

Choose and complete ONE of the following problems: Graded out of 10 points; beyond that is extra credit. If you attempt multiple problems make it clear which one you want graded; failure to do so will result in only the lowest score being graded.

1) Given that $x^2 + y^2 = 1$, find $\frac{dy}{dx}$. (10 points)

$$\begin{aligned}\frac{d}{dx}(x^2 + y^2) &= \frac{d}{dx}1 \\ 2x + 2y\frac{dy}{dx} &= 0 \\ 2y\frac{dy}{dx} &= -2x \\ \frac{dy}{dx} &= -\frac{2x}{2y} = -\frac{x}{y}\end{aligned}$$

2) Given that $x^3 + y^3 = 5x$, find $\frac{dy}{dx}$. (12 points)

$$\begin{aligned}\frac{d}{dx}(x^3 + y^3) &= \frac{d}{dx}5x \\ 3x^2 + 3y^2\frac{dy}{dx} &= 5 \\ 3y^2\frac{dy}{dx} &= 5 - 3x^2 \\ \frac{dy}{dx} &= \frac{5 - 3x^2}{3y^2}\end{aligned}$$

3) Given that $x^4 + y^4 = xy^2$, find $\frac{dy}{dx}$. (14 points)

$$\begin{aligned}\frac{d}{dx}(x^4 + y^4) &= \frac{d}{dx}xy^2 \\ 4x^3 + 4y^3\frac{dy}{dx} &= y^2 + 2xy\frac{dy}{dx} \\ 4y^3\frac{dy}{dx} - 2xy\frac{dy}{dx} &= y^2 - 4x^3 \\ \frac{dy}{dx}(4y^3 - 2xy) &= y^2 - 4x^3 \\ \frac{dy}{dx} &= \frac{y^2 - 4x^3}{4y^3 - 2xy}\end{aligned}$$