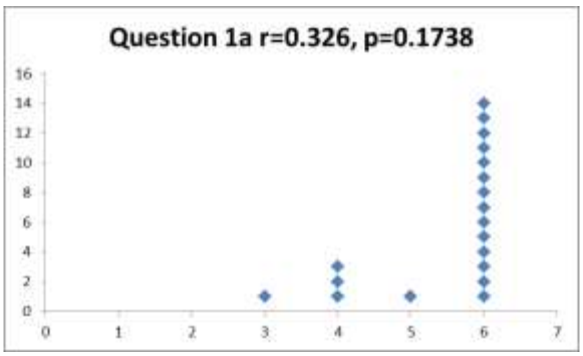
Name _____Solutions____ Calculus I, Test 3, Fall 2016

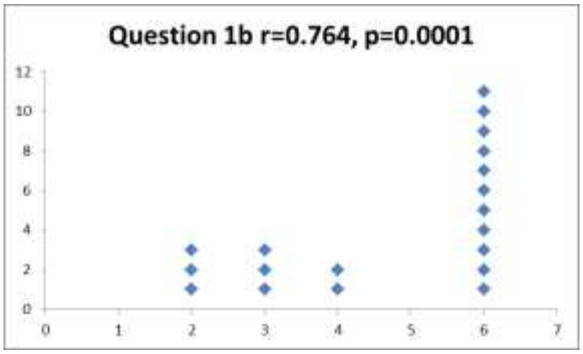
Please show all your work and circle your answer when appropriate. You do not need to simplify answers.

1) Find each of the integrals below. (6 points each)

$$\int_0^2 4x^3 dx = x^4|_0^2 = 2^4 - 0 = 16$$



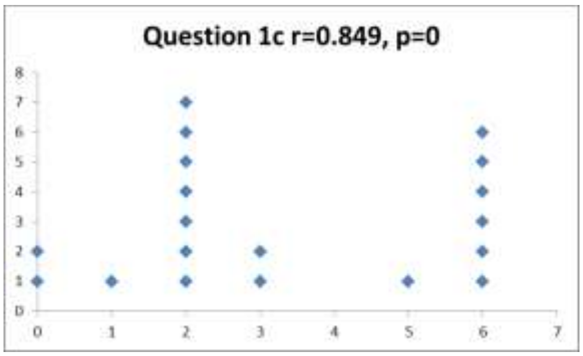
$$\int_{-2}^{-1} \frac{1}{x^3} dx = \int_{-2}^{-1} x^{-3} dx = \frac{x^{-2}}{-2}\Big|_{-2}^{-1} = -2\left(\frac{1}{x^2}\Big|_{-2}^{-1}\right) = -2\left(\frac{1}{(-1)^2} - \frac{1}{(-2)^2}\right) = -\frac{3}{8}$$



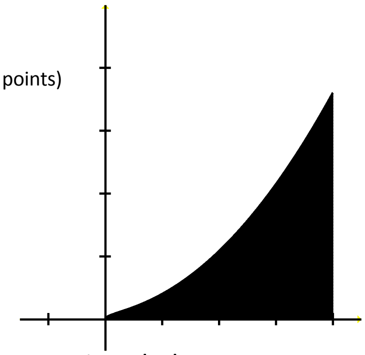
$$\int_0^{\ln(4)} \frac{e^x}{3+2e^x}dx = \frac{1}{2}\int_0^{\ln(4)} \frac{2e^x dx}{3+2e^x} = \frac{1}{2}\int_5^{11} \frac{du}{u} = \frac{1}{2}\ln(|u|)|_5^{11} = \frac{\ln(11)}{2} - \frac{\ln(5)}{2}$$

$u = 3 + 2e^x$
$du = 2e^x dx$

When $x = 0, u = 3 + 2 = 5$
When $x = \ln(4), u = 3 + 2 \cdot 4 = 11$

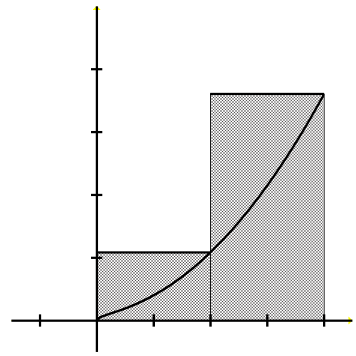


2) Illustrate (do not calculate) the area under the curve given below. (2 points)



3) Illustrate (do not calculate) an approximation to the area under the curve given below. (3 points)

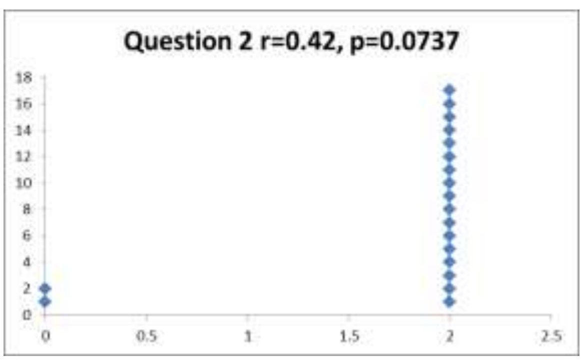
(There are multiple answers)

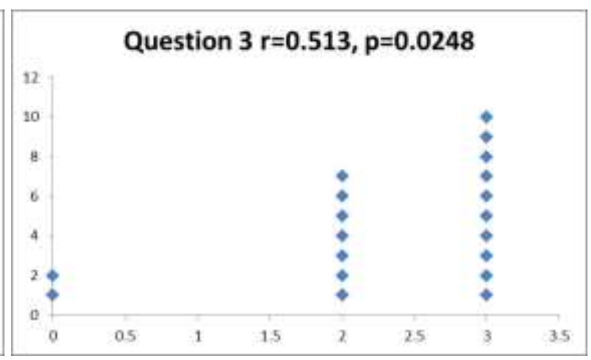


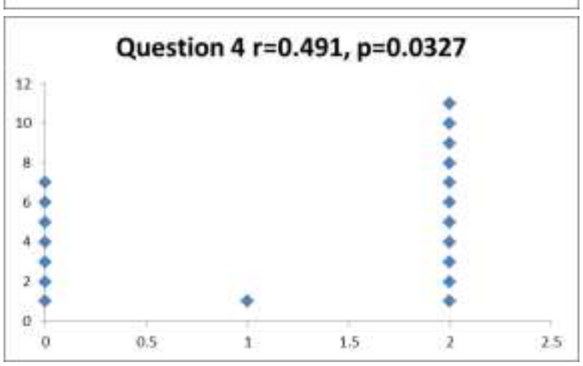
4) Calculate the approximation you illustrated in #3. (2 points)

(There are multiple answers, but each is unique and based on your answer to #3)

$1.1 \cdot 2 + 3.5 \cdot 2$



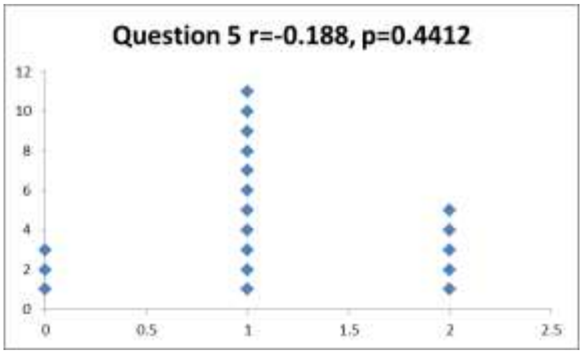




5) Is your approximation in #3 an overestimate or an underestimate? (1 point)

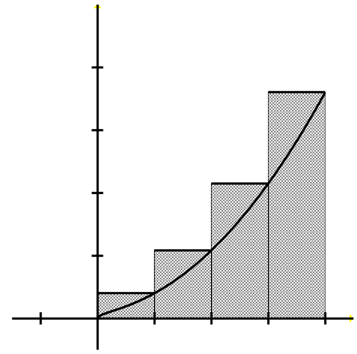
(Again based on #3)

Overestimate



6) Illustrate a better approximation than you came up with in #3. (2 points)

(Again based on #3)



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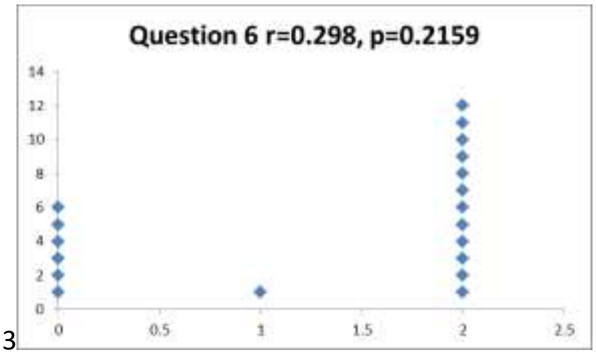
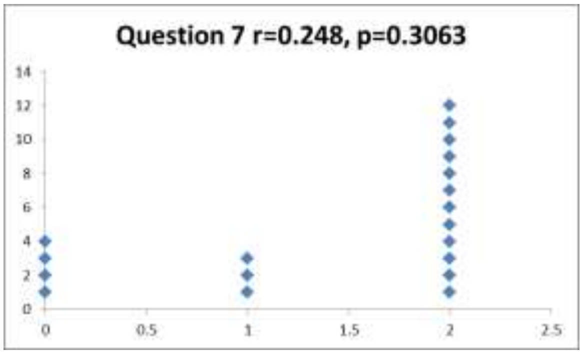
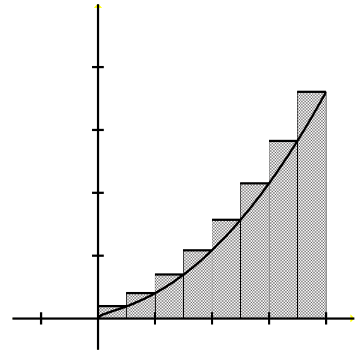


7) Illustrate an even better approximation than you came up with in the previous question. (2 points)

(Again based on #3)

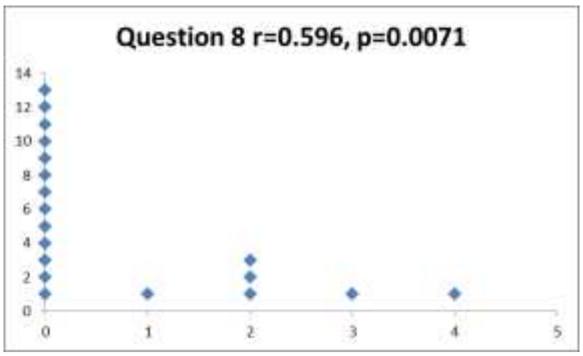




8) Write the integral below as a limit of a Riemann Sum. Do not calculate it. (4 points)

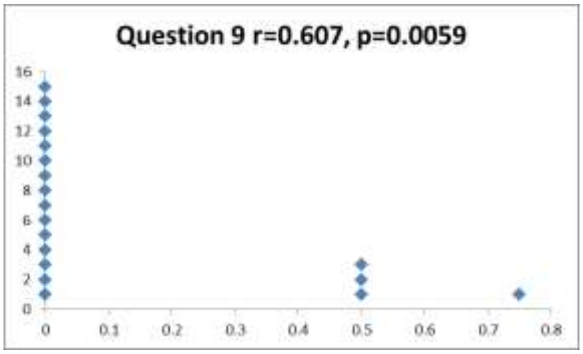
$$\int_0^1 x^2\, dx$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\left(\frac{k}{n}\right)^2$$



9) Evaluate $\sinh(3)$. (1 point)

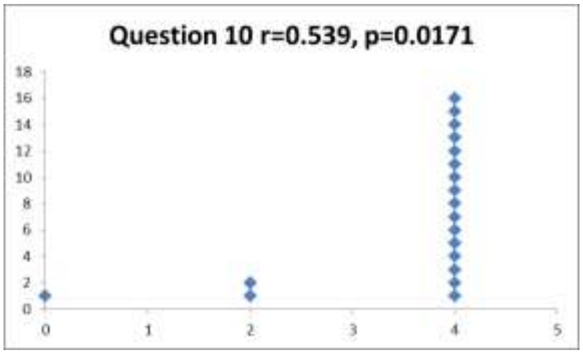
$$\frac{e^3 - e^{-3}}{2}$$



10) Evaluate the limit below. (4 points)

$$\lim_{x \to 0} \frac{e^x - 1}{x^2 + 3x}$$

$$\lim_{x \to 0} \frac{e^x - 1}{x^2 + 3x} = \lim_{x \to 0} \frac{e^x}{2x + 3} = \frac{1}{3}$$



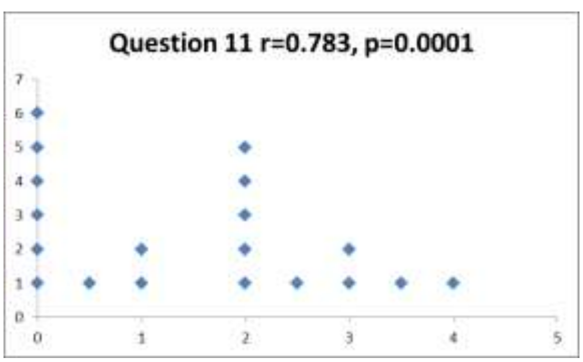
11) Evaluate the limit below. (4 points)

$$\lim_{x\to 0^+}(\sin(x))^{\tan(x)}$$

$$L = \lim_{x\to 0^+}(\sin(x))^{\tan(x)}$$

$$\begin{aligned}
\ln(L) &= \ln\left(\lim_{x\to 0^+}(\sin(x))^{\tan(x)}\right) \\
&= \lim_{x\to 0^+}\ln\left((\sin(x))^{\tan(x)}\right) \\
&= \lim_{x\to 0^+}\tan(x)\ln(\sin(x)) \\
&= \lim_{x\to 0^+}\frac{\ln(\sin(x))}{\frac{1}{\tan(x)}} \\
&= \lim_{x\to 0^+}\frac{\ln(\sin(x))}{\cot(x)} \\
&= \lim_{x\to 0^+}\frac{\frac{\cos(x)}{\sin(x)}}{-\csc^2(x)} \\
&= \lim_{x\to 0^+}\frac{\frac{\cos(x)}{\sin(x)}}{-\frac{1}{\sin^2(x)}} \\
&= \lim_{x\to 0^+}-\frac{\cos(x)}{\sin(x)}\cdot\frac{\sin^2(x)}{1} \\
&= \lim_{x\to 0^+}-\cos(x)\sin(x) \\
&= 0
\end{aligned}$$

$$L = e^0 = 1$$

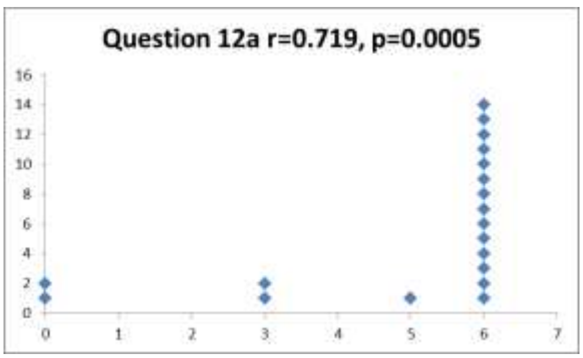


12) Find each of the integrals below. (6 points each)

$$\int x^3(x^4+16)dx = \frac{1}{4}\int (x^4+16)4x^3dx = \frac{1}{4}\int udu = \frac{1}{4}\frac{u^2}{2} + C = \frac{1}{8}(x^4+16)^2 + C$$

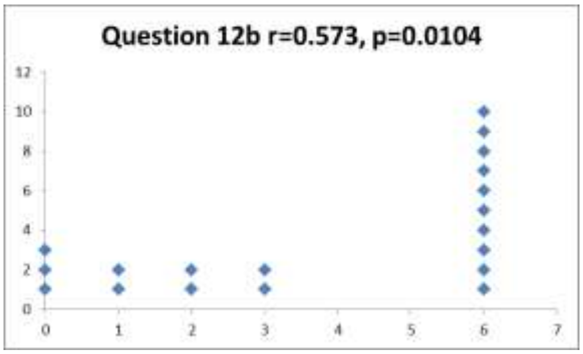
$u = x^4 + 16$

$du = 4x^3dx$



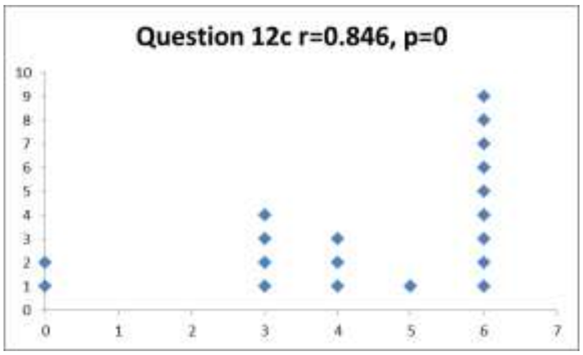
$$\int \sin(x)\cos(x)\,dx = \int u du = \frac{u^2}{2} + C = \frac{\sin^2(x)}{2} + C$$

$u = \sin(x)$
$du = \cos(x)\,dx$



$$\int \frac{e^{2x}}{2}dx = \frac{1}{2}\int \frac{e^{2x}}{2}2dx = \frac{1}{4}\int e^{2x}2dx = \frac{1}{4}\int e^{u}du = \frac{1}{4}e^{u} + C = \frac{1}{4}e^{2x} + C$$

$u = 2x$
$du = 2dx$

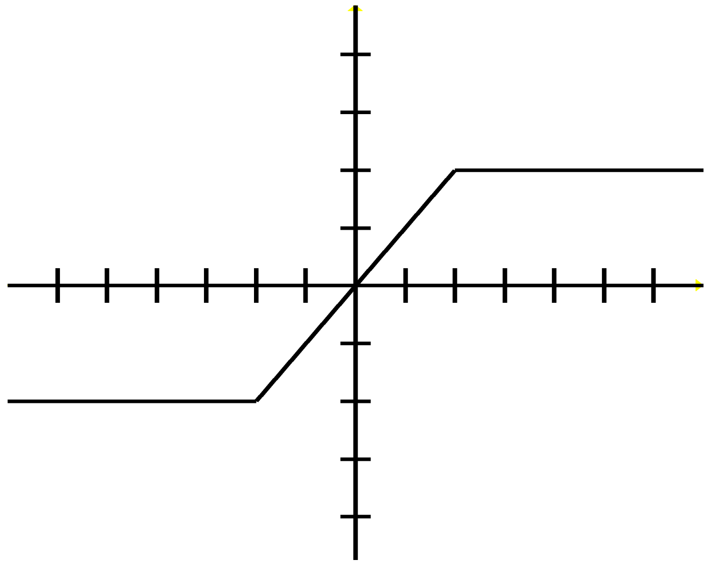


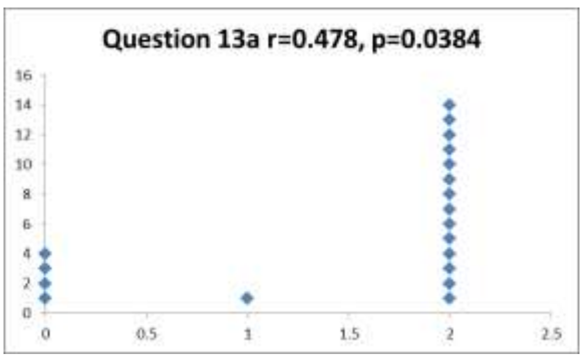
13) Use the graph of $y = f(x)$, below, and geometry to find each of the following. (2 points each)

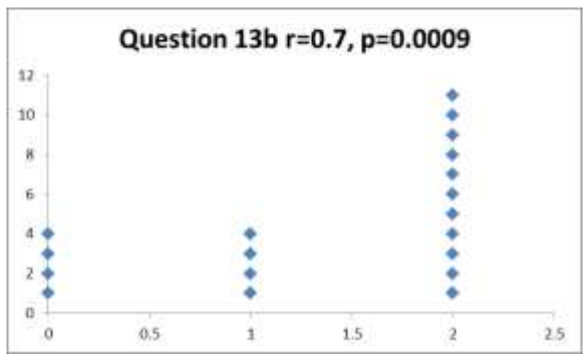
$$\int_{-2}^{2} f(x)dx = 0$$

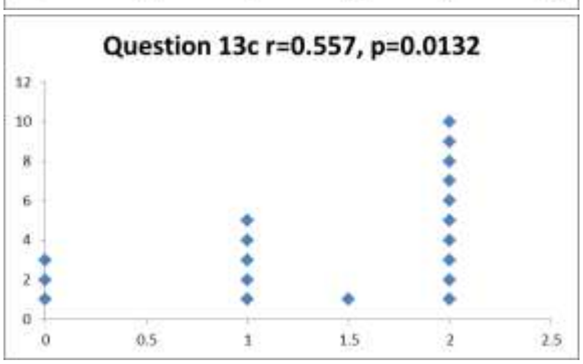
$$\int_{2}^{5} f(x)dx = 2 \cdot 3 = 6$$

$$\int_{-2}^{0} f(x)dx = -\frac{1}{2}2 \cdot 2 = -2$$





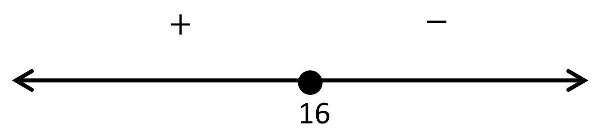




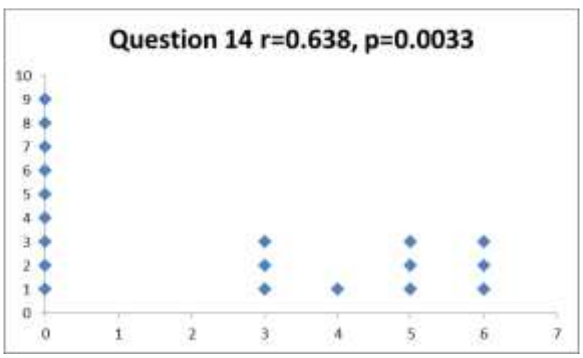
14) An entrepreneur rents batteries at Central Park to Pokémon Go players and has a profit function as given below. How many batteries should they rent to maximize their profit? The profit P is measured in dollars, while the variable b is measured in hundreds of batteries. $0 \leq b \leq 40$. (6 points)

$$P(b) = 32b - b^2$$

$$P'(b) = 32 - 2b$$
$$32 - 2b = 0$$
$$b = 16$$



The maximum value occurs when they rent 1600 batteries.

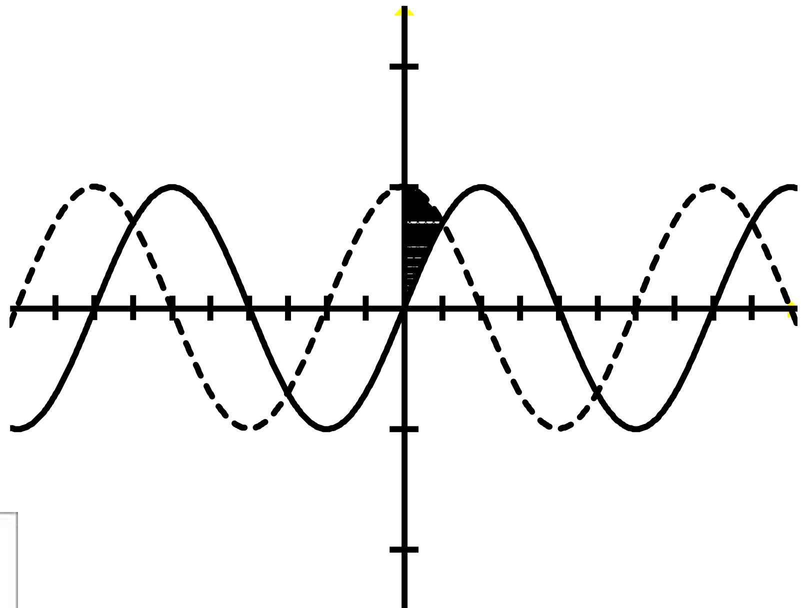


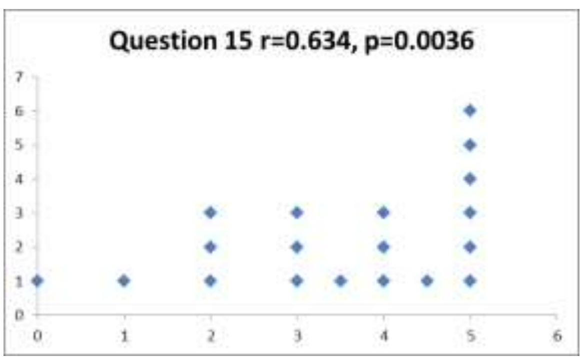
TAKE NOTE: The rest of the test asks you to set up some integrals. Do not calculate them.

15) Shade in and set up the integral to find the area between $y = \sin(x)$ and $y = \cos(x)$ on $\left[0, \frac{\pi}{4}\right]$.

(5 points)

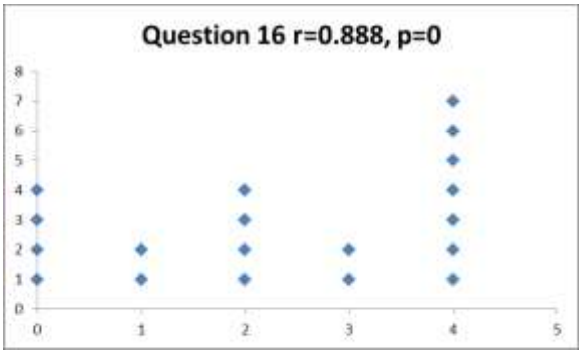
$$\int_{0}^{\pi/4} \cos(x) - \sin(x)\, dx$$





16) Set up the integral to find the arc length of $y = \sin(x)$ between $x = 0$ and $x = \pi$. (4 points)

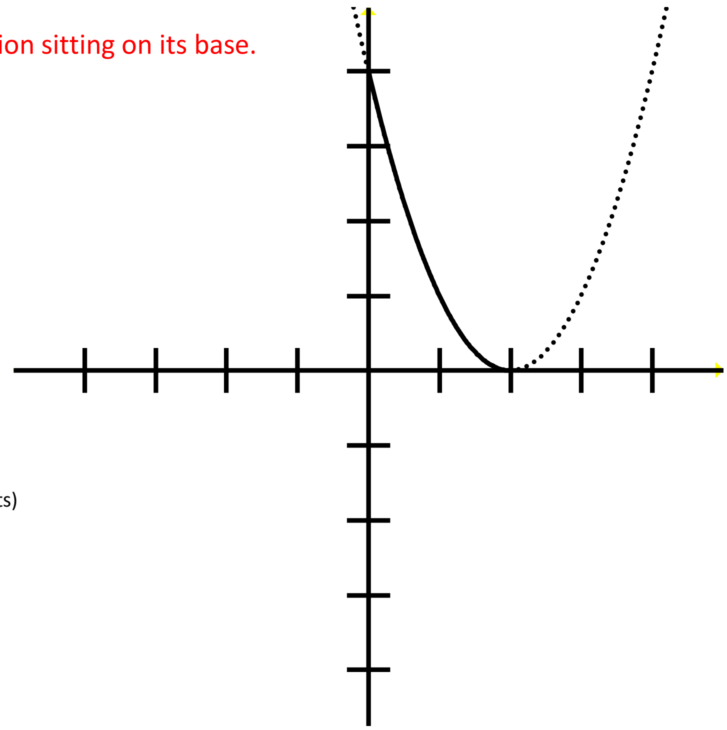
$$\int_0^{\pi} \sqrt{1 + \cos^2(x)}\, dx$$



17) The function $y = x^2 - 2x - 3$ with $0 \leq x \leq 2$ will be rotated around the y-axis.

(a) Describe the shape created. (2 points)

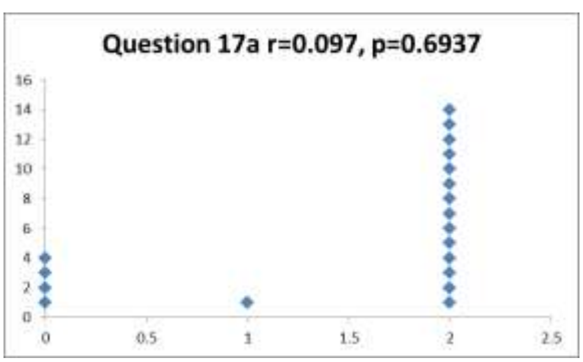
It's a sort of curved cone oriented in the usual position sitting on its base.

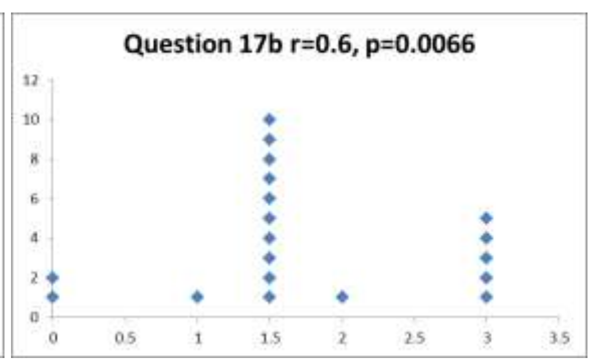


(b) Set up an integral to find the volume. (3 points)

$$\int_0^2 2\pi x(x^2 - 2x - 3)dx$$

(Cylindrical shell method)

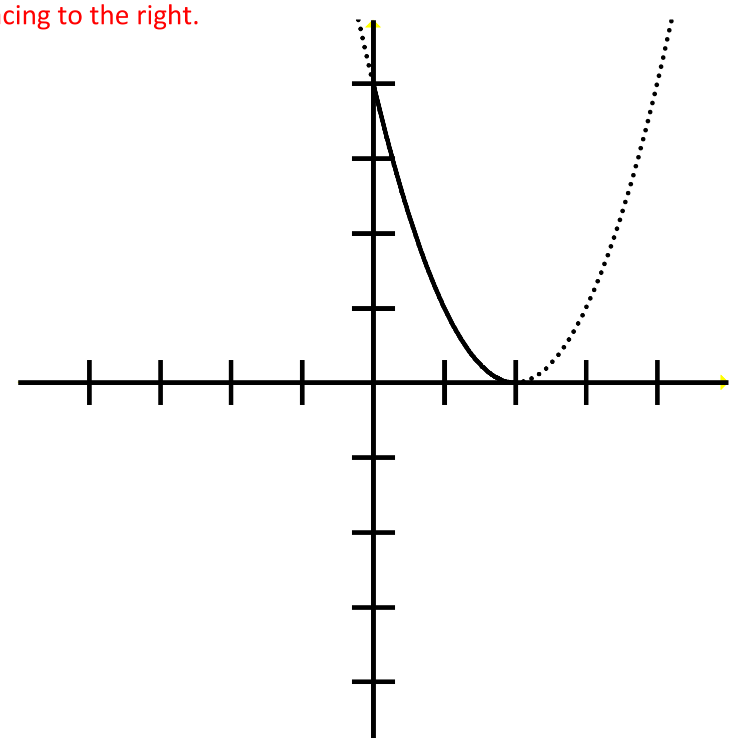




18) The function $y = x^2 - 2x - 3$ with $0 \le x \le 2$ will be rotated around the x-axis.

(a) Describe the shape created. (2 points)

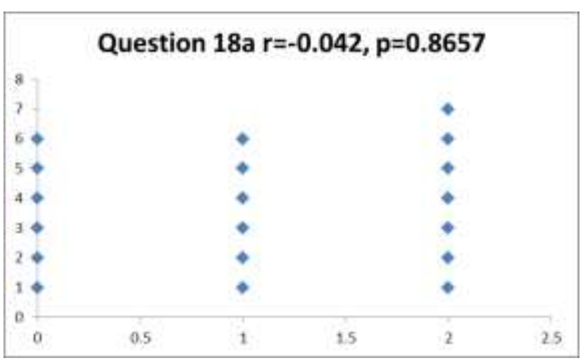
Again it's a curved cone-like object, but this time it's facing to the right.

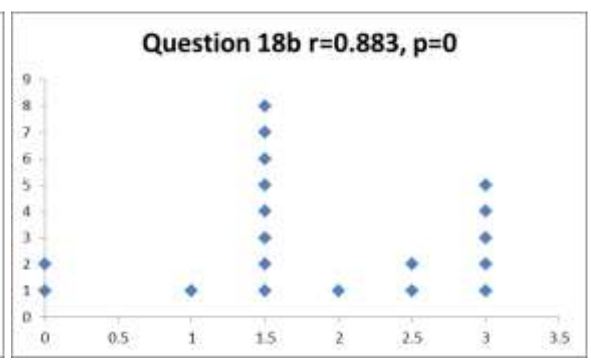


(b) Set up an integral to find the volume. (3 points)

$$\int_0^2 \pi(x^2 - 3x - 2)^2 dx$$

(disc method)

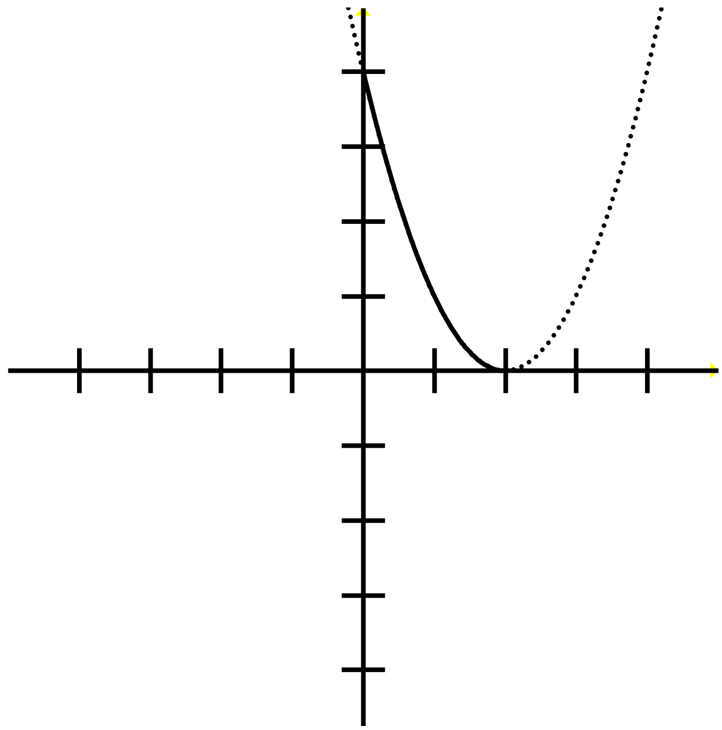


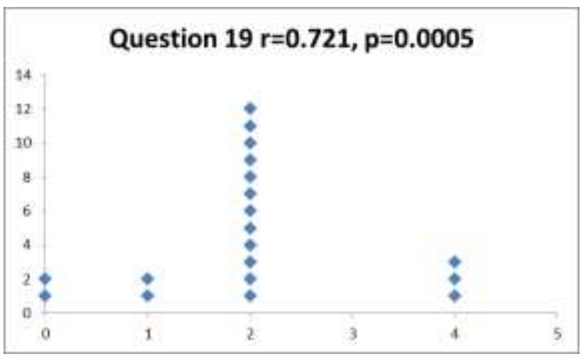


19) The function $y = x^2 - 2x - 3$ with $0 \le x \le 2$ will be rotated around the y-axis.

(a) Set up an integral to find the surface area, ignoring the base. (4 points)

$$\int_0^2 2\pi x\sqrt{1+(2x-2)^2}\,dx$$





20) The function $y = x^2 - 2x - 3$ with $0 \le x \le 2$ will be rotated around the x-axis.

(a) Set up an integral to find the surface area, ignore the base. (4 points)

$$\int_0^2 2\pi(x^2 - 2x - 3)\sqrt{1 + (2x-2)^2}\,dx$$