Name $\qquad$ Solutions $\qquad$ Quiz 8, Calculus 1

A hot-air balloon is rising vertically at a rate of $10 \mathrm{ft} / \mathrm{s}$. A motorcycle (traveling in a straight line on a horizontal road) is going to pass directly beneath it. When the motorcyclist is 50 feet from the location of the balloon, we know the following information:

- The balloon is 150 feet off the ground
- The distance between the balloon and motorcycle is decreasing at a rate of $25 \mathrm{ft} / \mathrm{s}$.

How fast is the motorcycle moving?

$$
\begin{gathered}
\frac{d y}{d t}=10 \\
y=150 \\
\frac{d L}{d t}=-25 \\
x=50
\end{gathered}
$$

The illustration of the scenario. Note that all the information in blue changes as the motorcycle moves. Hence this information cannot be plugged into the equation until after you take the derivative.

Using the Pythagorean theorem, we get an equation and take the derivative:

$$
\begin{gathered}
x^{2}+y^{2}=L^{2} \\
\frac{d}{d t}\left(x^{2}+y^{2}\right)=\frac{d}{d t}\left(L^{2}\right) \\
2 x \frac{d x}{d t}+2 y \frac{d y}{d t}=2 L \frac{d L}{d t} \\
x \frac{d x}{d t}+y \frac{d y}{d t}=L \frac{d L}{d t}
\end{gathered}
$$

No we plug in our information at the moment we're interested in, and solve for $\frac{d x}{d t}$.

$$
\begin{gathered}
50 \frac{d x}{d t}+150 \cdot 10=\sqrt{50^{2}+150^{2}} \cdot-25 \\
\frac{d x}{d t}=\frac{-25 \sqrt{50^{2}+150^{2}}-1500}{50}
\end{gathered}
$$

The motorcyclist is traveling $\frac{25 \sqrt{50^{2}+150^{2}}+1500}{50}$ feet per second toward the balloon.

