Name ___Solutions____ Calculus I, Test 2, Fall 2016

Non-calculator portion. Please show all your work and circle your answer when appropriate. You do not need to simplify answers unless the problem specifies to do so.

1) Find the derivative of $f(x) = \frac{x^3 - x}{x}$. (4 points)

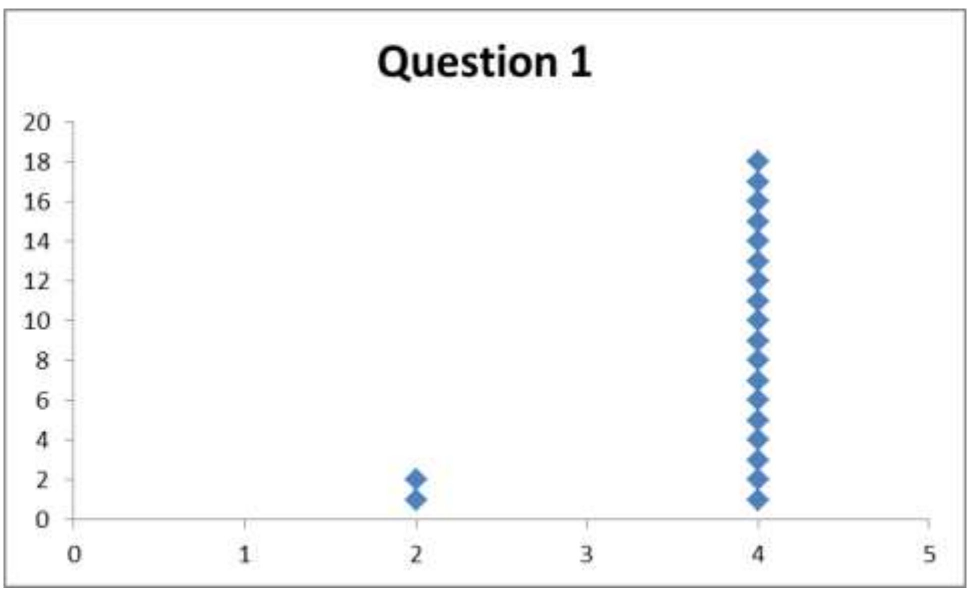
If you use quotient rule:

$$f'(x) = \frac{(3x^2 - 1)x - (x^3 - x)}{x^2}$$

If you simplified first:

$$f(x) = x^2 - 1$$
$$f'(x) = 2x$$

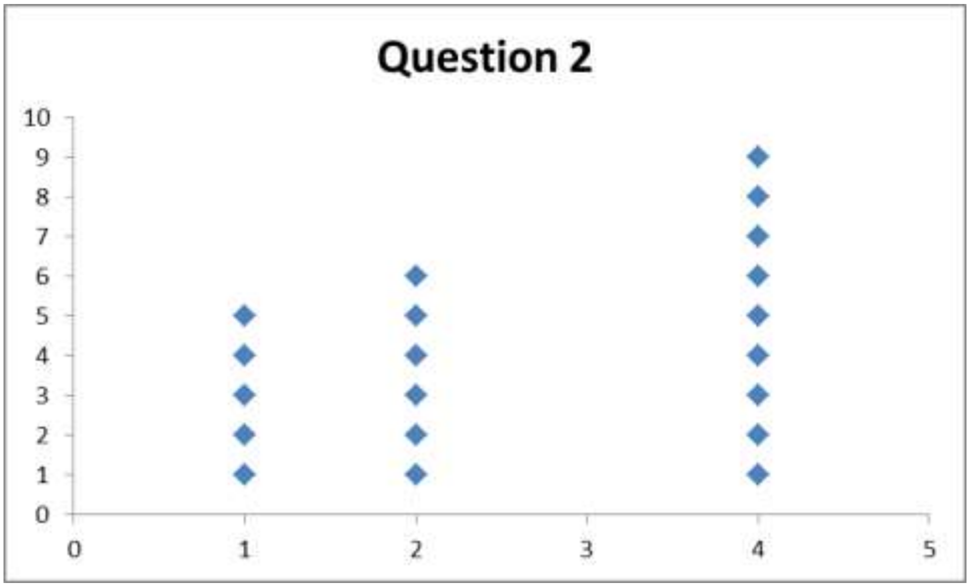
That would be accepted for full credit. However, 2 bonus points were awarded if you also noticed that $x \neq 0$.



2) Find the derivative of $f(x) = e^3$. (4 points)

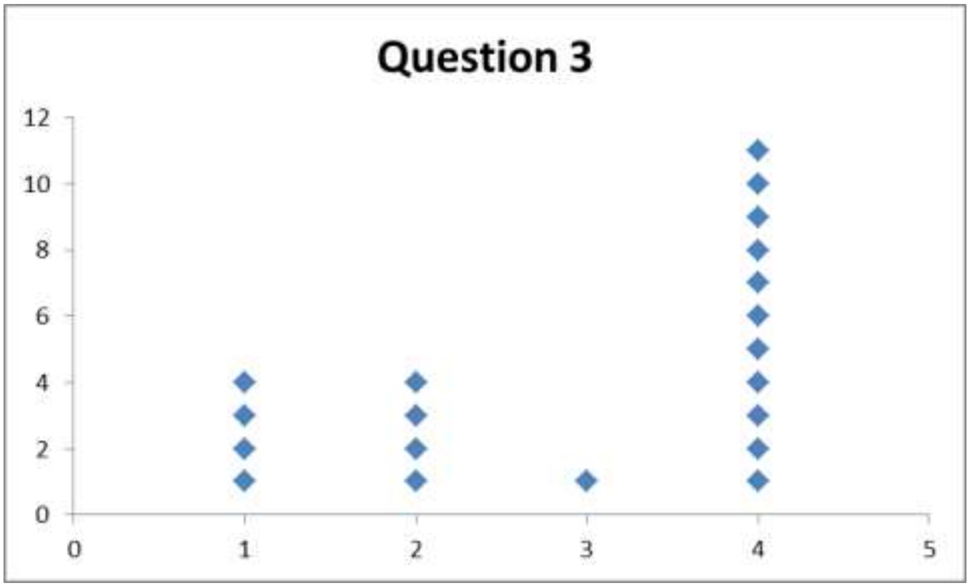
Note that e^3 is a constant, and the derivative of a constant is zero. Hence:

$$f'(x) = 0$$



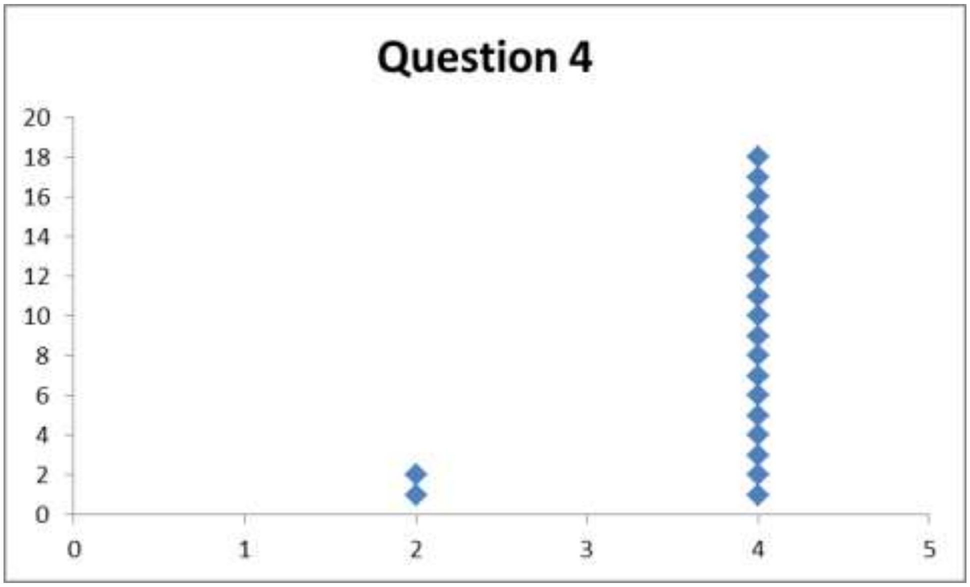
3) Find the derivative of $f(x) = 2^{\sin(x)}$. (4 points)

$$f'(x) = 2^{\sin(x)} \ln(2) \cos(x)$$



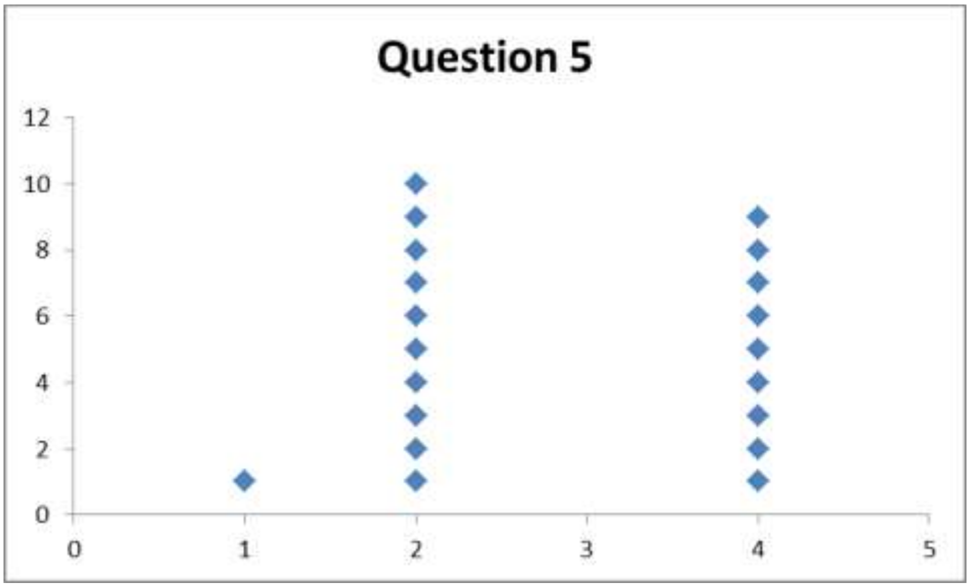
4) Find the second derivative of $f(x) = x^5 + 6x^2 + 7$. (4 points)

$$f'(x) = 5x^4 + 12x$$
$$f''(x) = 20x^3 + 12$$



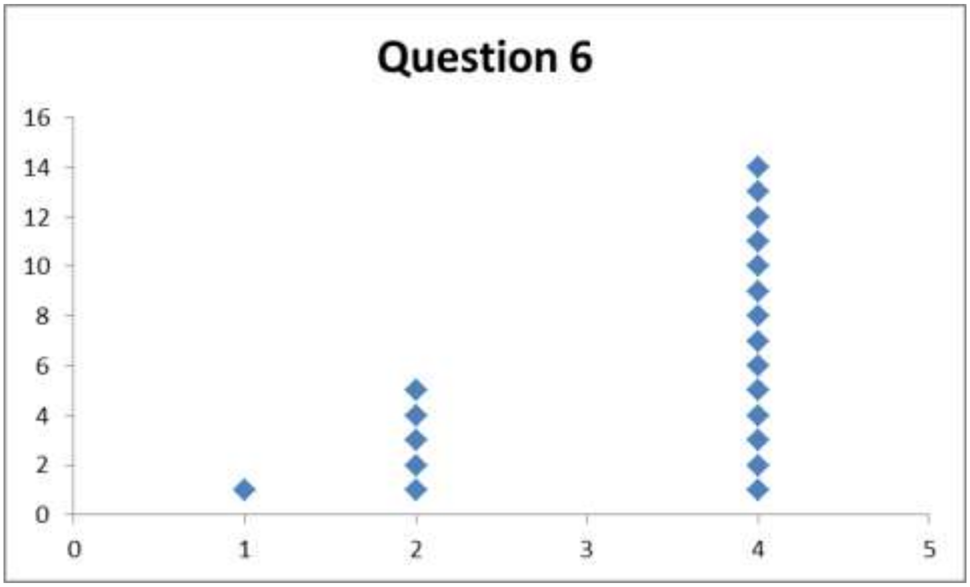
5) Find the 12th derivative of $f(x) = e^{2x}$. (4 points)

$$f'(x) = 2e^{2x}$$
$$f''(x) = 2^2 e^{2x}$$
$$\vdots$$
$$f^{(12)} = 2^{12} e^{2x}$$



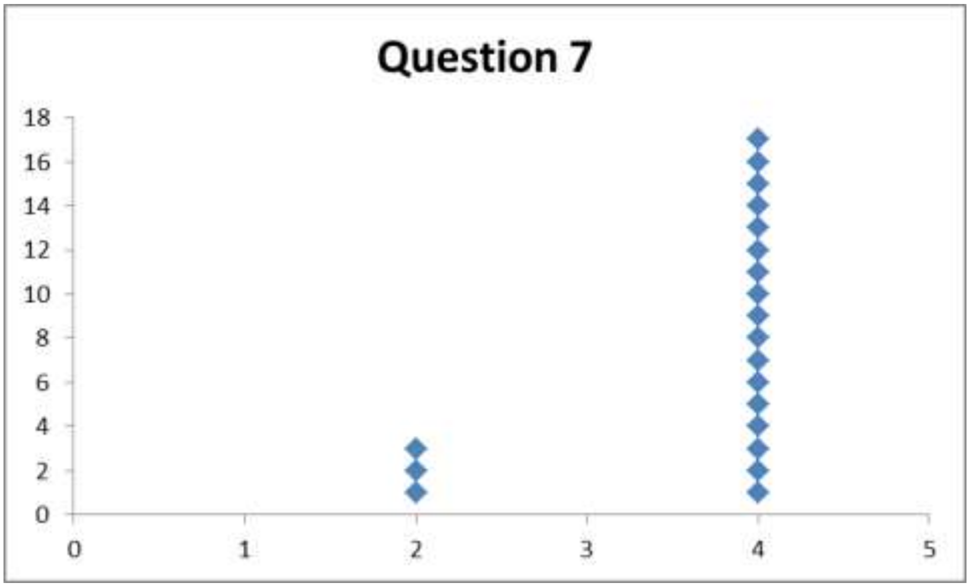
6) Find the derivative of $f(x) = \cos^3(x)$. (4 points)

$$f'(x) = -3\cos^2(x)\sin(x)$$



7) Find the derivative of $f(x) = x^5 \tan(x)$. (4 points)

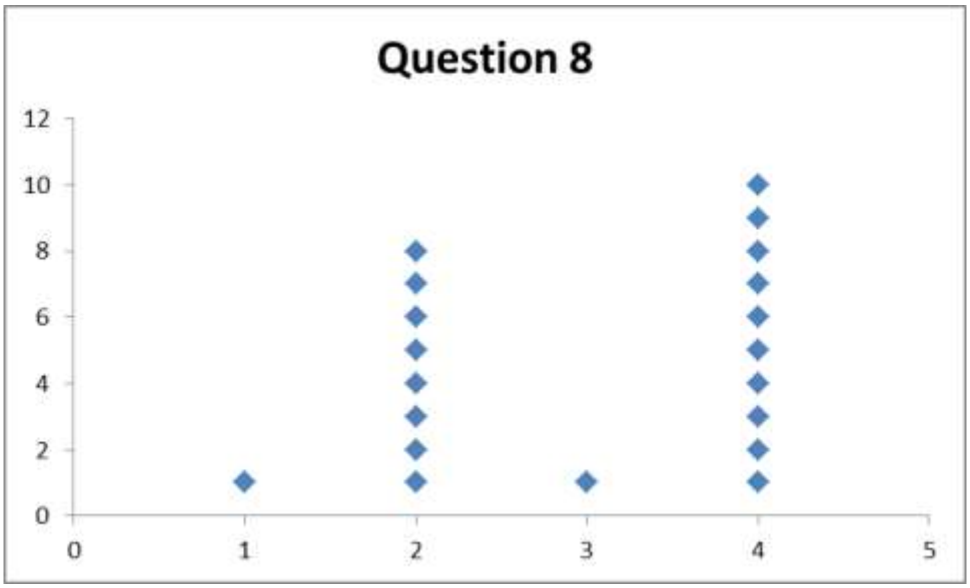
$$5x^4 \tan(x) + x^5 \sec^2(x)$$



8) Given the function $f(x) = \frac{x^2}{3^x}$, find $f'(5)$. (4 points)

$$f'(x) = \frac{2x3^x - x^2 3^x \ln(3)}{(3^x)^2}$$

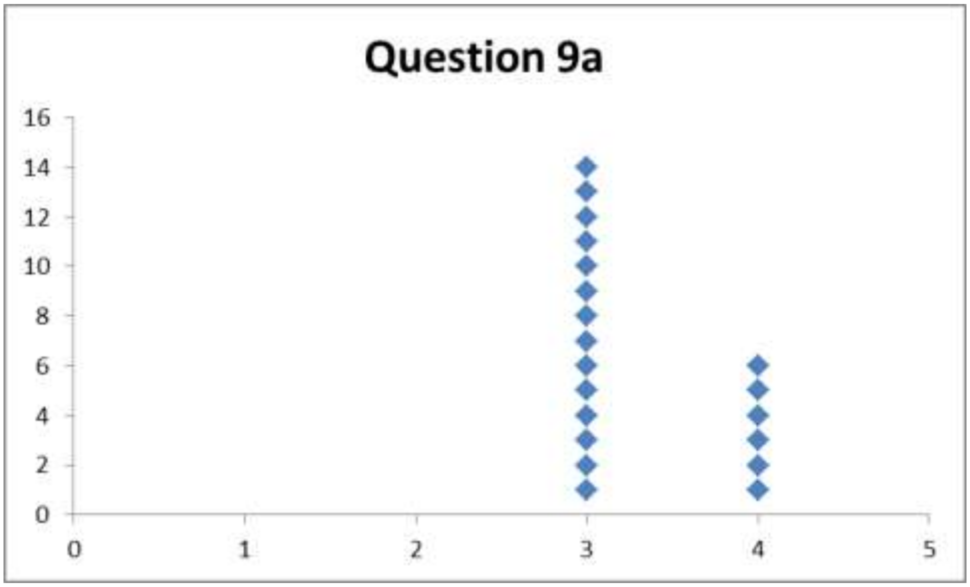
$$f'(5) = \frac{10 \cdot 3^5 - 5^2 3^5 \ln(3)}{(3^5)^2}$$



9) An object is moving horizontally. The position, in feet, after t seconds is given by the function $f(t) = 2t^2 - 6t + 18$ for $0 \leq t \leq 30$.

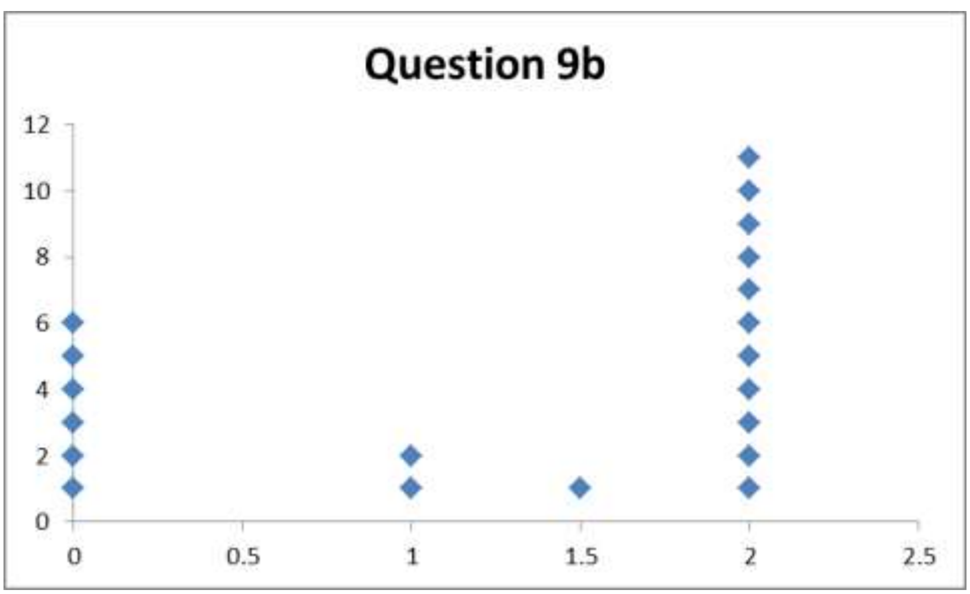
(a) Determine the velocity function of the object. (4 points)

$f'(t) = 4t - 6$ feet per second



(b) When is the velocity of the object 14 feet per second? (2 points)

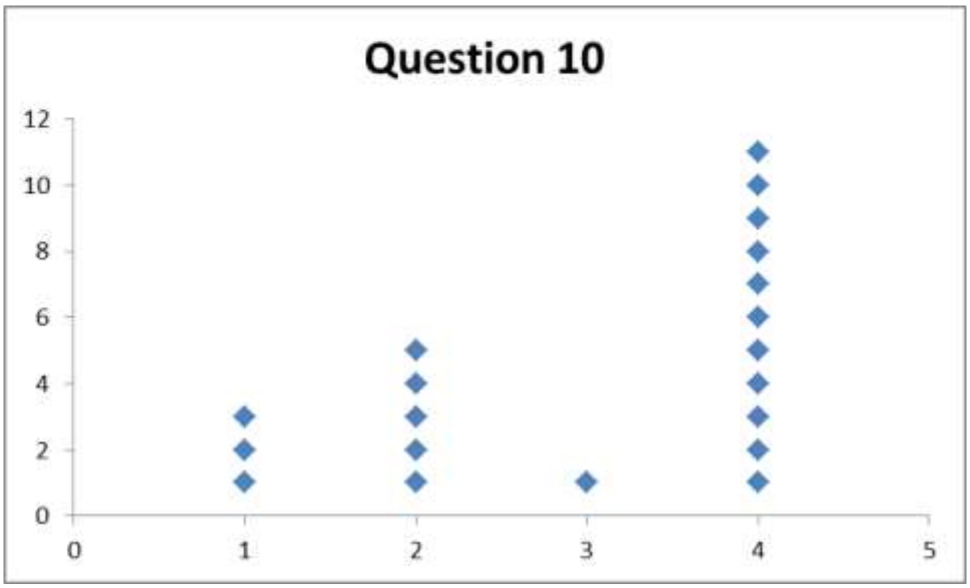
$f'(t) = 14$
$4t - 6 = 14$
$t = 5$ seconds



10) Find the following: (4 points)

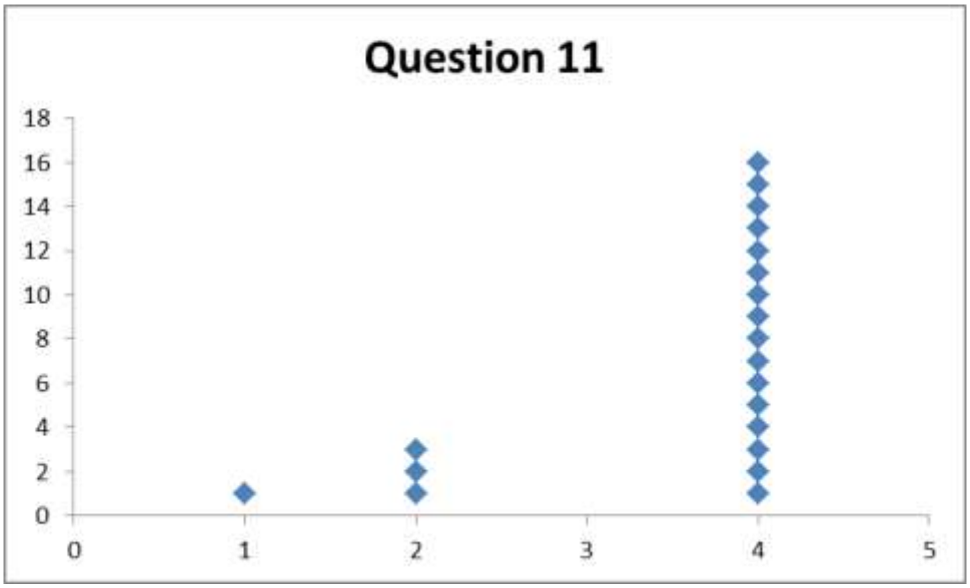
$$\frac{d}{dx}e^{e^x}$$

$$\frac{d}{dx}e^{e^x} = e^{e^x}e^x$$



11) Find the derivative of $f(x) = (2x^6 - 3x^3 + 3)^{25}$. (4 points)

$$f'(x) = 25(2x^6 - 3x^3 + 3)^{24}(12x^5 - 9x^2)$$



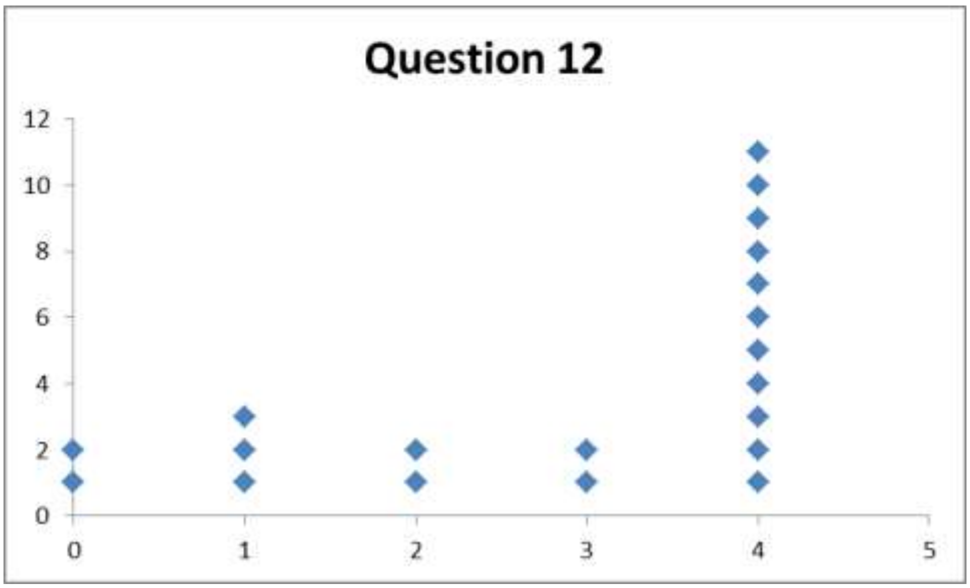
12) Use calculus to find the vertex of the parabola $y = x^2 - 6x + 18$. (4 points)

The vertex of the parabola is at its max or min (min in this case)

$$y' = 2x - 6 = 0$$
$$x = 3$$

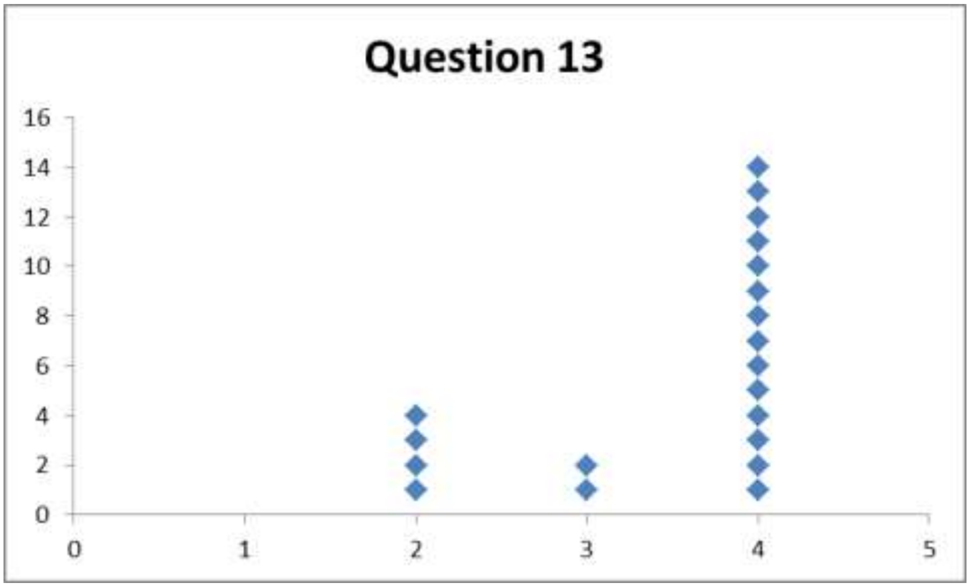
The vertex is then:

$$(3, 3^2 - 6 \cdot 3 + 18) = (3,9)$$



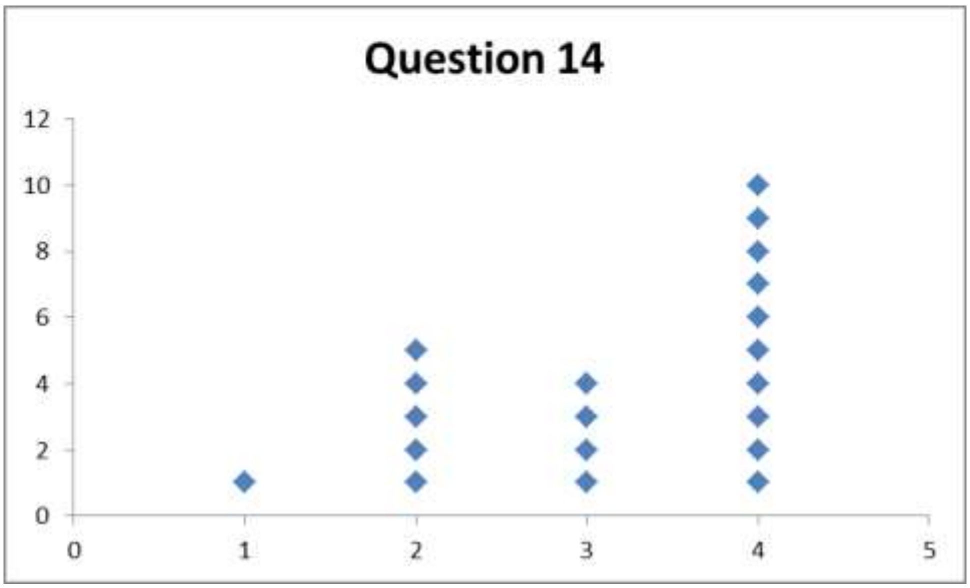
13) Given $x^2 + y^3 = 5x$, find $\frac{dy}{dx}$. (4 points)

$$2x + 3y^2 \frac{dy}{dx} = 5$$
$$\frac{dy}{dx} = \frac{5 - 2x}{3y^2}$$



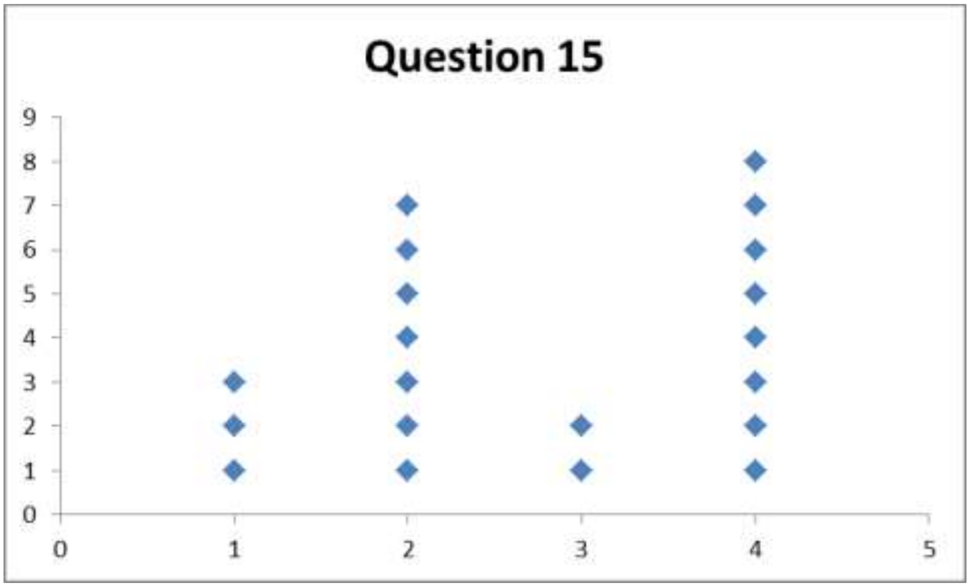
14) Given $x^2 + y^3 = 5x$, find $\frac{dx}{dy}$. (4 points)

$$2x\frac{dx}{dy} + 3y^2 = 5\frac{dx}{dy}$$
$$3y^2 = 5\frac{dx}{dy} - 2x\frac{dx}{dy}$$
$$\frac{dx}{dy} = \frac{3y^2}{5-2x}$$

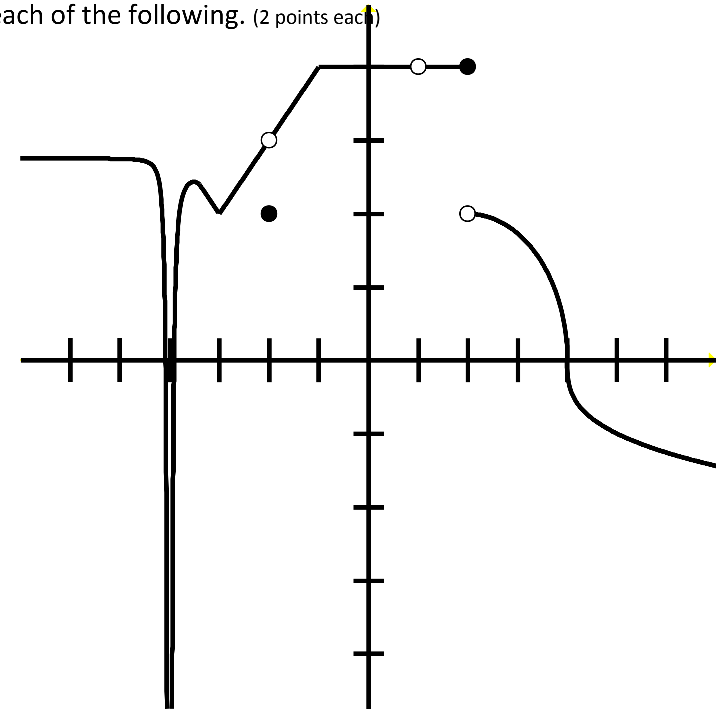


15) Find the derivative of $f(x) = \tan^{-1}(e^{4x})$. (4 points)

$$f'(x) = \frac{1}{1 + (e^{4x})^2} \cdot 4e^{4x}$$



16) Use the graph of $y = f(x)$ below to estimate each of the following. (2 points each)



a) $f'(-6)$

-0.001, or anything slightly negative.

(True answer: -0.0009)

b) $f'(-3)$

Does not exist

c) $f'(-1.5)$

1

(True answer: 1)

d) $f'(3.999)$

$-4{,}000{,}000$ or anything large and negative.

(True answer: -31.61)

e) $f''(3)$

-1 or anything negative.

(True answer: -0.769)

f) The x-value of an inflection point.

$x = 4$

g) The absolute maximum of $f(x)$.

$y = 4$

h) A maximizer corresponding to the previous answer.

Any of the many points that $f(x) = 4$ such as $x = -1, 0, 2$.

i) A relative minimum of $f(x)$ that is not the absolute maximum.

$y = 2$

j) The minimizer corresponding to the previous answer.

$x = -3$ (Or $x = -2$)

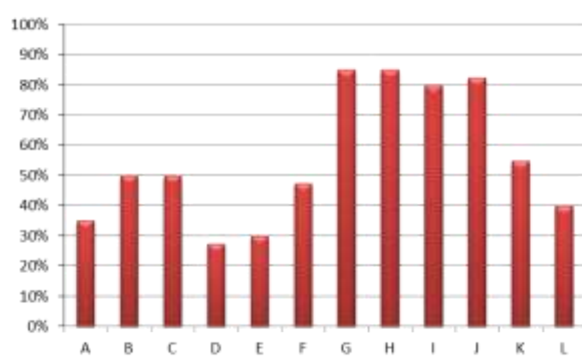
k) An interval where $f(x)$ is increasing.

There are many answers. One is $(-2, -1)$

l) An interval where $f(x)$ is concave up.

Any interval contained in $(4, \infty)$

(Technically you could also choose an interval contained in $(-3.5, -2)$, but that is kind of skirting around the idea of what concavity is, because it is piecewise linear on that interval.)



Technology portion: After you tear off and turn in the non-calculator portion, you may take out your technology and finish this portion. Again, please circle your answer.

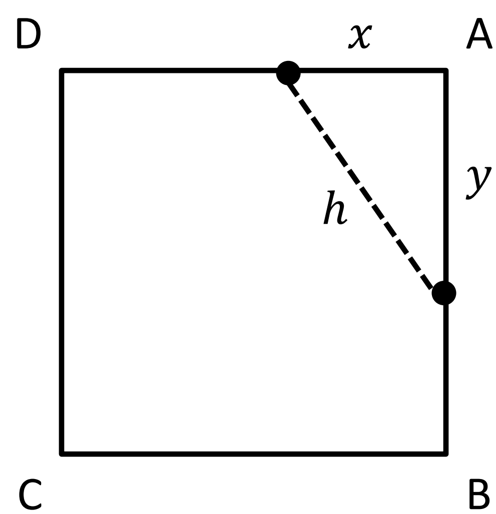
17) A 90-inch square has two bugs on it. In clockwise order, the corners are labelled A, B, C, and D. One bug is located on corner A and the other is on corner B. At exactly $2:30$ today, the bug on corner B will start walking to corner A at 20 in/min. At this same time the bug at corner A will also start walking toward corner D at 18 in/min.

How fast is the distance between the two bugs changing at 2:32? (6 points)

$x^2 + y^2 = h^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2h\frac{dh}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = h\frac{dh}{dt}$



$x = 36$

$y = 50$

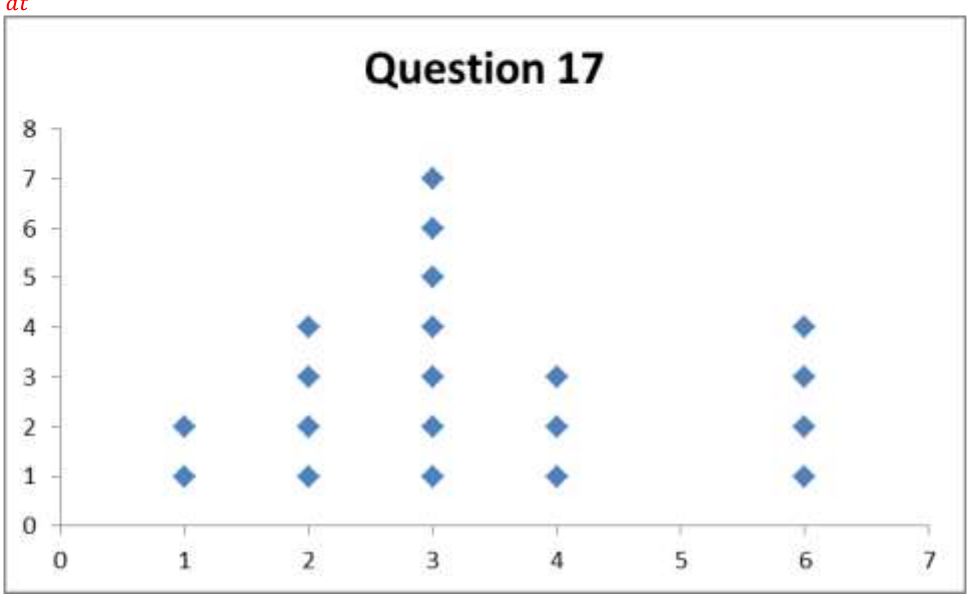
$h = \sqrt{50^2 + 36^2}$

$\frac{dx}{dt} = 18$

$\frac{dy}{dt} = -20$

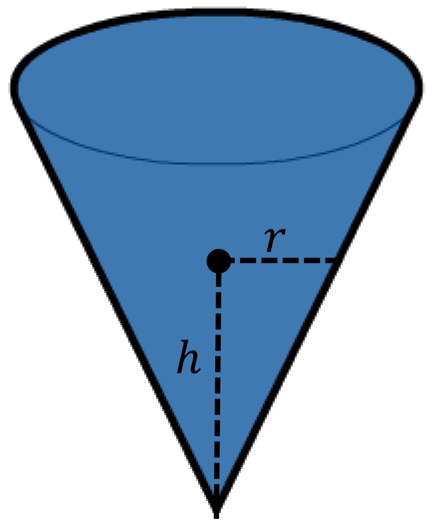
$36 \cdot 18 + 50 \cdot (-20) = \sqrt{50^2 + 36^2}\frac{dh}{dt}$

$\frac{dh}{dt} = 5.71$ in/s



18) A water tank has is shaped like an upside down cone. It's radius is 4 feet wide and it has a height of 8 feet. It's supposed to hold water, but actually there's a small hole in the bottom. Water is draining out at a rate of 2 ft^3/min. How quickly is the water level dropping when the water is 3 feet deep? (8 points)

From similar triangles we know that $\frac{r}{4} = \frac{h}{8}$, so $r = \frac{h}{2}$.



$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$
$$V = \frac{1}{12}\pi h^3$$
$$\frac{dV}{dt} = \frac{3}{12}\pi h^2 \frac{dh}{dt}$$
$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = -2$$
$$h = 3$$
$$-2 = \frac{\pi}{4} 3^2 \frac{dh}{dt}$$
$$\frac{dh}{dt} = -\frac{8}{9\pi} = -2.83\ in/s$$