A hot-air balloon is rising vertically at a rate of 10 ft/s. A motorcycle (traveling in a straight line on a horizontal road) is going to pass directly beneath it. When the motorcyclist is 50 feet from the location of the balloon, we know the following information:

- The balloon is 150 feet off the ground •
- The distance between the balloon and motorcycle is decreasing at a rate of 25 ft/s. •

How fast is the motorcycle moving?

$$y = 150$$

$$\frac{dy}{dt} = 10$$

$$x = 50$$

$$L = \sqrt{50^2 + 150^2}$$

$$\frac{dL}{dt} = -25$$

The illustration of the scenario. Note that all the information in blue changes as the motorcycle moves. Hence this information *cannot* be plugged into the equation until after you take the derivative.

Using the Pythagorean theorem, we get an equation and take the derivative:

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(L^2)$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2L\frac{dL}{dt}$$
$$x\frac{dx}{dt} + y\frac{dy}{dt} = L\frac{dL}{dt}$$

 $x^2 + y^2 = L^2$

No we plug in our information at the moment we're interested in, and solve for $\frac{dx}{dt}$.

$$50\frac{dx}{dt} + 150 \cdot 10 = \sqrt{50^2 + 150^2} \cdot -25$$
$$\frac{dx}{dt} = \frac{-25\sqrt{50^2 + 150^2} - 1500}{50}$$

The motorcyclist is traveling $\frac{25\sqrt{50^2+150^2}+1500}{50}$ feet per second toward the balloon.