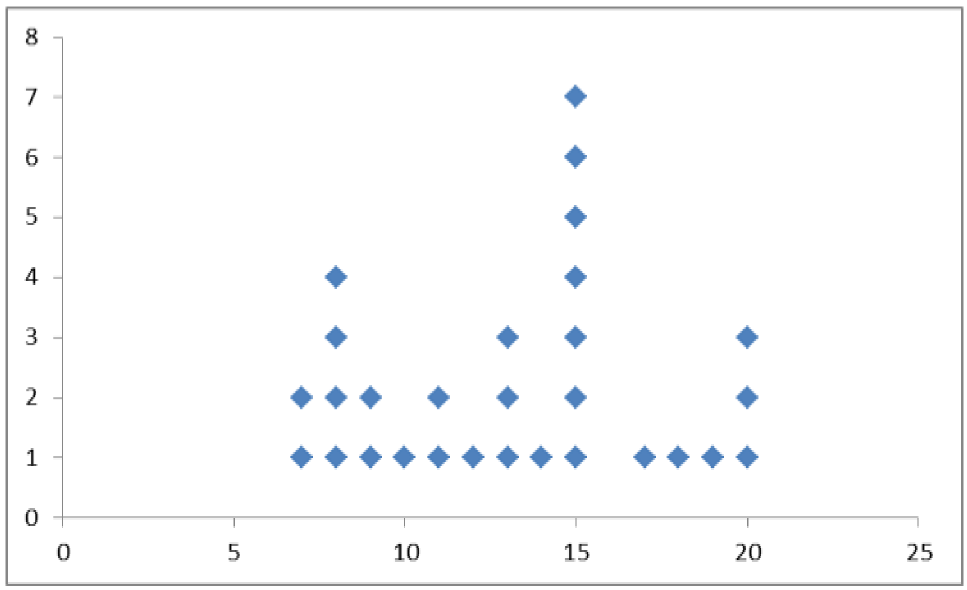
$$= \frac{\sin(x) - \sin^3(x)}{\sin(x)\cos^2(x)}$$

$$= \frac{\sin(x)\,(1 - \sin^2(x))}{\sin(x)\cos^2(x)}$$

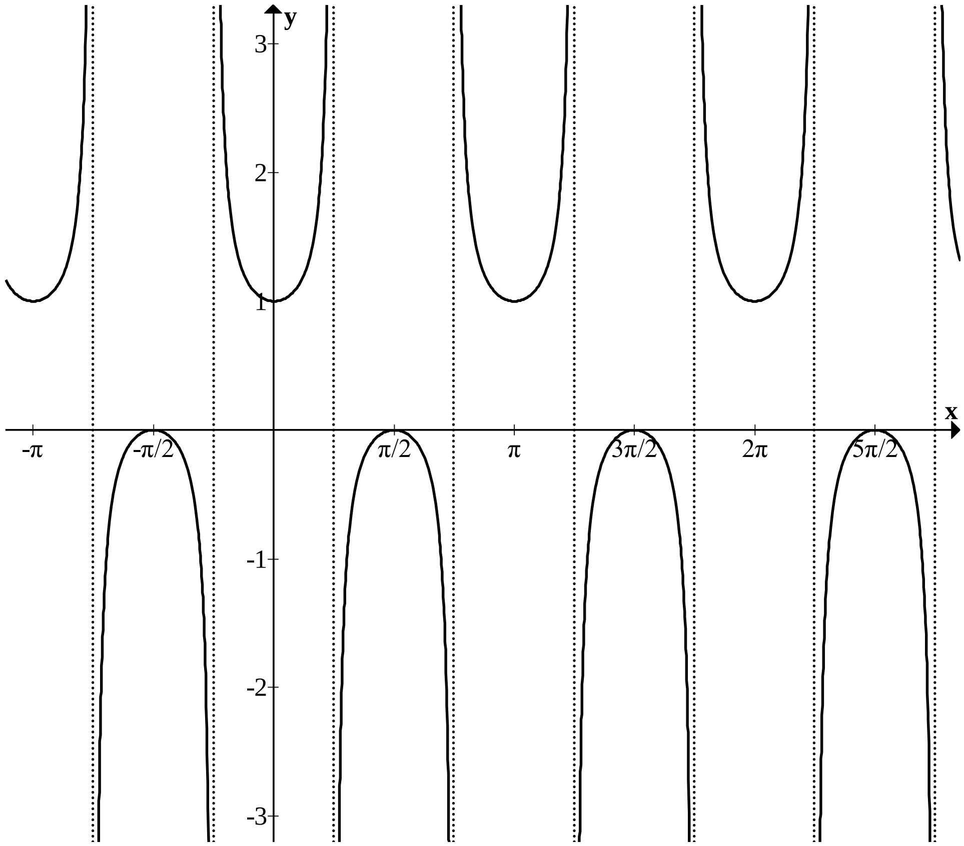
$$= \frac{1 - \sin^2(x)}{\cos^2(x)}$$

$$= \frac{\cos^2(x)}{\cos^2(x)}$$

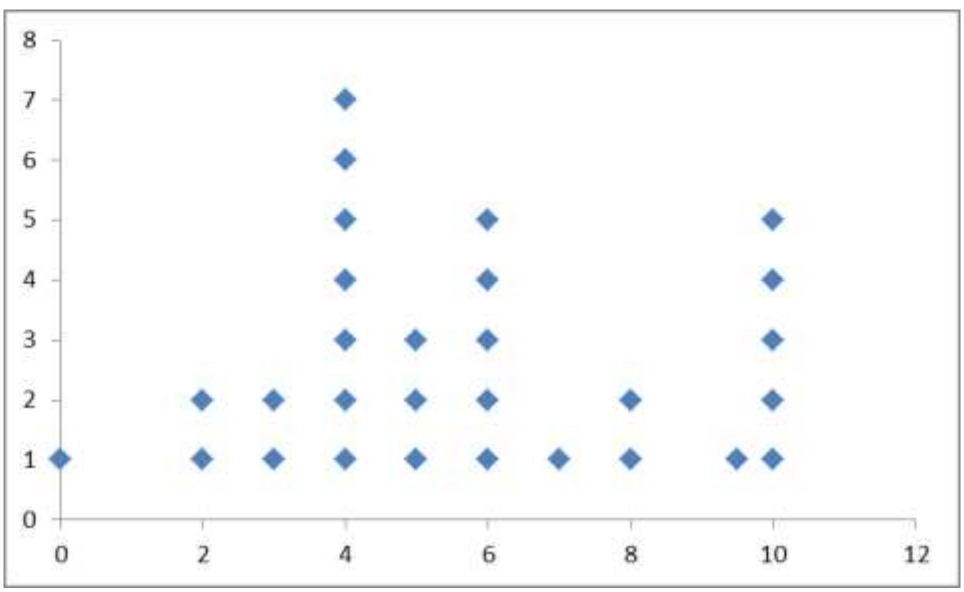
$$= 1$$



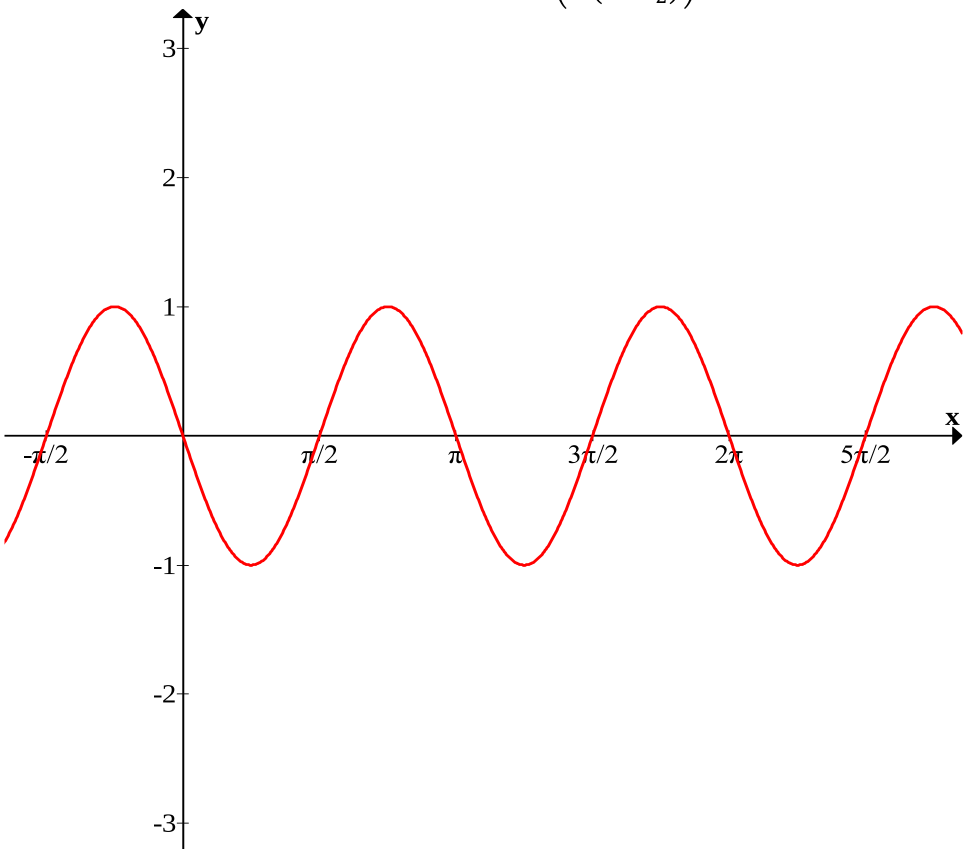
Find an equation for the graph shown below. Circle your answer. (10 points)

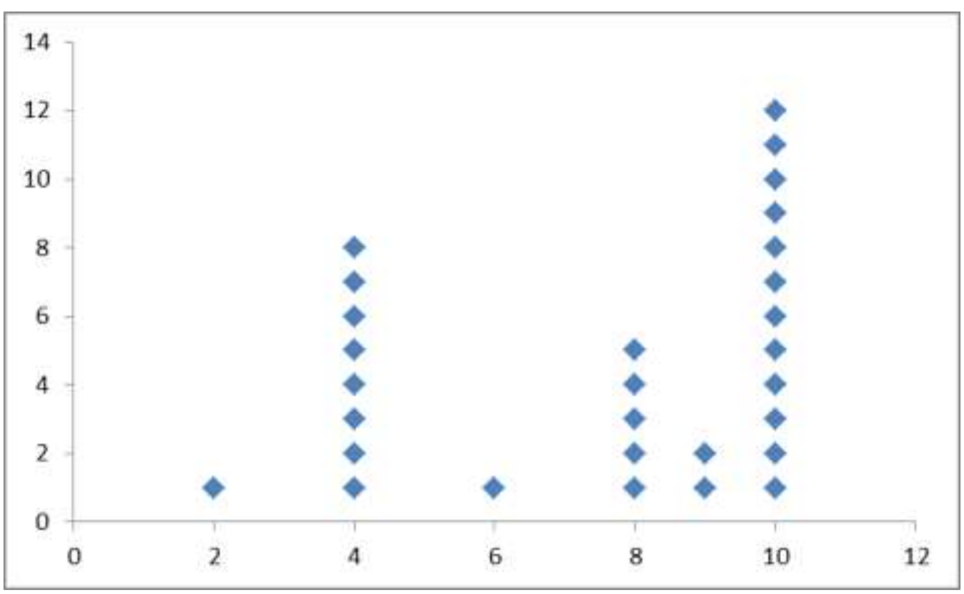


$y = \frac{1}{2}\sec(2x) + \frac{1}{2}$

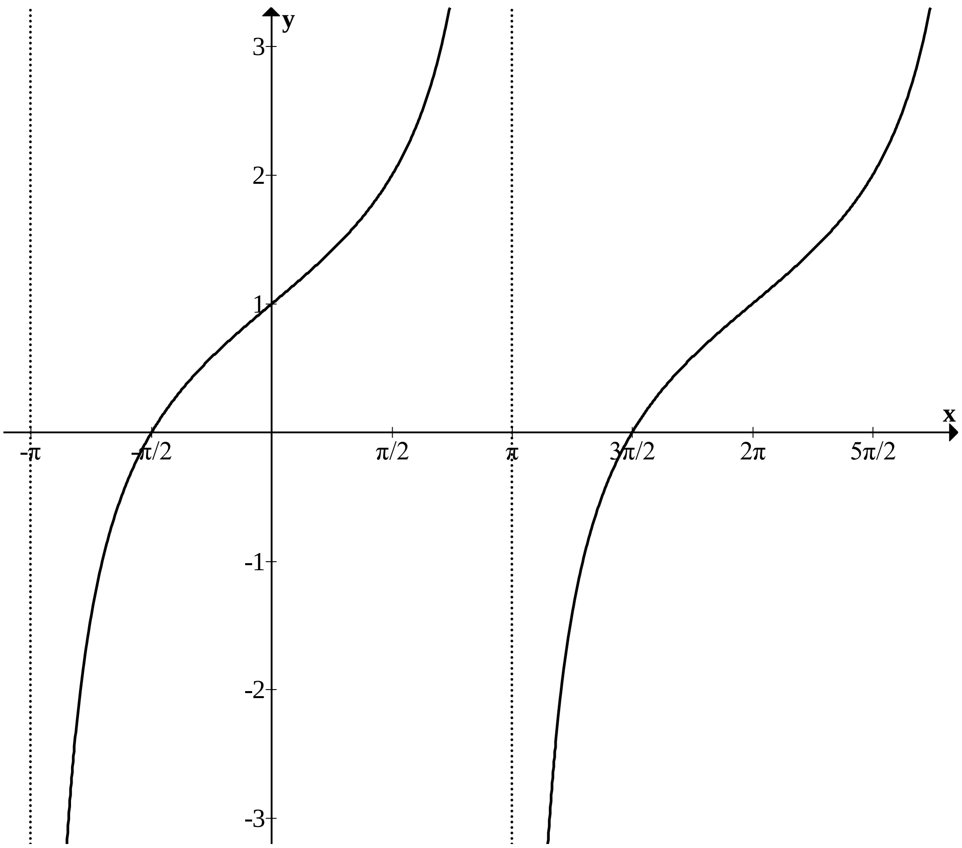


6) On the axis below, sketch the function $y = \sin\left(2\left(x - \frac{\pi}{2}\right)\right)$. (10 points)

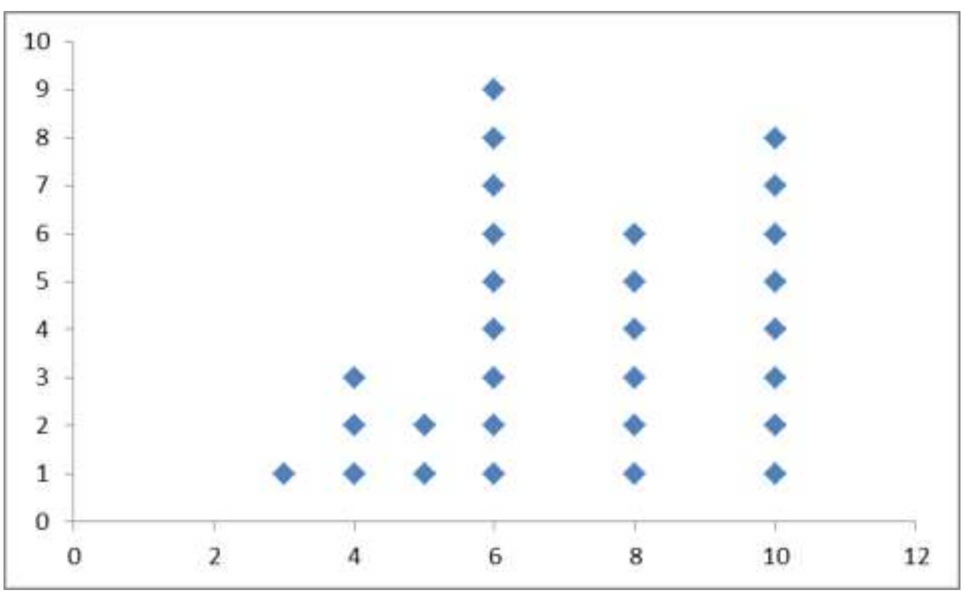




7) Find an equation for the graph shown below. Circle your answer. (10 points)



$y = \tan\left(\frac{x}{2}\right) + 1$



8) On the axis below, sketch the function $y = 2\cos(x + 457\pi) - 1$. (10 points)