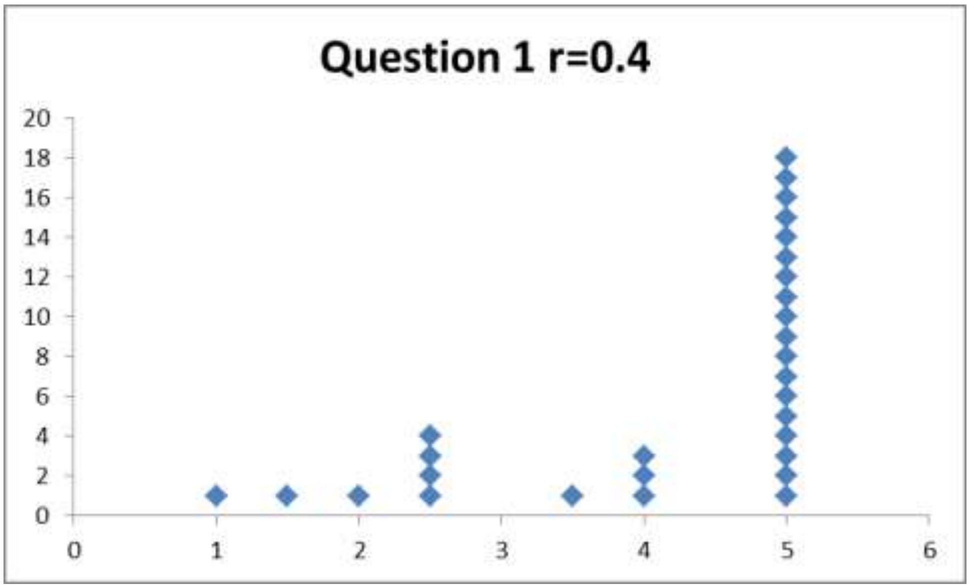
Name __Solutions____ Trigonometry, Test 3, 4/21/2015

Please show your work, circle your answer, and leave all numbers as fractions.
No calculators are allowed.

1) Simplify $(\cos(x) - 1)(\cos(x) + 1)$ by expanding it into a sum of terms. (5 points)

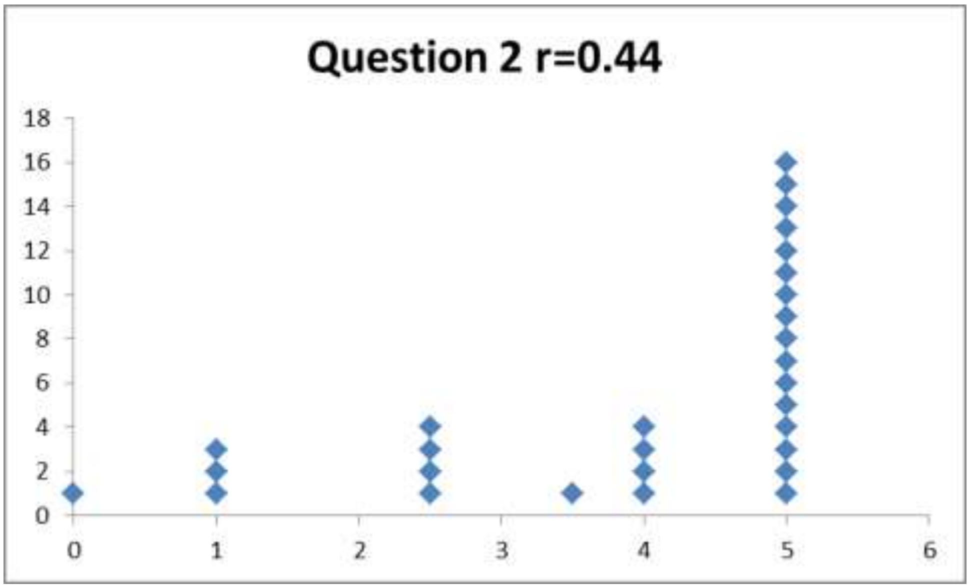
$$(\cos(x) - 1)(\cos(x) + 1) = \cos^2(x) - \cos(x) + \cos(x) - 1 = \cos^2(x) - 1 = -\sin^2(x)$$



2) Find the number below by expressing it as a single reduced fraction. (5 points)

$$\frac{\frac{1}{2}+\frac{5}{7}}{\frac{1}{2}}$$

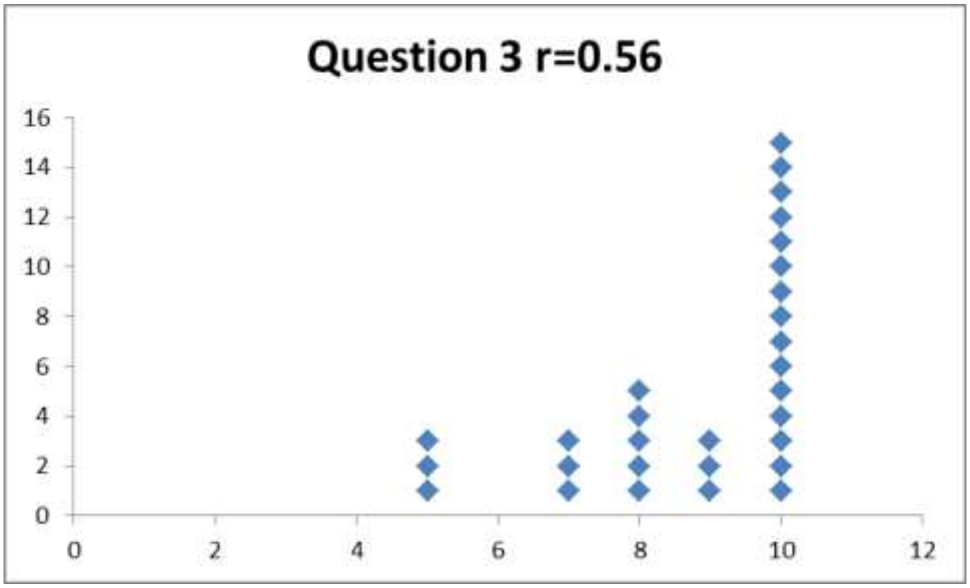
$$\frac{\frac{1}{2}+\frac{5}{7}}{\frac{1}{2}}=\frac{\frac{7}{14}+\frac{10}{14}}{\frac{1}{2}}=\frac{\frac{17}{14}}{\frac{1}{2}}=\frac{17}{14}\cdot\frac{2}{1}=\frac{17}{7}$$



3) Verify the identity below. (10 points)

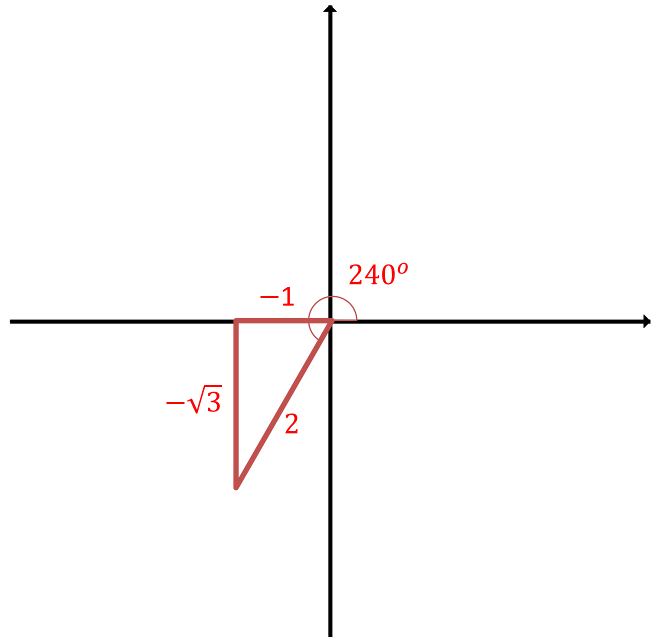
$$\cos(x)\tan(x) = \sin(x)$$

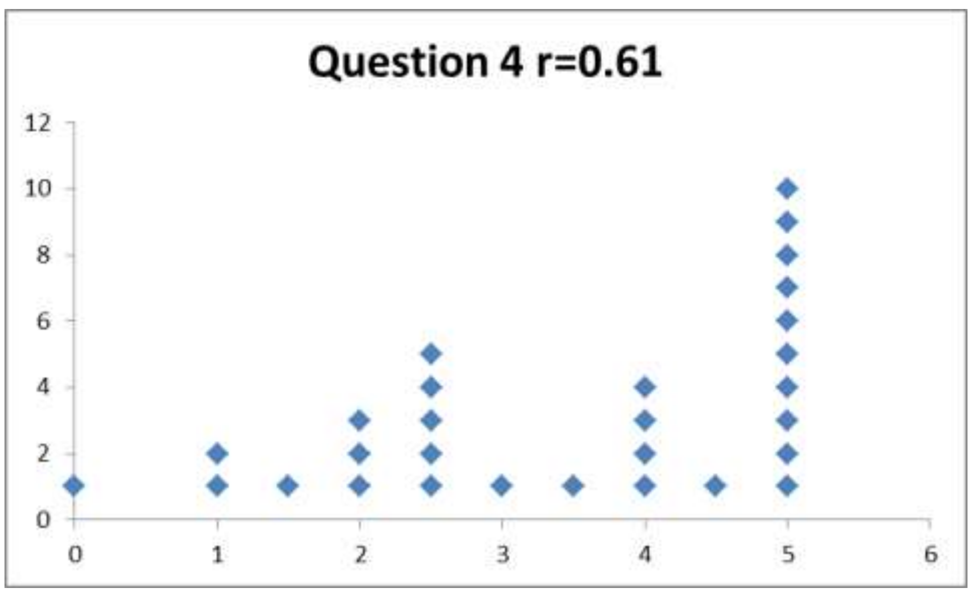
$$\cos(x)\tan(x) = \cos(x) \cdot \frac{\sin(x)}{\cos(x)} = \sin(x)$$



4) Find $\sin(240^o)$. (5 points)

$\sin(240^o) = -\frac{\sqrt{3}}{2}$

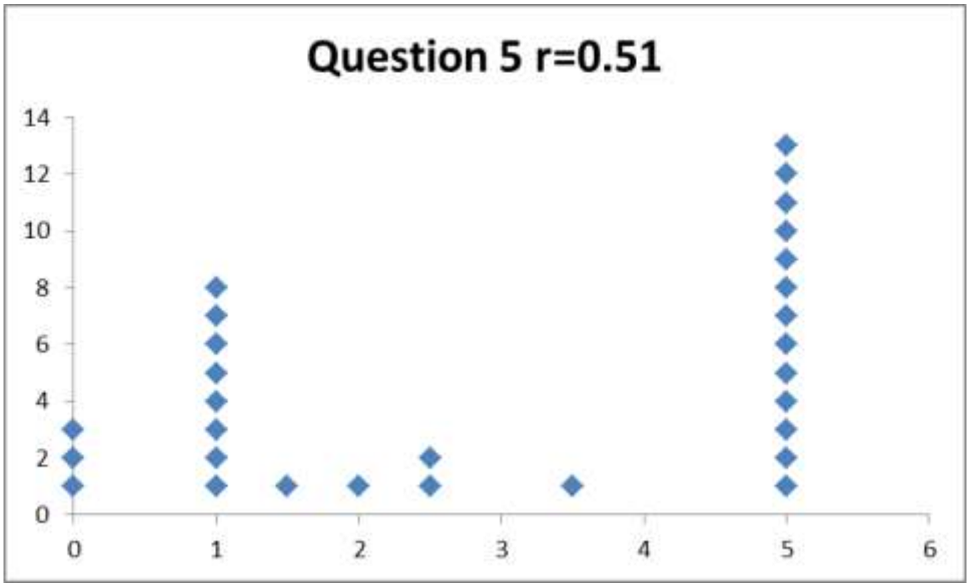




5) Find the number below by expressing it as a single reduced fraction. (5 points)

$$\frac{\frac{1}{3}}{6}$$

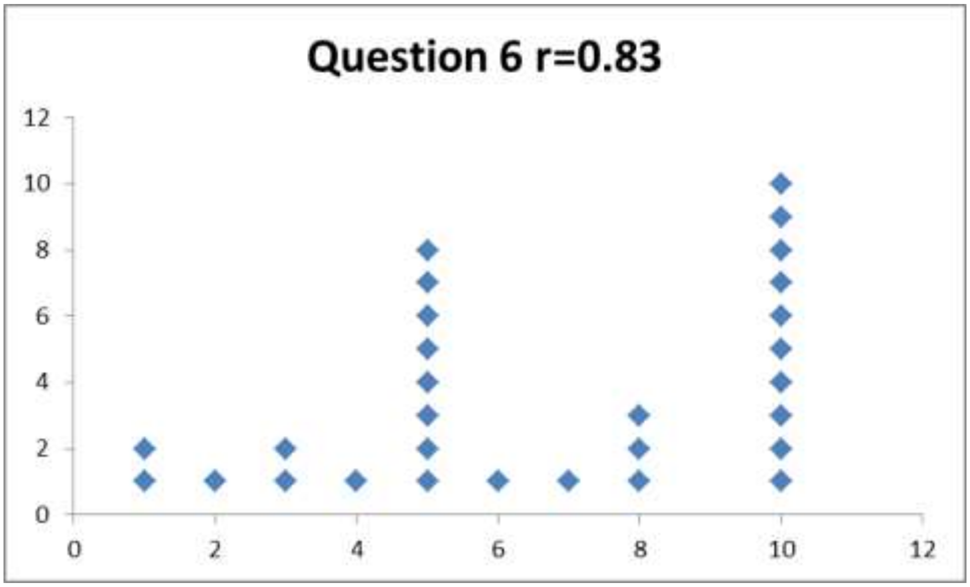
$$\frac{\frac{1}{3}}{6} = \frac{\frac{1}{3}}{\frac{6}{1}} = \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$$



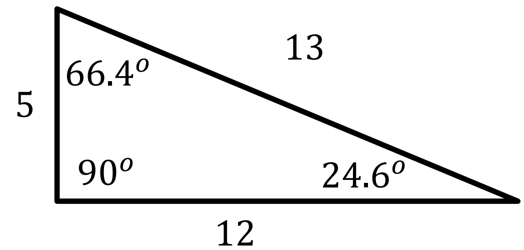
6) Verify the identity below. (10 points)

$$\tan(x) + \cot(x) = \sec(x)\csc(x)$$

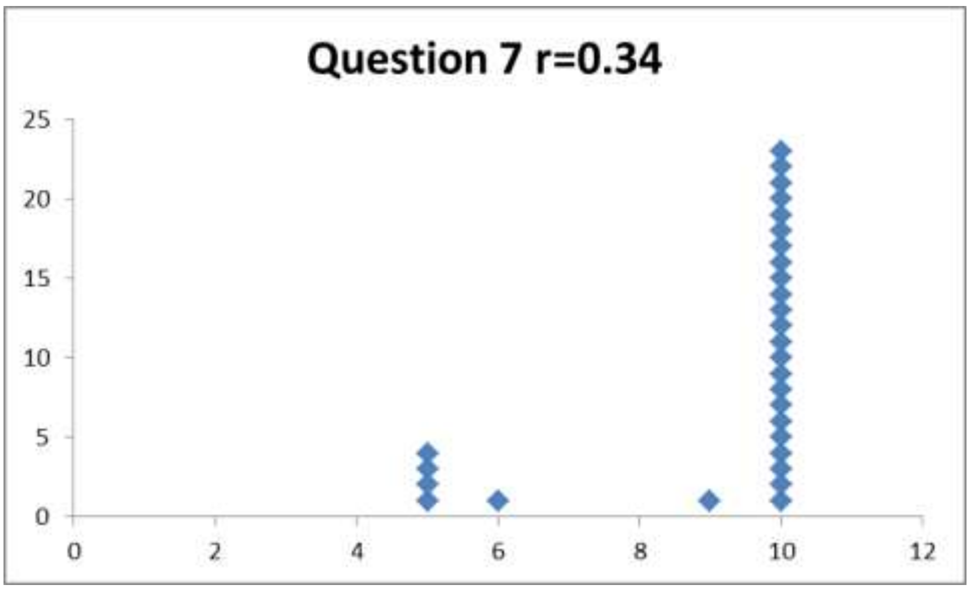
$$\tan(x) + \cot(x) = \frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)} = \frac{\sin^2(x)}{\cos(x)\sin(x)} + \frac{\cos^2(x)}{\cos(x)\sin(x)} = \frac{\sin^2(x) + \cos^2(x)}{\cos(x)\sin(x)} = \frac{1}{\cos(x)\sin(x)} = \sec(x)\csc(x)$$



7) Use the triangle given here to find $\sin(24.6^o)$. (10 points)



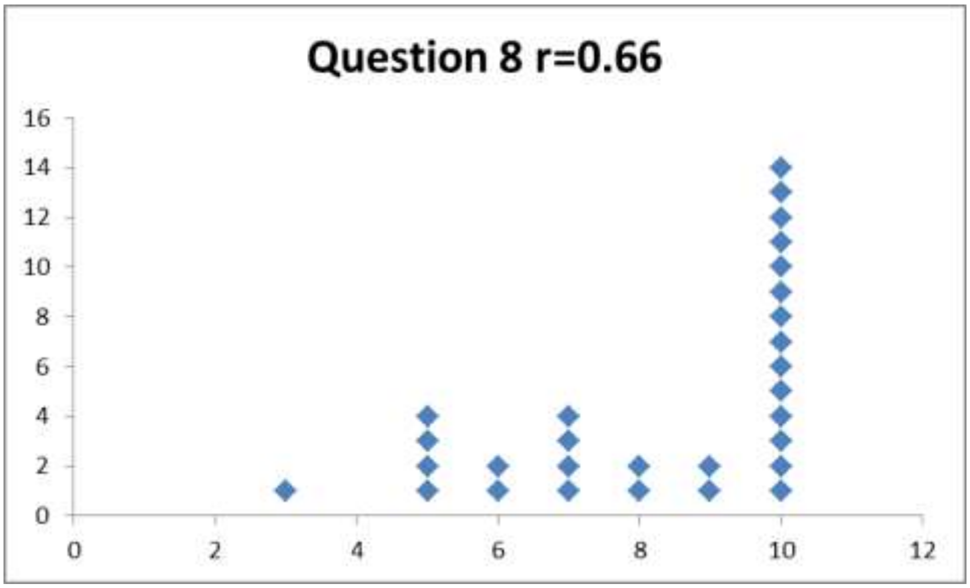
$\frac{5}{13}$



8) Verify the identity below. (10 points)

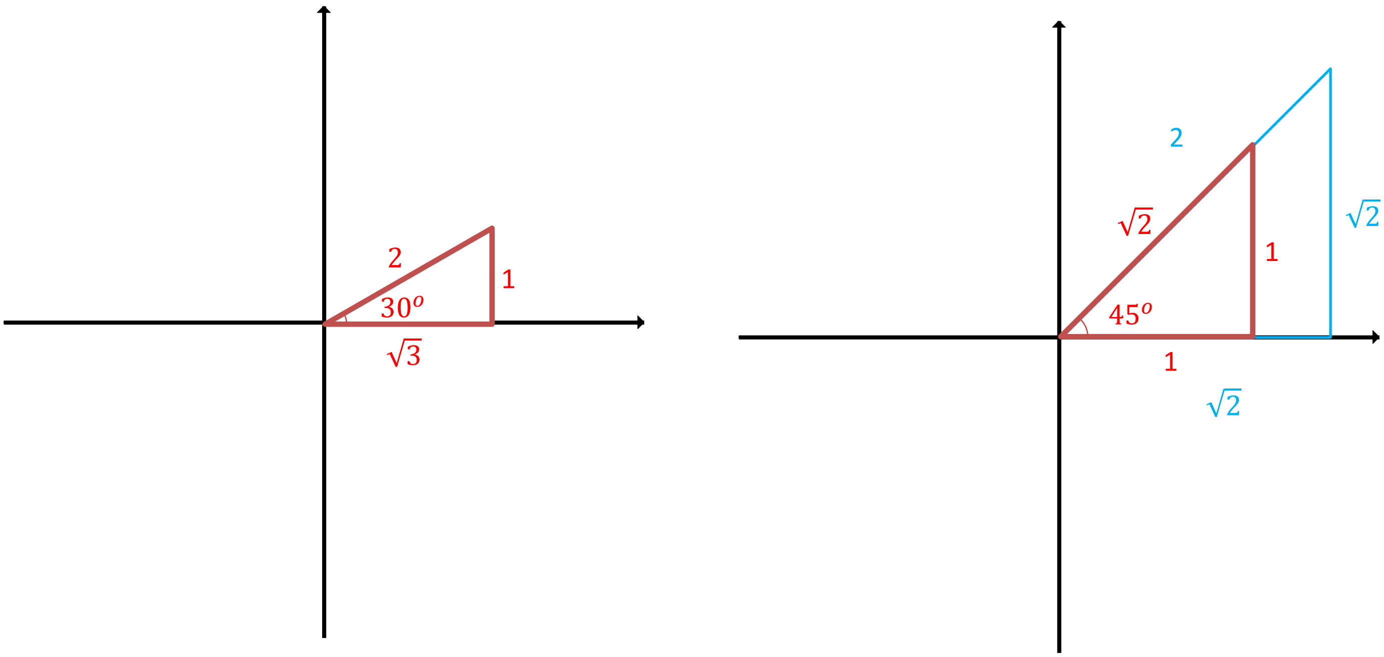
$$\sin^3(x)\csc(x) + \cos^3(x)\sec(x) = 1$$

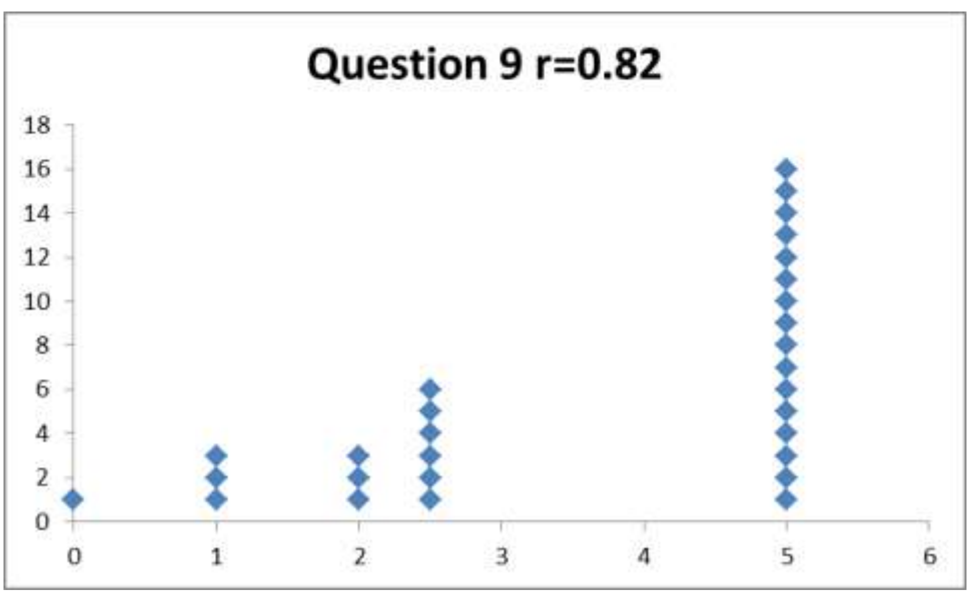
$$\sin^3(x)\csc(x) + \cos^3(x)\sec(x) = \sin^3(x)\cdot\frac{1}{\sin(x)} + \cos^3(x)\cdot\frac{1}{\cos(x)} = \sin^2(x) + \cos^2(x) = 1$$



9) Find $\sin(15^o)$. (5 points)

$$\sin(15^o) = \sin(45^0 - 30^o) = \sin(45^o)\cos(30^o) - \cos(45^o)\sin(30^o)$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

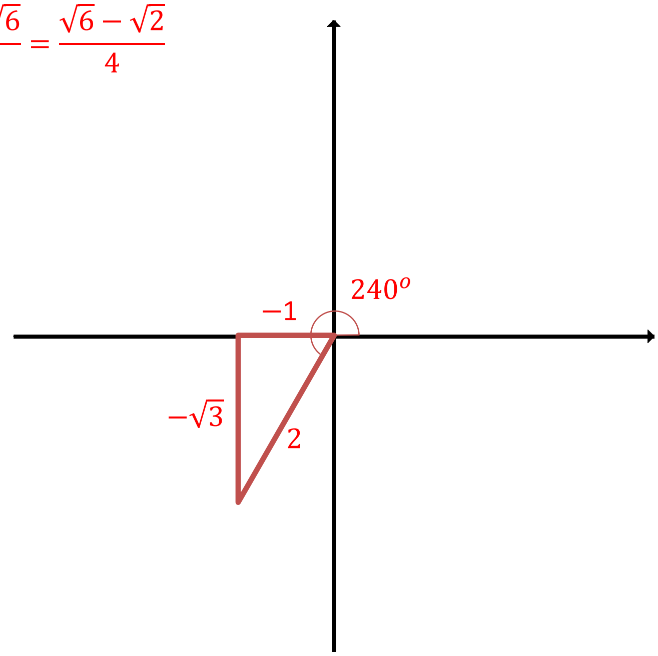


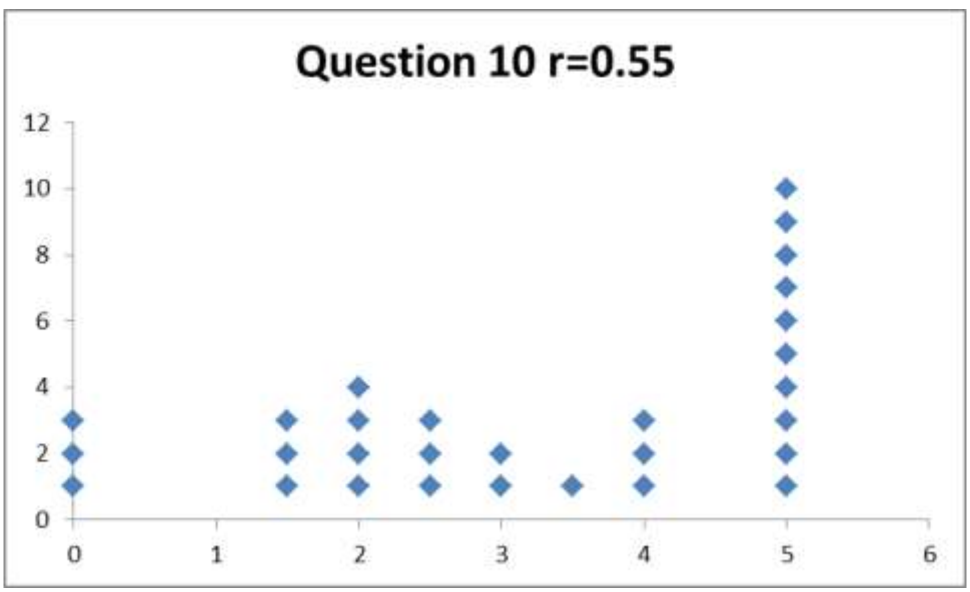


10) Find $\cos(285^o)$. (5 points)

$$\cos(285^o) = \cos(240^o + 45^o) = \cos(240^o)\cos(45^o) - \sin(240^o)\sin(45^o)$$

$$-\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{-\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{-\sqrt{2}+\sqrt{6}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

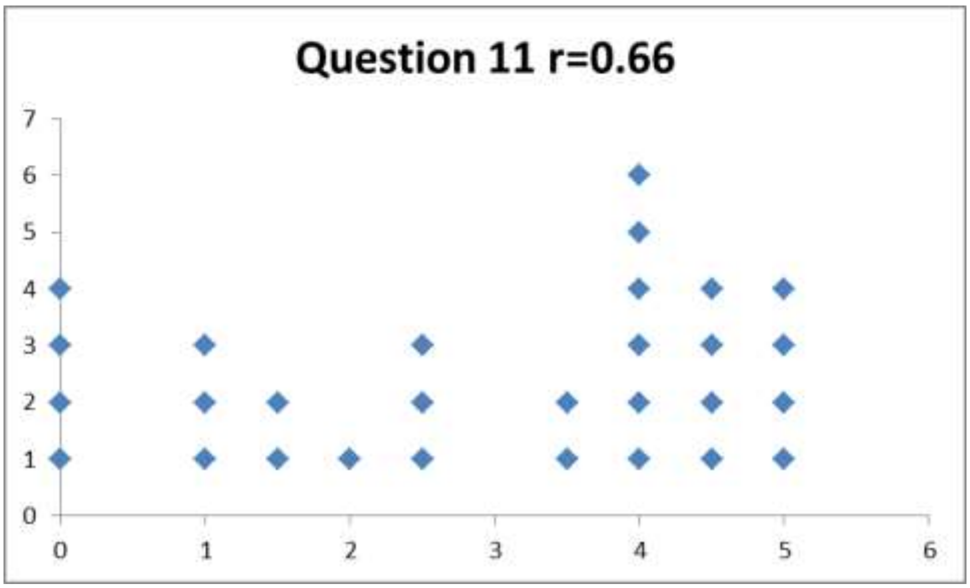




11) Find $\tan\left(\frac{7\pi}{12}\right)$. (5 points)

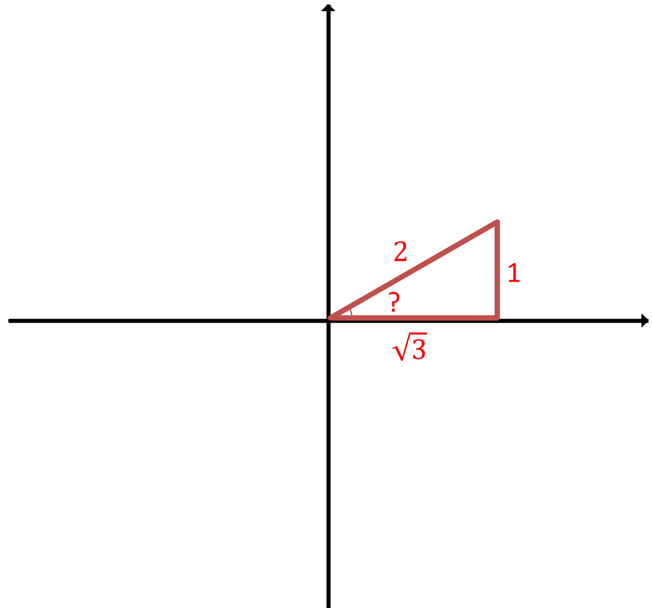
$$\tan\left(\frac{7\pi}{12}\right)=\tan\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)=\tan\left(\frac{\pi}{4}+\frac{\pi}{3}\right)=\frac{\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\pi}{3}\right)}{1-\tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}=\frac{1+\sqrt{3}}{1-1\cdot\sqrt{3}}=\frac{1+\sqrt{3}}{1-\sqrt{3}}$$

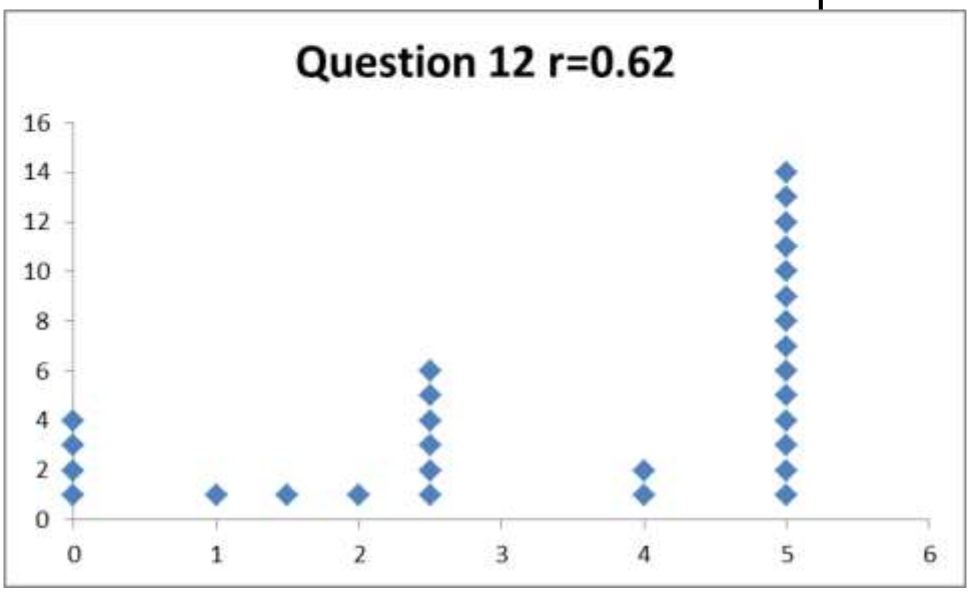
$$=\frac{1+\sqrt{3}}{1-\sqrt{3}}\cdot\frac{(1+\sqrt{3})}{(1+\sqrt{3})}=\frac{(1+\sqrt{3})}{(1-\sqrt{3})}\cdot\frac{(1+\sqrt{3})}{(1+\sqrt{3})}=\frac{1+2\sqrt{3}+3}{1-3}=\frac{4+2\sqrt{3}}{-2}=-2-\sqrt{3}$$



12) Find $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$. (5 points)

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^o \text{ or } \frac{\pi}{6}$





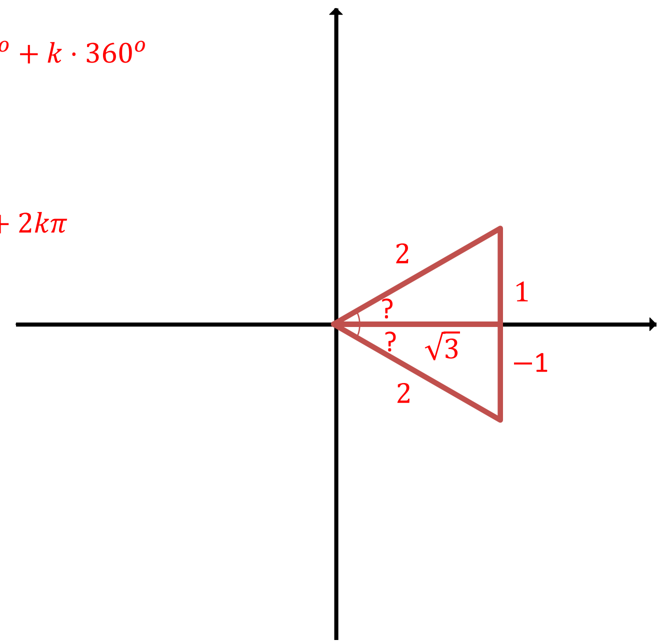
13) Solve the equation below for x. (10 points)

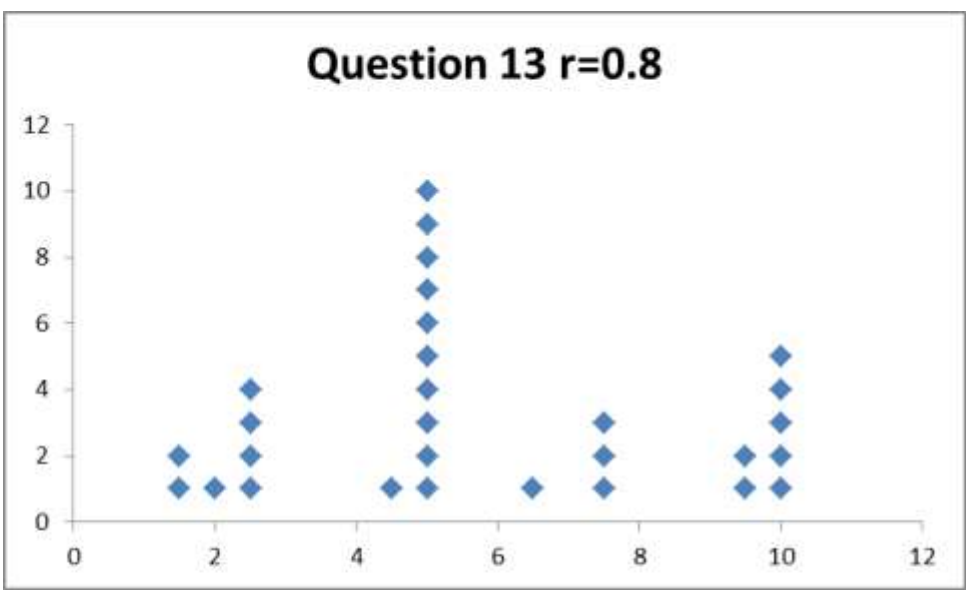
$$\cos(x) = \frac{\sqrt{3}}{2}$$

$x = 30^o + k \cdot 360^o$ and $330^o + k \cdot 360^o$

OR

$x = \frac{\pi}{6} + 2k\pi$ and $\frac{11\pi}{6} + 2k\pi$



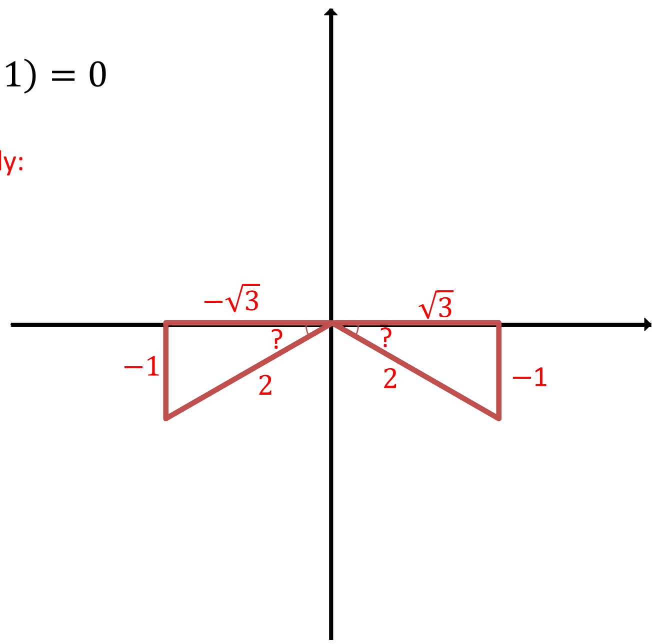


14) Solve the equation below for x. (10 points)

$$(\sin(x) - 1)(2\sin(x) + 1) = 0$$

Either factor could be zero, so we solve each of these individually:

$$\sin(x) = 1$$
$$x = 90^o + k \cdot 360^o$$



$$2\sin(x) + 1 = 0$$
$$\sin(x) = -\frac{1}{2}$$
$$x = 330^o + k \cdot 360^o$$
$$x = 210^o + k \cdot 360^o$$