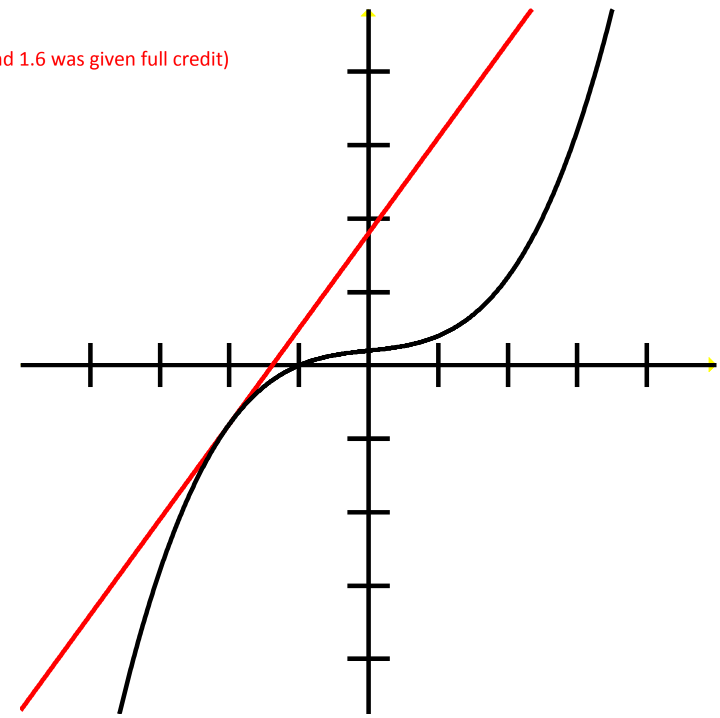
Name __Solutions____ Calculus I, Test 1, Fall 2016

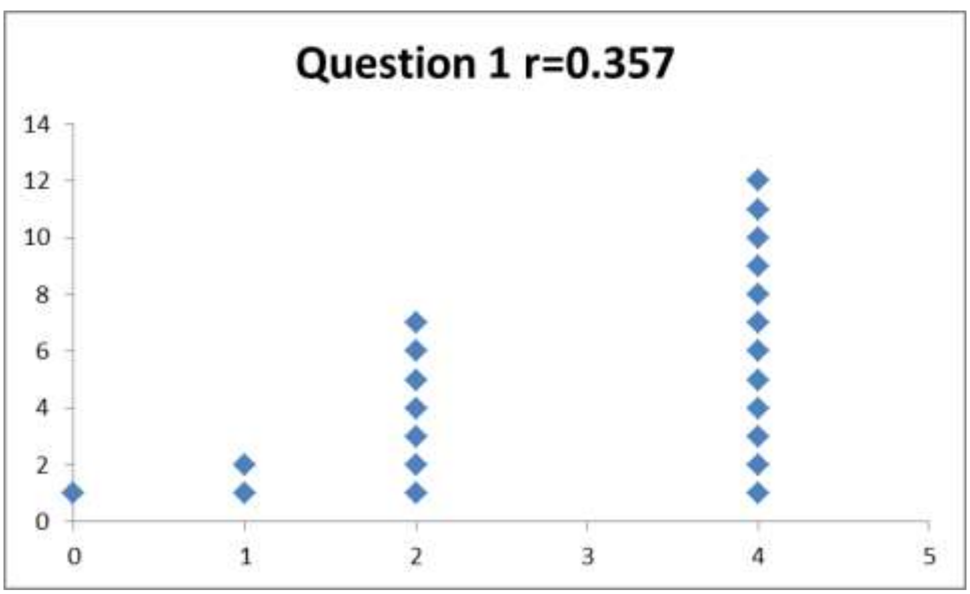
Non-calculator portion. Please show all your work and circle your answer when appropriate. You do not need to simplify answers unless the problem specifies to do so.

1) Estimate the instantaneous rate of change of the function graphed below at $x = -2$. (4 points)

As the tangent line on the graph illustrates, the slope of this line appears to be about 1.2.

(The actual answer would be 1.3, so my guess was close. Anything between 1 and 1.6 was given full credit)



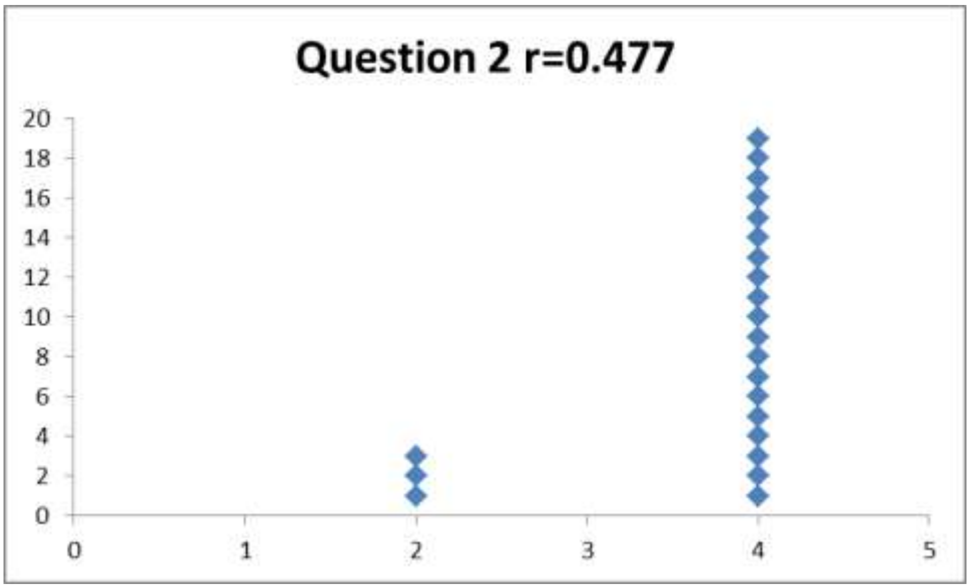


2) Using the function graphed to the right, find $\lim_{x \to -1} f(x)$.

(4 points)

Looking at the graph, we see that:

$\lim_{x \to -1} f(x) = 0$



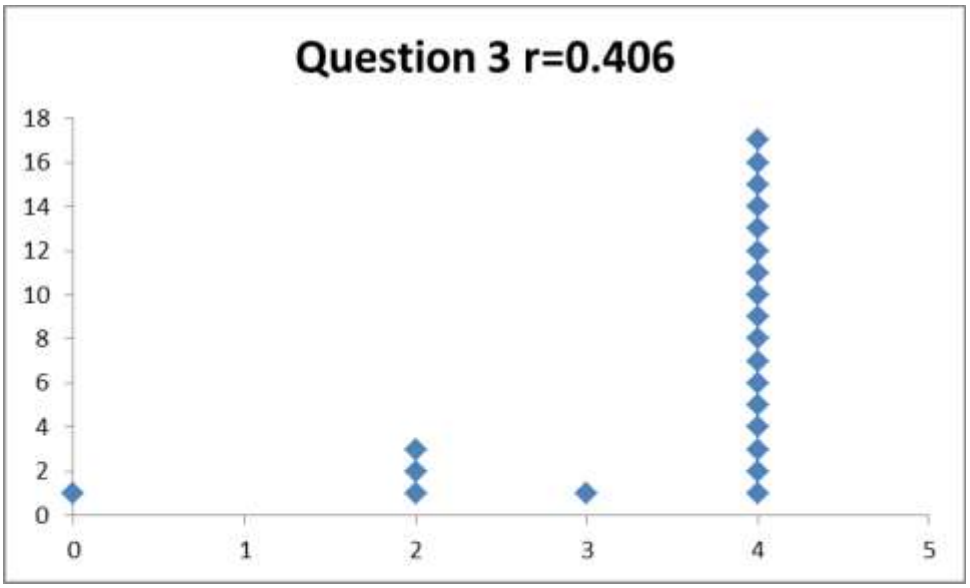
3) Suppose $f(x)$ and $g(x)$ are polynomials with $f(0) = 4$ and $\lim_{x\to 0} \frac{f(x)}{g(x)} = 10$. Find $g(0)$. (4 points)

Assuming $g(0) \neq 0$, this is a continuous function, so we get:

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = 10$$

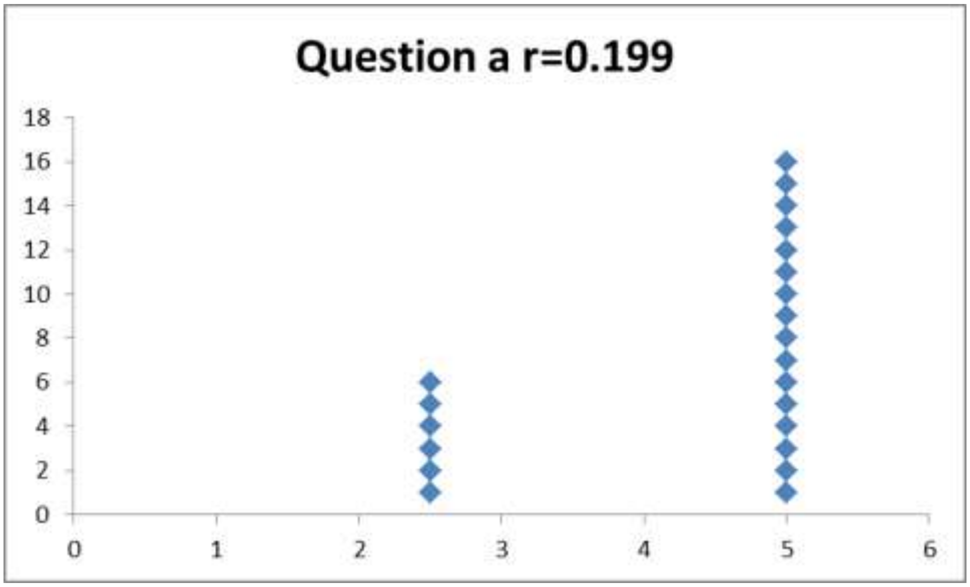
$$\frac{4}{g(0)} = 10$$

$$g(0) = \frac{4}{10}$$

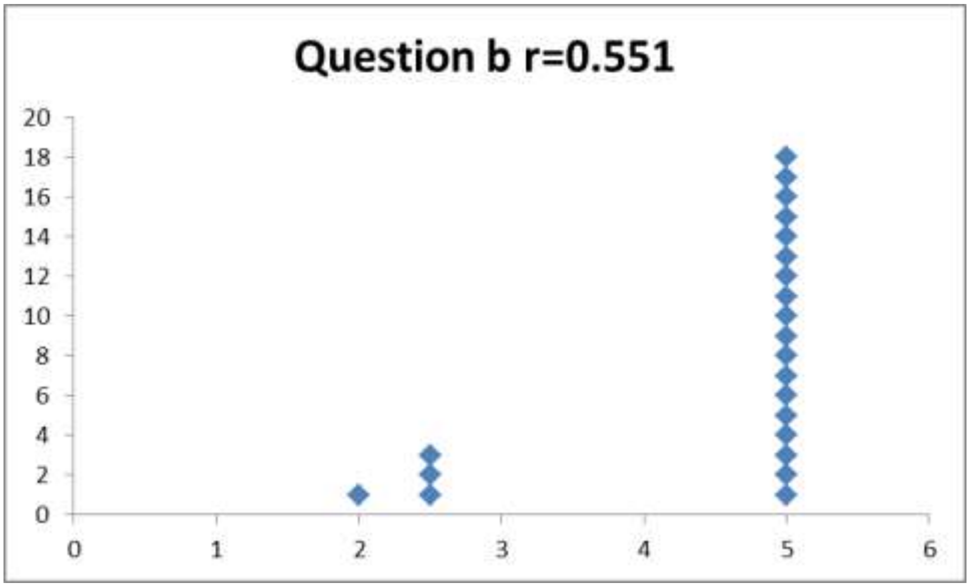


4) Find each of the following limits. (5 points each)

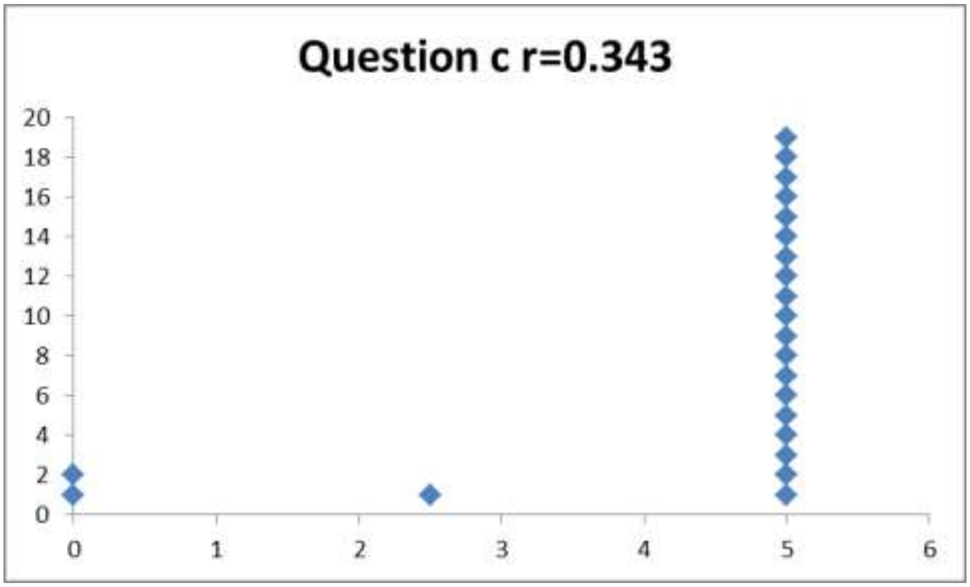
a) $\lim_{x \to 5^+} \frac{x^2-16}{x-5} = \infty$



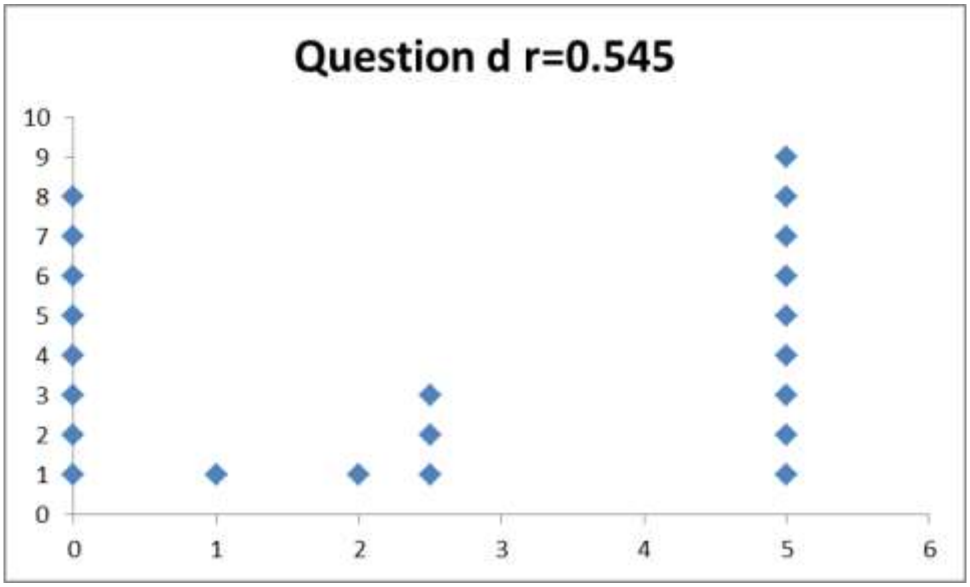
b) $\lim_{x\to 2}\frac{2x^2-2x-4}{x-2} = \lim_{x\to 2}\frac{2(x-2)(x+1)}{x-2} = \lim_{x\to 2} 2(x+1) = 2\cdot 3 = 6$



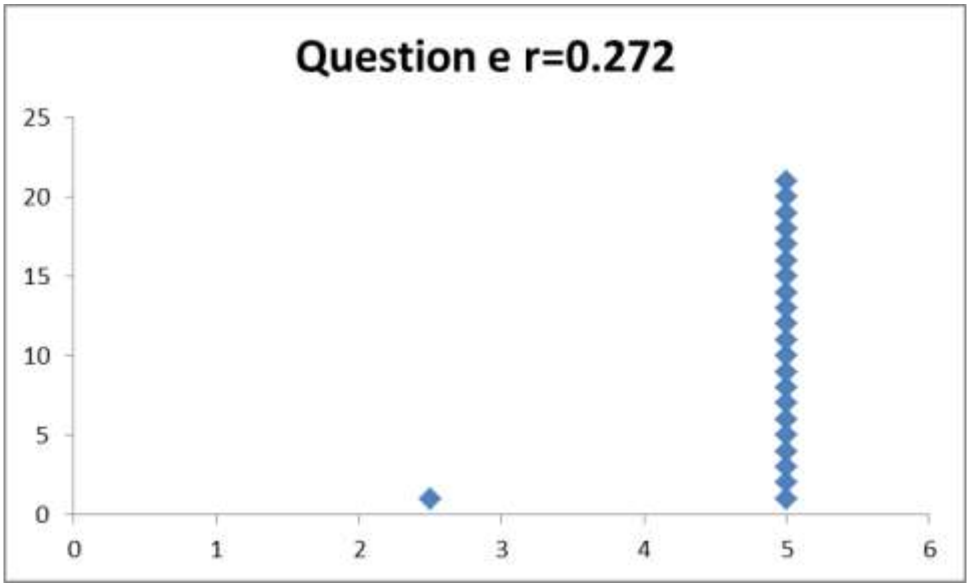
c) $\lim_{x \to \infty} \frac{x^2 - 25}{x - 5} = \infty$



d) $\lim_{x\to 3} \tan(x)\cos(x) = \lim_{x\to 3} \frac{\sin(x)}{\cos(x)}\cos(x) = \lim_{x\to 3} \sin(x) = \sin(3)$



e) $\lim_{x \to 3} \frac{x^2 - x - 2}{x^2 - 4} = \frac{3^2 - 3 - 2}{3^2 - 4} = \frac{4}{5}$



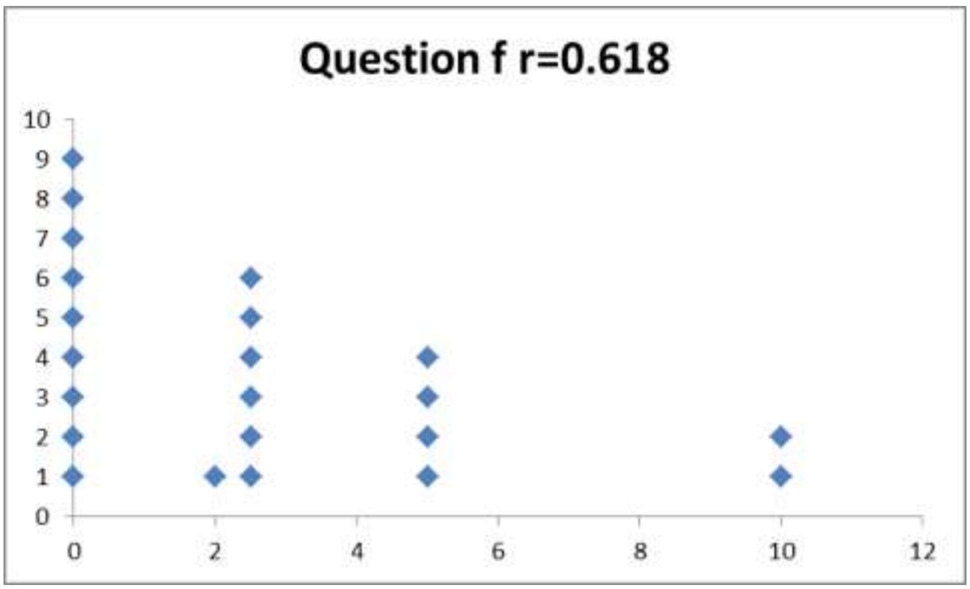
f) $\lim_{x \to -\infty} \frac{x^6+3x^3+x^2+1}{x^4+3^x+2^x+1} = \infty$

You could unfortunately get the correct answer by doing the problem completely wrong. Hence on this problem:

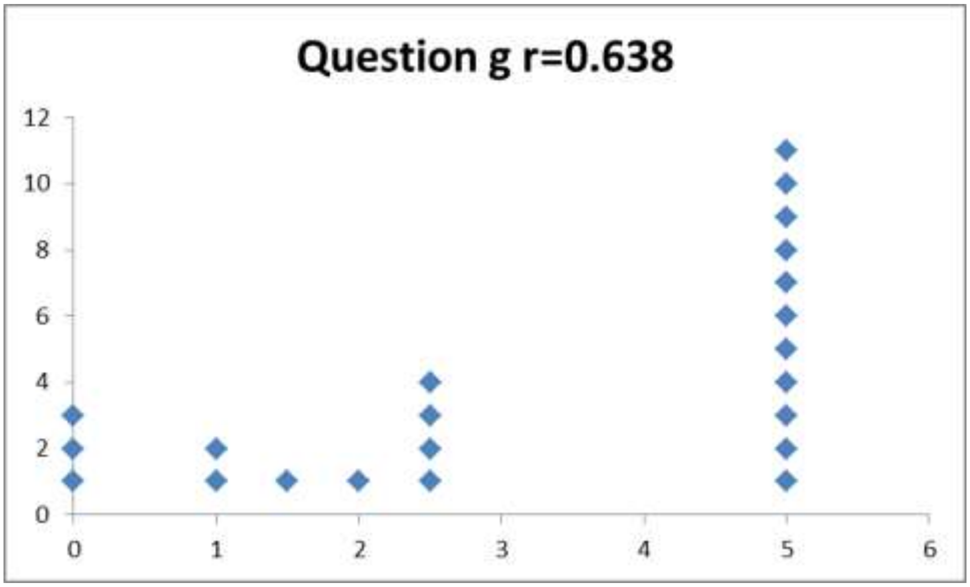
Right answer = Full credit

Right answer using correct math = Full credit + 5 bonus points

Some papers have red and blue marks where I accidently graded it wrong the first time because I had the wrong answer on the solutions.

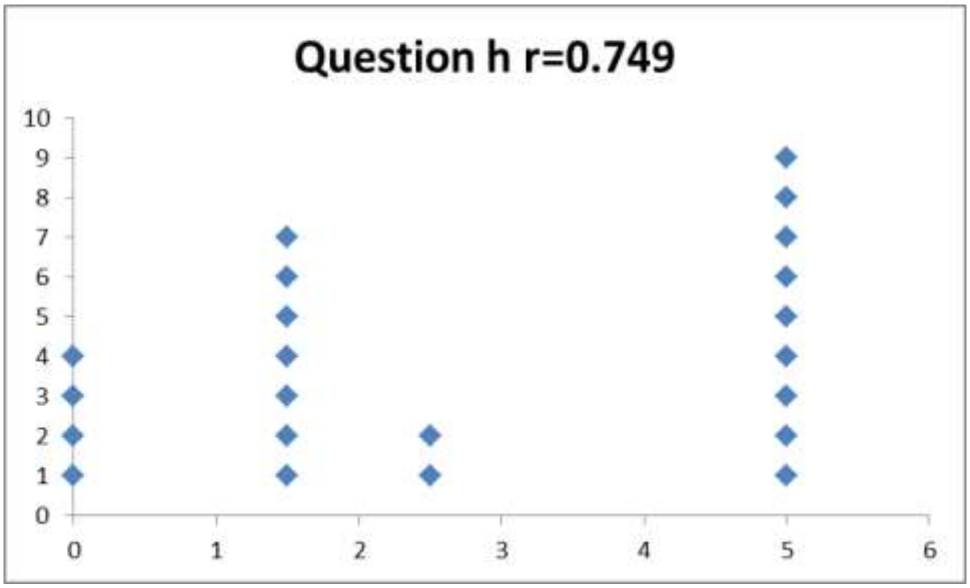


g) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x \to 9} \frac{(\sqrt{x}-3)}{(x-9)} \cdot \frac{(\sqrt{x}+3)}{(\sqrt{x}+3)} = \lim_{x \to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)} = \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{3+3} = \frac{1}{6}$

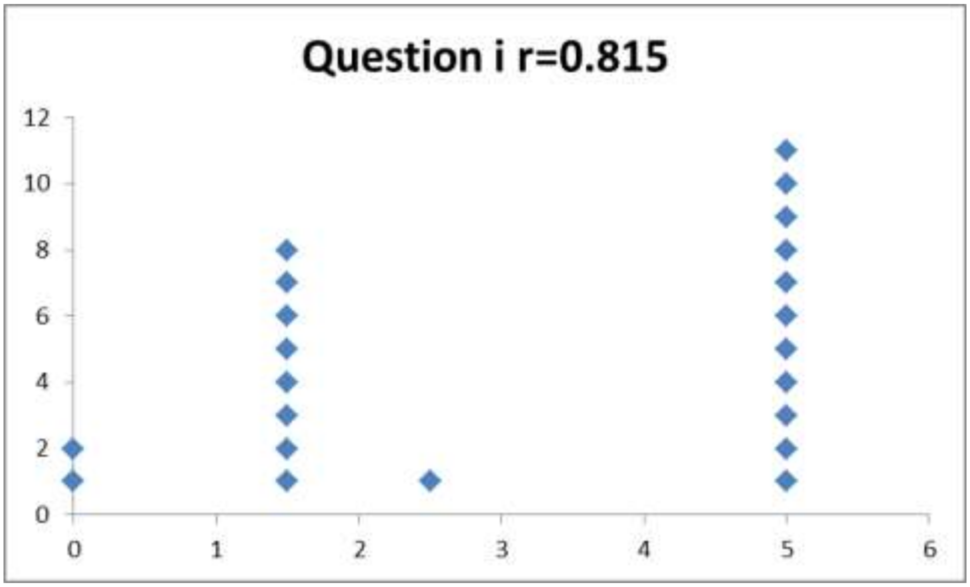


h) $\lim_{x \to 2^-} \frac{x^2+ax}{bx+1} = \frac{4+2a}{2b+1}$

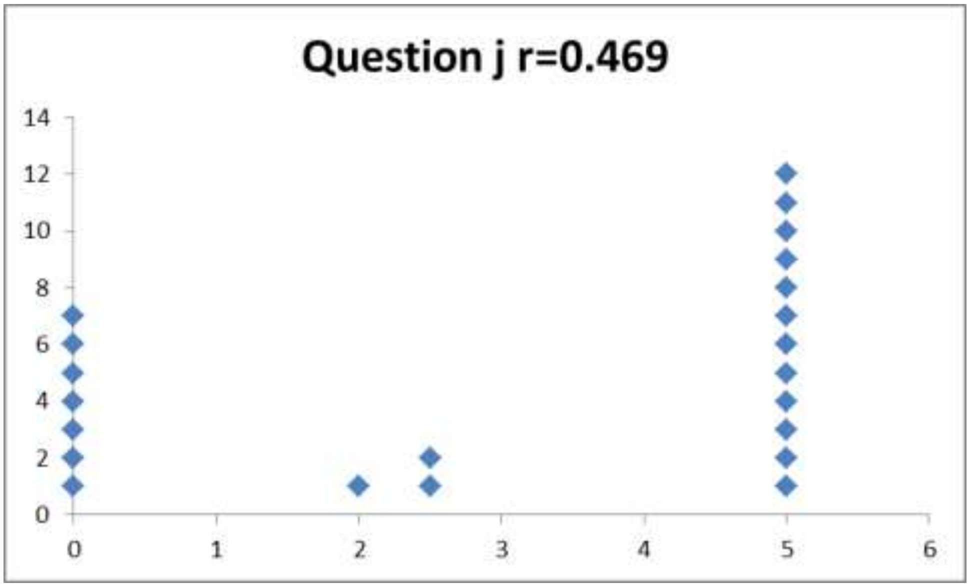
(Assuming $b \neq -\frac{1}{2}$. Extra credit would have been awarded had anyone pointed this out … but nobody did ☹)



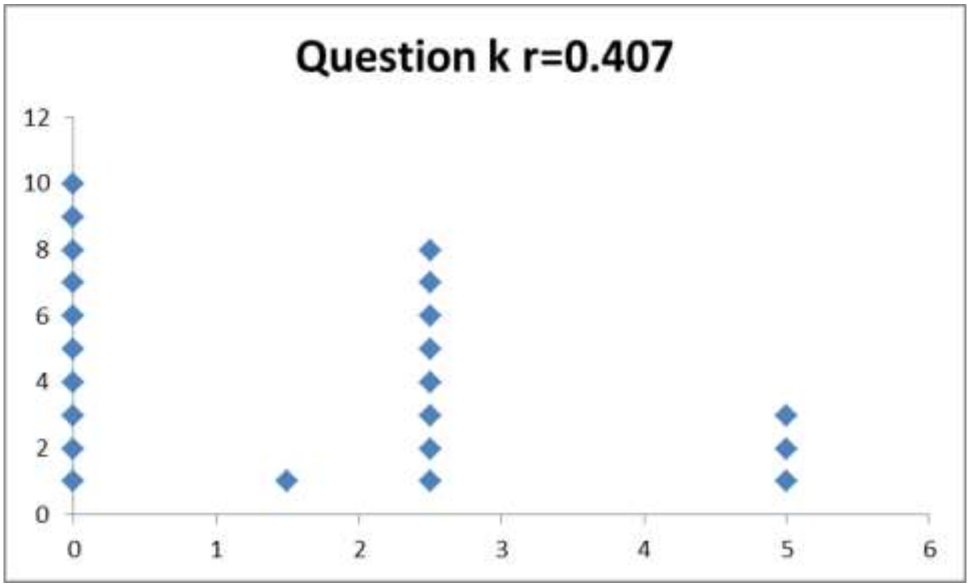
i) $\lim_{x \to a} \frac{x^4+2x}{5} = \frac{a^4+2a}{5}$



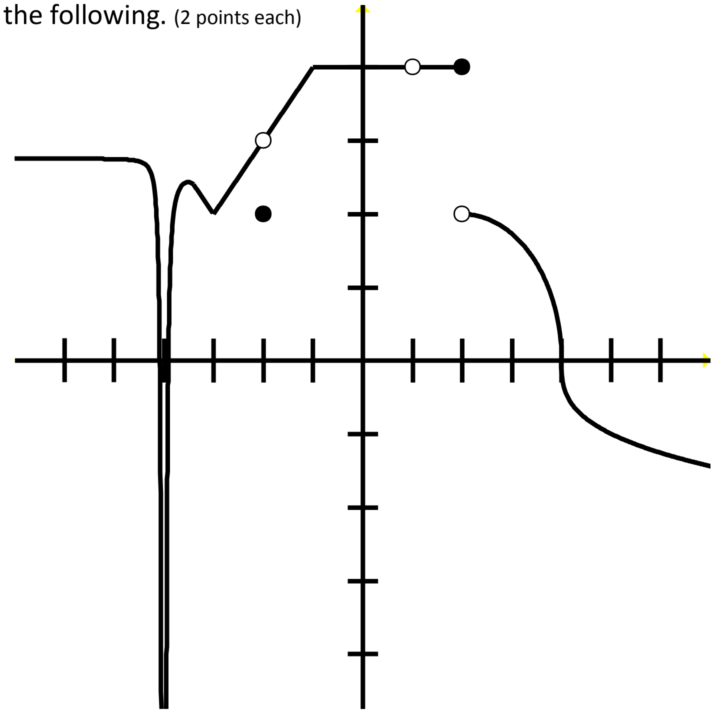
j) $\lim_{x \to -\infty} \frac{x^2+\sin(x)}{x^2-\cos(x)} = 1$



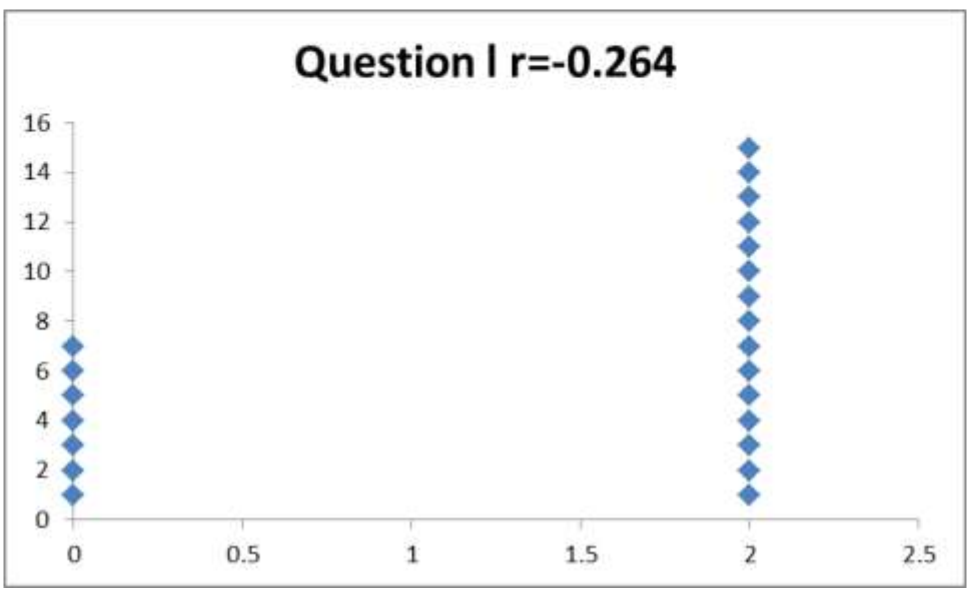
k) $\lim_{x\to\infty} \sqrt{x} - \sqrt{x-1} = \lim_{x\to\infty} \left(\sqrt{x} - \sqrt{x-1}\right) \cdot \frac{(\sqrt{x}+\sqrt{x-1})}{(\sqrt{x}+\sqrt{x-1})} = \lim_{x\to\infty} \frac{x-(x-1)}{\sqrt{x}+\sqrt{x-1}} = \lim_{x\to\infty} \frac{1}{\sqrt{x}+\sqrt{x-1}} = 0$



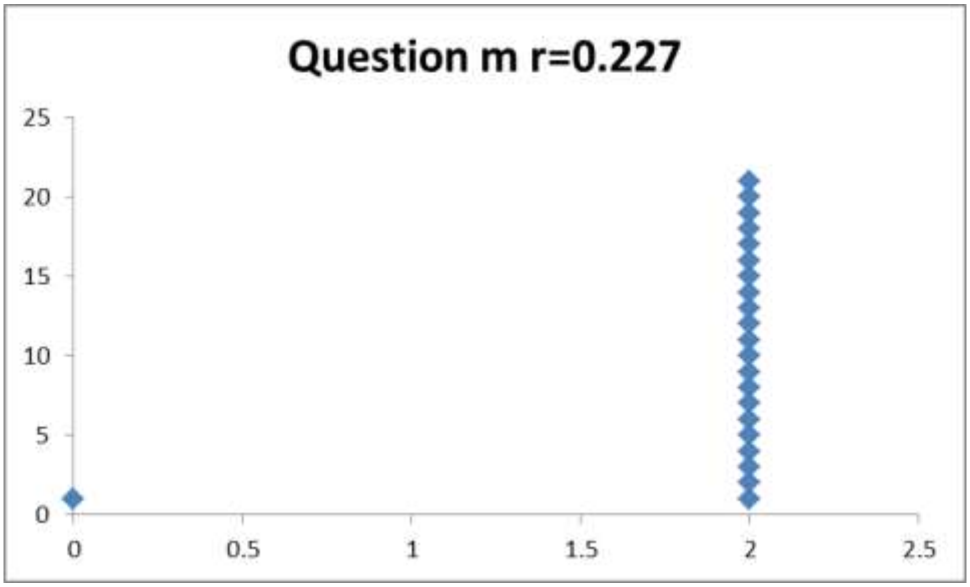
5) Use the graph of $y = f(x)$ below to find each of the following. (2 points each)



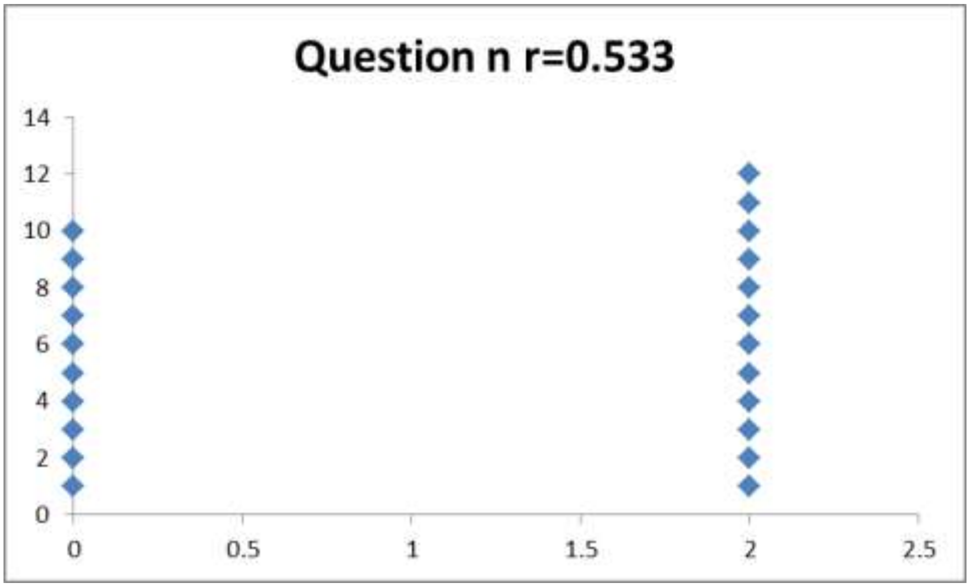
I) $\lim_{x \to -4^+} f(x) = -\infty$



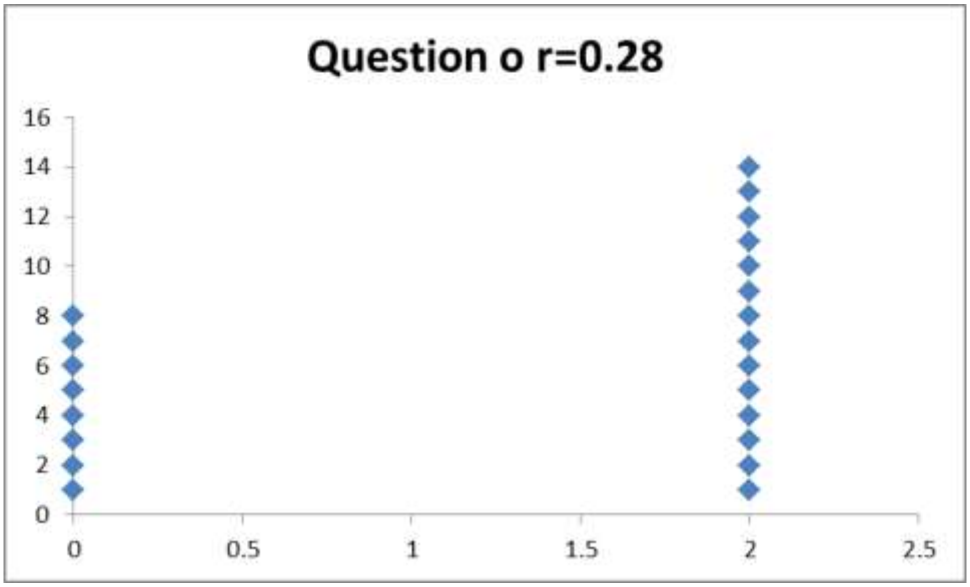
m) $\lim_{x \to -3^-} f(x) = 2$



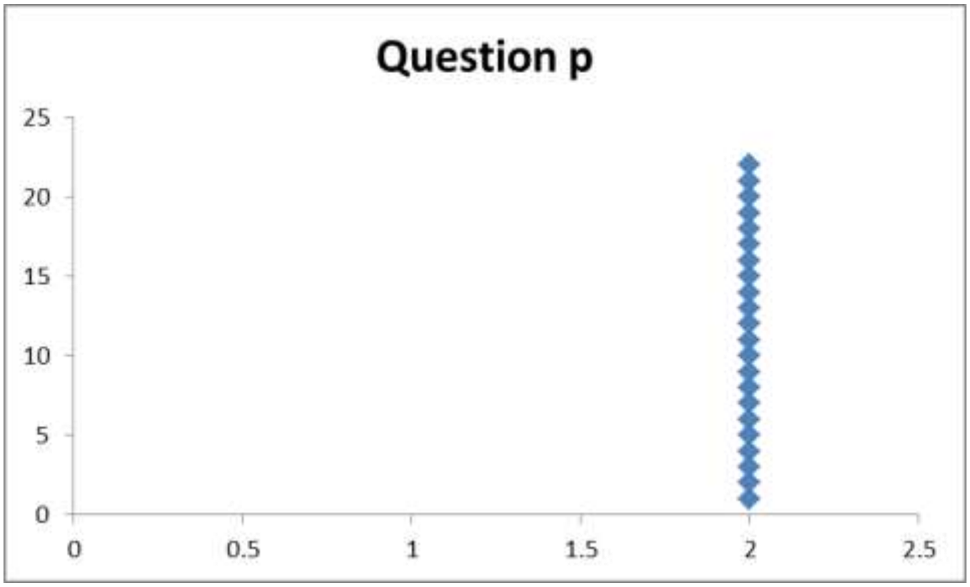
n) $\lim_{x \to -2} f(x) = 3$



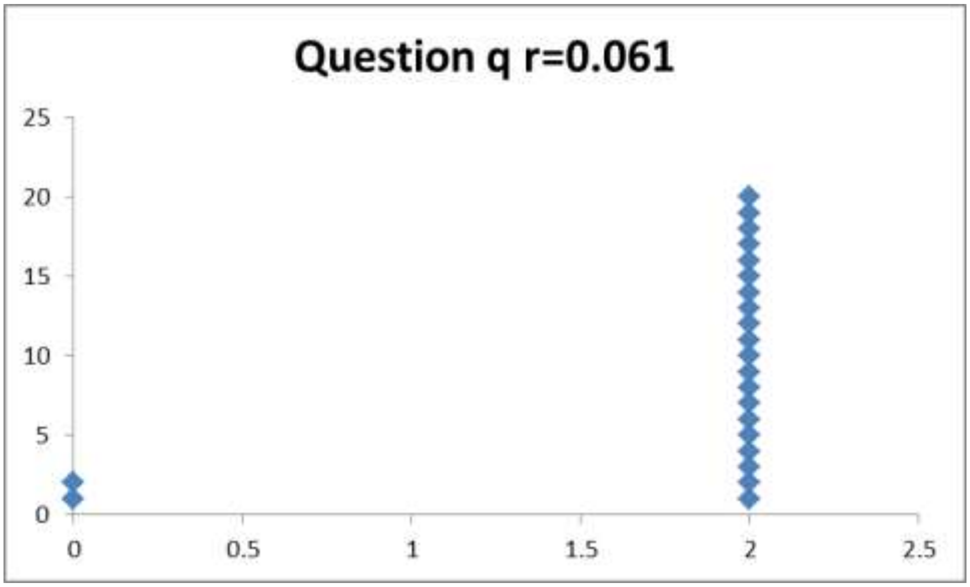
o) $\lim_{x \to 1} f(x) = 4$



p) $\lim_{x \to 2} f(x)$ DNE (Does not exist)

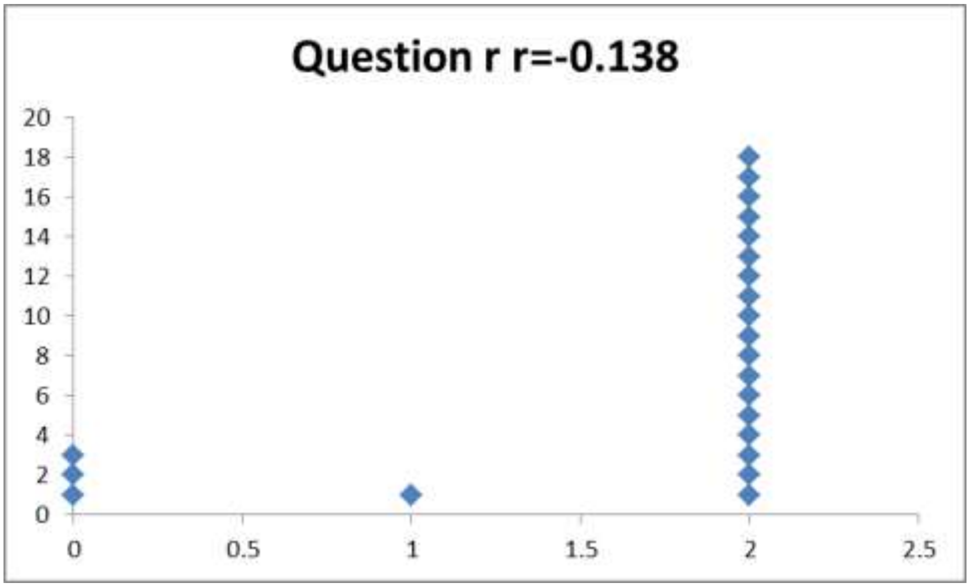


q) $\lim_{x \to 2^-} f(x) = 4$



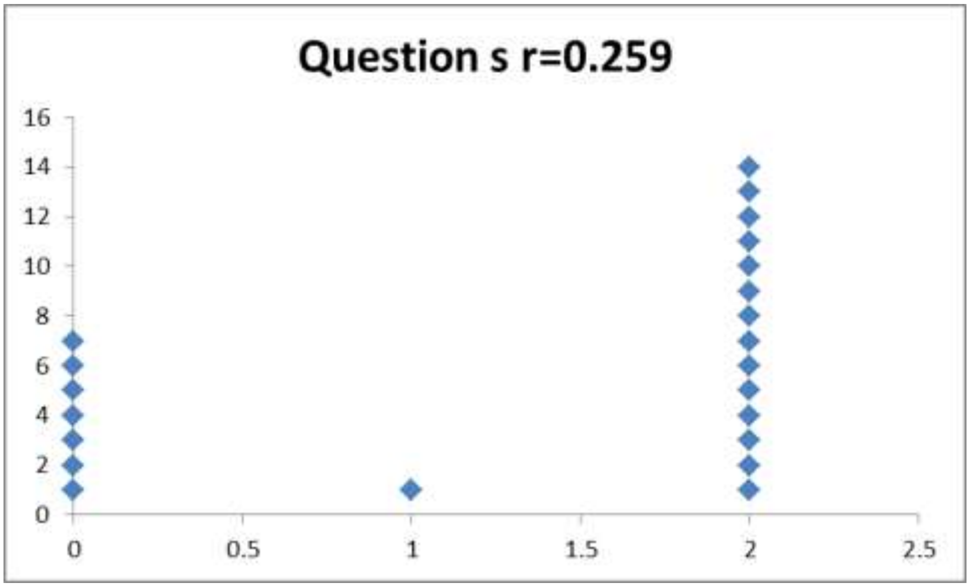
r) A vertical asymptote

$x = -4$



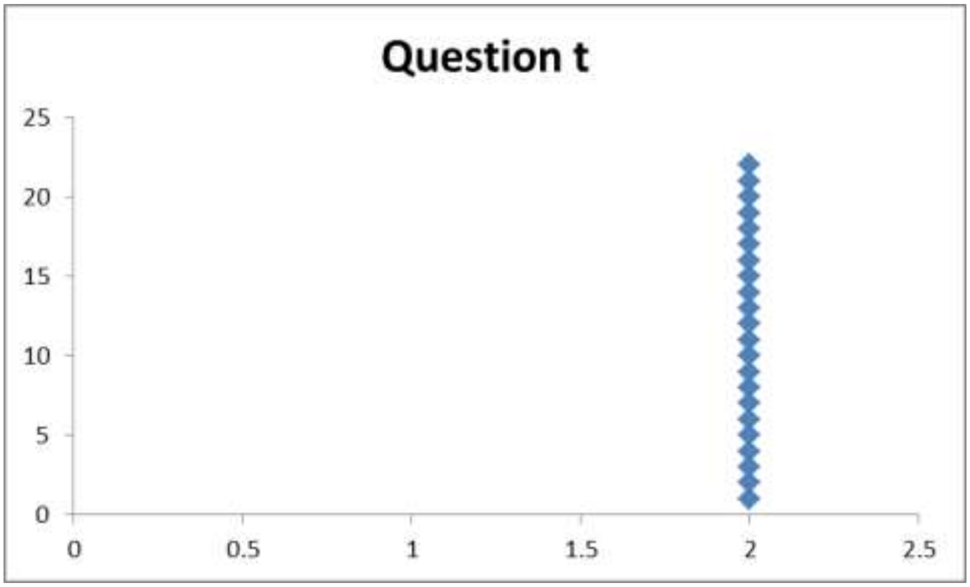
s) A horizontal asymptote

$y = 3$



t) An x-value where $f(x)$ is not continuous.

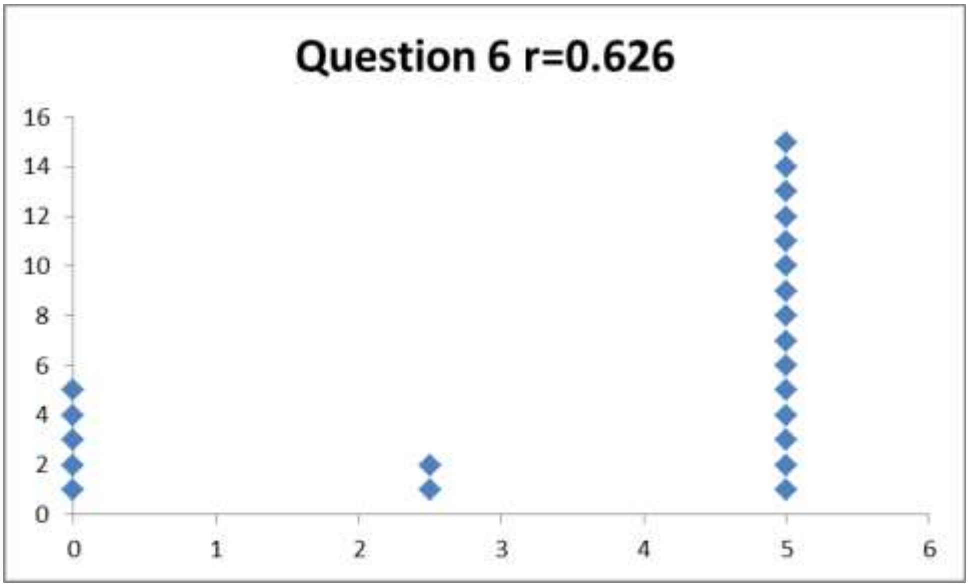
$x = -4$, or any of $-2, 1, 2$.



Technology portion: After you tear off and turn in the non-calculator portion, you may take out your technology and finish this portion. Again, please circle your answer.

6) Find $\lim_{x \to \infty} \frac{e^{2x}}{3^x} = \infty$

(5 points)

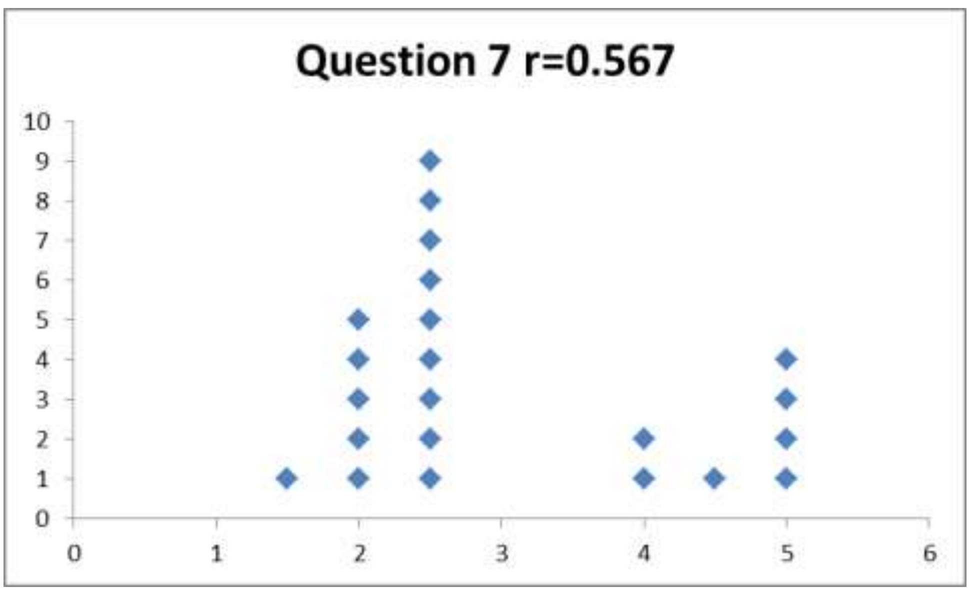


7) A projectile is shot out of a bunker. Its distance from the bunker is given by $y = 1000\ln(1+10^6 x)$ where x is the time since the shot was fired, measured in seconds and y is measured in feet. Estimate the instantaneous velocity of the projectile after 5 seconds. (5 points)

x	$\frac{f(x)-f(5)}{x-5}$
4	223.14
4.5	210.72
4.9	202.03
4.99	200.20
4.999	200.012
4.9999	200.0012

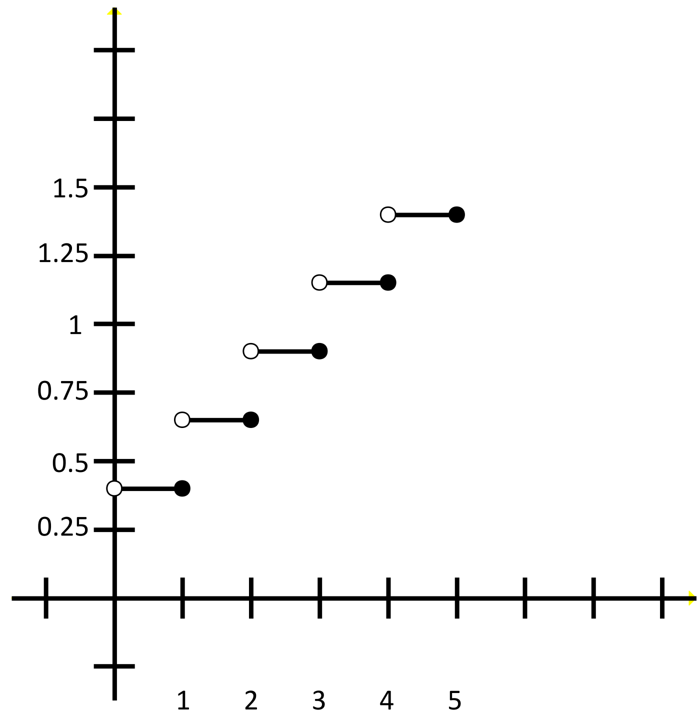
Based on the table above I estimate that the limit is 200 feet per second.

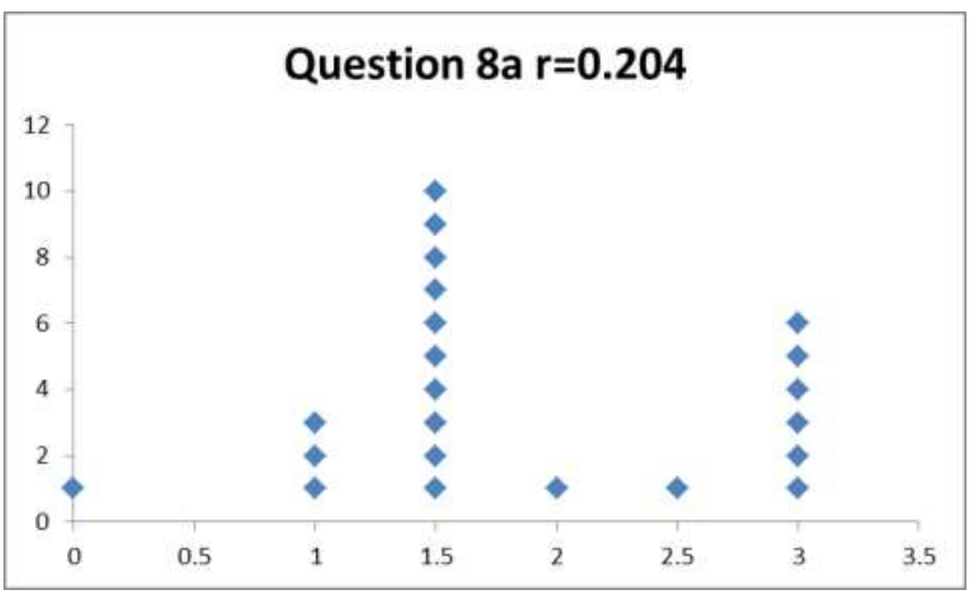
(Interestingly enough, our estimate is not quite accurate. The true value, as we'll be able to figure out in the coming weeks, is exactly 199.99996 feet per second.)



8) Assume that postage for sending a first-class letter is $0.40 for the first ounce, plus $0.25 for each additional ounce. Postage is not prorated, meaning for instance a 1.5 ounce letter requires the same postage as a 2 ounce letter.

a) Graph the function $p = f(w)$ that gives the postage p for sending a letter that weighs w ounces, for $0 < w \le 5$. (3 points)

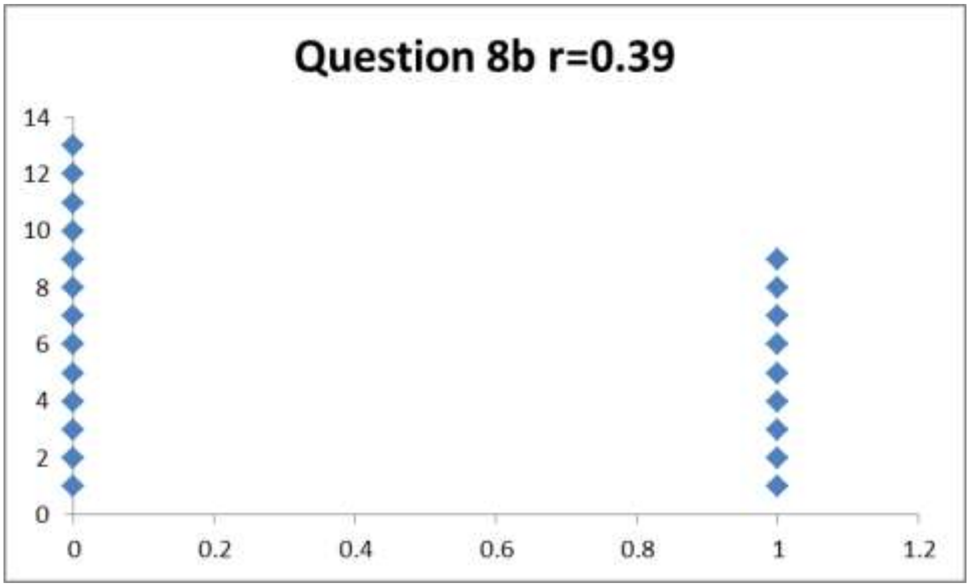




b) Find $\lim_{w \to 3.3} f(w) = 0.4 + 0.25 \cdot 3 = 1.15$

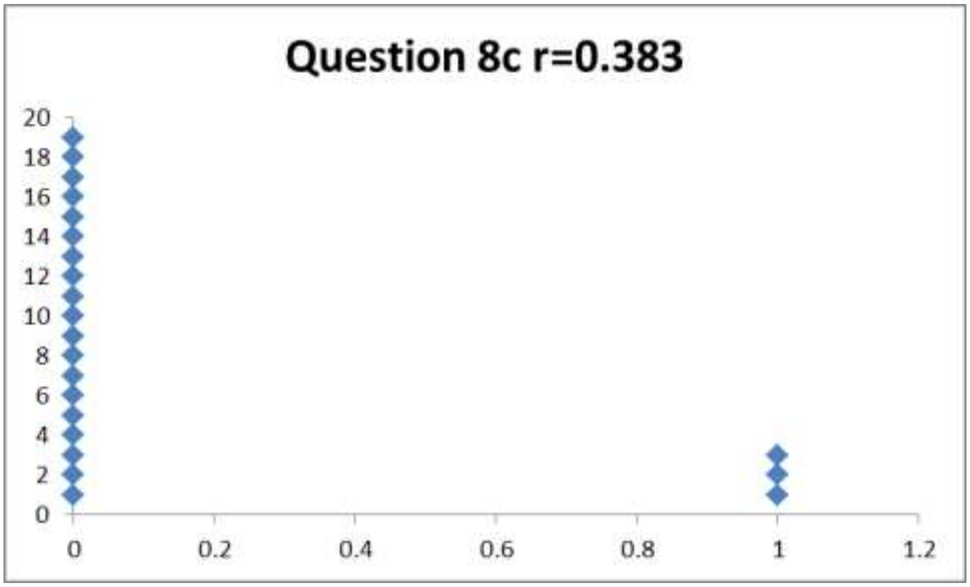
(1 point)

$1.15



c) Find $\lim_{w \to 4} f(w)$ DNE (Does not exist)

(1 point)



Mathematical Grammar: Points taken off for notational, poor illustration of work, etc:

These were marked in pink
Instead of taking points off of the problem itself, they're pooled together with a cap at -5% – because you might make the same mistake on a dozen problems, such as not writing limit or equating numbers and variables.

However, that cap will increase for subsequent tests:

- 5% cap on test 1
- 10% cap on test 2
- 15% cap on test 3
- 20% cap on the final exam

In particular, if you're adding up your grade to verify the final score, add up the points for all the problems, and then subtract the points deducted in pink – but don't subtract more than 5 even if more are marked.