Name __ Solutions ________________________________

Trigonometry, Test 2, 3/17/2015

Please show your work, circle your answer, and round all decimals to two decimal places.

No calculators are allowed. When graphing a function:

- Graph as much of the function as the axis provides.
- Identify which graph is your answer if you have multiple graphs on the given axis.
- Use dotted lines to denote any asymptotes (if applicable).

1) Graph the function \( y = \sin\left(x - \frac{\pi}{2}\right) + 3 \). (10 points)

There should be 5 marks corresponding to each of these positioned according to the part of the equation that determines the characteristic of the graph.

Points were deducted when the graphs were too far accurate: for instance if it had corners or flat sections.

If you're still plotting points to graph these functions I did give full credit as long as you came up with the correct graph, but you should know that you’ll be handicapped in the future if you have to rely on points every time you need a graph.
2) Graph the function $y = \tan \left( \frac{1}{3} x \right)$. (10 points)

1 point – amplitude (right ball park)
3 points – tangent graph (2/3 if cotangent)
3 points – period
1 point – phase shift
1 point – vertical shift
1 point – asymptotes
3) Find an equation for the graph shown below. (10 points)

\[ y = 2 \cos(3x) - 1 \]

1 point – amplitude
4 points – sine/cosine graph & phase shift
3 points – period
1 point – vertical shift
1 point – equation
4) Label the giant tick mark on the x-axis, then find an equation for the graph shown. (10 points)

![Graph with tick marks labeled and an equation provided]

\[ y = \sec \left( 2 \left( x - \frac{\pi}{3} \right) \right) \]

3 points - Secant/cosecant
1 point - realistic value
3 points – phase shift
3 points – period
5) Find an equation for an asymptote of \( y = \csc(x) \). (5 points)

\[ x = 0 \]

OR \( x = \pi, x = 2\pi, x = 3\pi, ..., x = k\pi, ... \)

OR \( x = -\pi, x = -2\pi, ... \)
6) The period of our moon is 28 days. If Dr. Evil uses his “Evil Ray” to reduce the period to 14 days, describe how its speed changes. (5 points)

The speed will be increasing, because we made it revolve around the Earth in half the time.

(You might say it’s moving twice as fast. Full credit was given for this because that’s what we would intuitively think. However, physics tells us that if it went twice as fast it would fly out of orbit. To reduce the period we also have to change the altitude. Long story short, the speed would actually increase by about 26%)
7) Find \( \cos \left( \frac{7\pi}{3} \right) \). Note that the next problem will illustrate this. (3 points)

\[
\cos \left( \frac{7\pi}{3} \right) = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} = \frac{-\sqrt{3}}{2}
\]

8) Illustrate your answer to the previous question by drawing and labeling a diagram on the axis below. (7 points)
9) For the past 7 years, a manager of “Bucks Beefy Burgers” has noticed that the utility bill reaches its maximum at $600 in January, and then decreases gradually to a minimum of $400 during the summer. After that it continues to increase until reaching $600 again the following January.

(a) What is the period of the utility bill at Bucks Beefy Burgers? (5 points)

1 year

(b) What is the amplitude of the utility bill at Bucks Beefy Burgers? (5 points)

$100
10) With some rather bizarre but creative science and math, astronomers used the “wobble” depicted here to discover a star.

(a) What is the period of the wobble? (5 points)

Approximately 2 years.

(b) Should the wobble be modeled with a sine graph? If so, find the equation of the sine function. (5 points)

No Sine has more symmetry. Just look at how quickly it increases versus how quickly it decreases.

You might be able to model it with a tangent function though...
11) Simplify the expression shown below. (5 points)

\[
\sin(x) + \frac{\cos^2(x)}{\sin(x)}
\]

\[
\sin(x) + \frac{\cos^2(x)}{\sin(x)} = \frac{\sin^2(x)}{\sin(x)} + \frac{\cos^2(x)}{\sin(x)} = \frac{\sin^2(x) + \cos^2(x)}{\sin(x)} = \frac{1}{\sin(x)} = \csc(x)
\]
12) Simplify the expression shown below. (5 points)

\[
\frac{5}{7} + \frac{2}{3}
\]

\[
\frac{5}{7} + \frac{2}{3} = \frac{5 \cdot 3}{7 \cdot 3} + \frac{2 \cdot 7}{3 \cdot 7} = \frac{15 + 14}{21} = \frac{29}{21}
\]
13) Find all the missing side lengths and angles on the triangle shown below. (10 points)

\[ 5\sqrt{3} = \sqrt{75} \]