Name _____ Calculus I, Test 3, Fall 2016

Please show all your work and circle your answer when appropriate. You do not need to simplify answers.

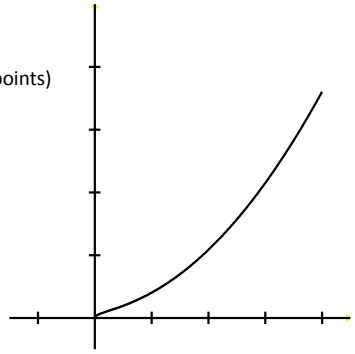
1) Find each of the integrals below. (6 points each)

$$\int_0^2 4x^3\,dx$$

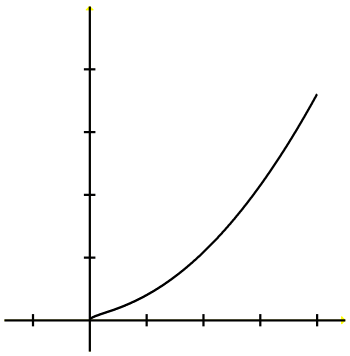
$$\int_{-2}^{-1} \frac{1}{x^3}\,dx$$

$$\int_0^{\ln(4)} \frac{e^x}{3+2e^x}\,dx$$

2) Illustrate (do not calculate) the area under the curve given below. (2 points)



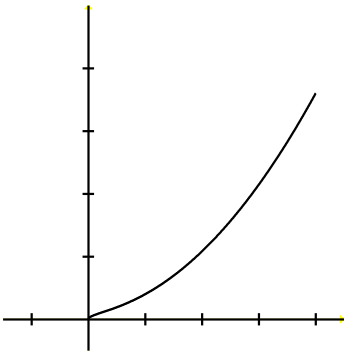
3) Illustrate (do not calculate) an approximation to the area under the curve given below. (3 points)



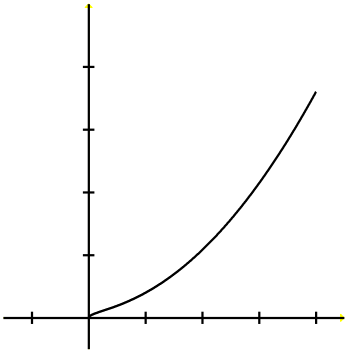
4) Calculate the approximation you illustrated in #3. (2 points)

5) Is your approximation in #3 an overestimate or an underestimate? (1 point)

6) Illustrate a better approximation than you came up with in #3. (2 points)



7) Illustrate an even better approximation than you came up with in the previous question. (2 points)



8) Write the integral below as a limit of a Riemann Sum. Do not calculate it. (4 points)

$$\int_0^1 x^2\,dx$$

9) Evaluate $\sinh(3)$. (1 point)

10) Evaluate the limit below. (4 points)

$$\lim_{x\to 0}\frac{e^x-1}{x^2+3x}$$

11) Evaluate the limit below. (4 points)

$$\lim_{x \to 0^+} (\sin(x))^{\tan(x)}$$

12) Find each of the integrals below. (6 points each)

$$\int x^3(x^4 + 16)dx$$

$$\int \sin(x)\cos(x)\,dx$$

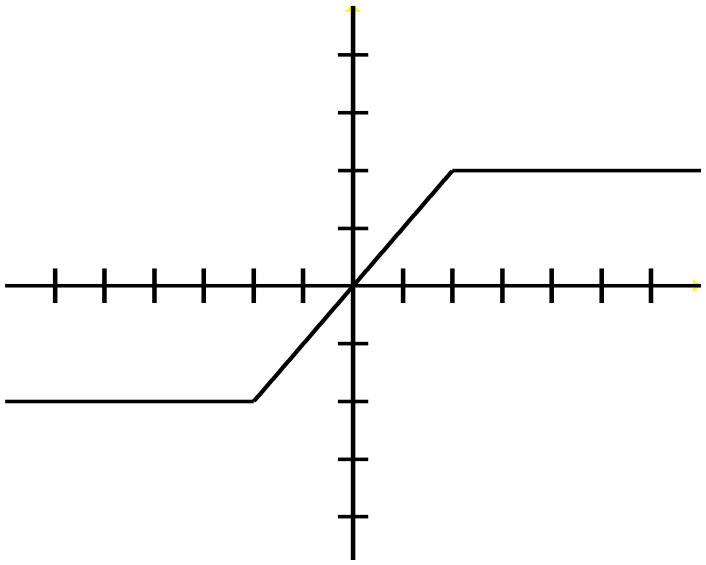
$$\int \frac{e^{2x}}{2}\,dx$$

13) Use the graph of $y = f(x)$, below, and geometry to find each of the following. (2 points each)

$\int_{-2}^{2} f(x)dx$

$\int_{2}^{5} f(x)dx$

$\int_{-2}^{0} f(x)dx$

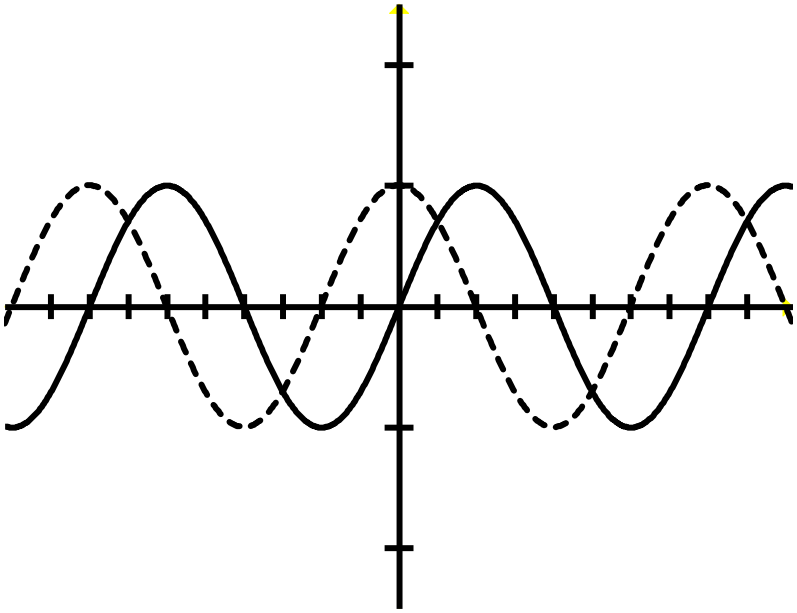


14) An entrepreneur rents batteries at Central Park to Pokémon Go players and has a profit function as given below. How many batteries should they rent to maximize their profit? The profit P is measured in dollars, while the variable b is measured in hundreds of batteries. $0 \leq b \leq 40$. (6 points)

$$P(b) = 32b - b^2$$

TAKE NOTE: The rest of the test asks you to set up some integrals. Do not calculate them.

15) Shade in and set up the integral to find the area between $y = \sin(x)$ and $y = \cos(x)$ on $\left[0, \frac{\pi}{4}\right]$.
(5 points)

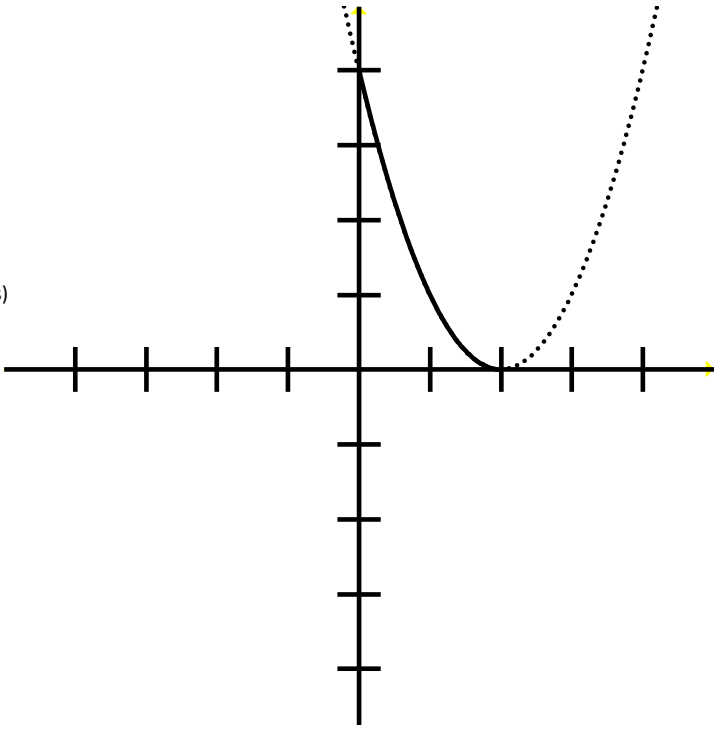


16) Set up the integral to find the arc length of $y = \sin(x)$ between $x = 0$ and $x = \pi$. (4 points)

17) The function $y = x^2 - 2x - 3$ with $0 \leq x \leq 2$ will be rotated around the y-axis.

(a) Describe the shape created. (2 points)

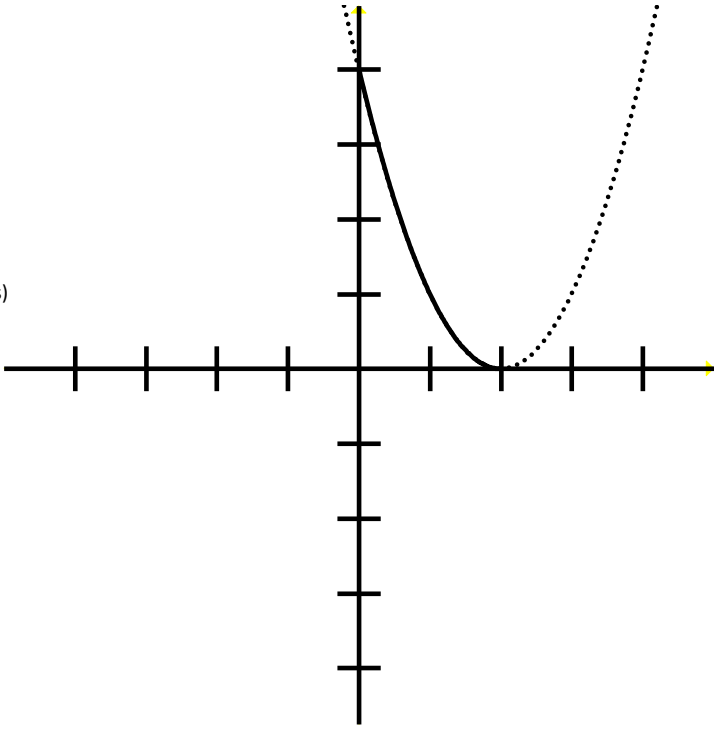
(b) Set up an integral to find the volume. (3 points)



18) The function $y = x^2 - 2x - 3$ with $0 \leq x \leq 2$ will be rotated around the x-axis.

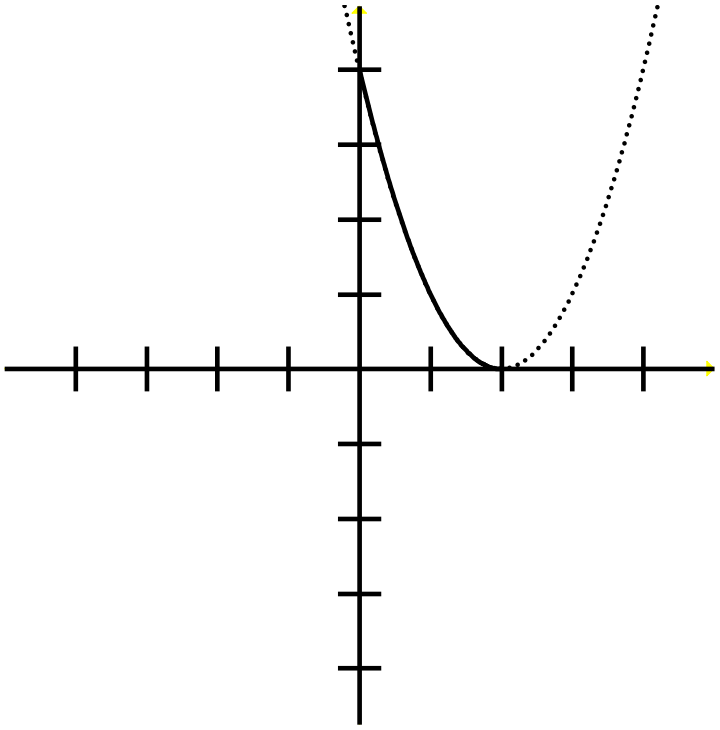
(a) Describe the shape created. (2 points)

(b) Set up an integral to find the volume. (3 points)



19) The function $y = x^2 - 2x - 3$ with $0 \leq x \leq 2$ will be rotated around the y-axis.

(a) Set up an integral to find the surface area, ignoring the base. (4 points)



20) The function $y = x^2 - 2x - 3$ with $0 \leq x \leq 2$ will be rotated around the x-axis.

(a) Set up an integral to find the surface area, ignore the base. (4 points)