A hot-air balloon is rising vertically at a rate of 10 ft/s. A motorcycle (traveling in a straight line on a horizontal road) is going to pass directly beneath it. When the motorcyclist is 50 feet from the location of the balloon, we know the following information:

- The balloon is 150 feet off the ground
- The distance between the balloon and motorcycle is decreasing at a rate of 25 ft/s.

How fast is the motorcycle moving?

Using the Pythagorean theorem, we get an equation and take the derivative:

\[ x^2 + y^2 = L^2 \]

\[ \frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(L^2) \]

\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2L \frac{dL}{dt} \]

\[ \frac{x}{\frac{dx}{dt}} + \frac{y}{\frac{dy}{dt}} = \frac{L}{\frac{dL}{dt}} \]

No we plug in our information at the moment we’re interested in, and solve for \( \frac{dx}{dt} \).

\[ 50 \frac{dx}{dt} + 150 \cdot 10 = \sqrt{50^2 + 150^2} \cdot -25 \]

\[ \frac{dx}{dt} = \frac{-25\sqrt{50^2 + 150^2} - 1500}{50} \]

The motorcyclist is traveling \( \frac{25\sqrt{50^2 + 150^2} + 1500}{50} \) feet per second toward the balloon.