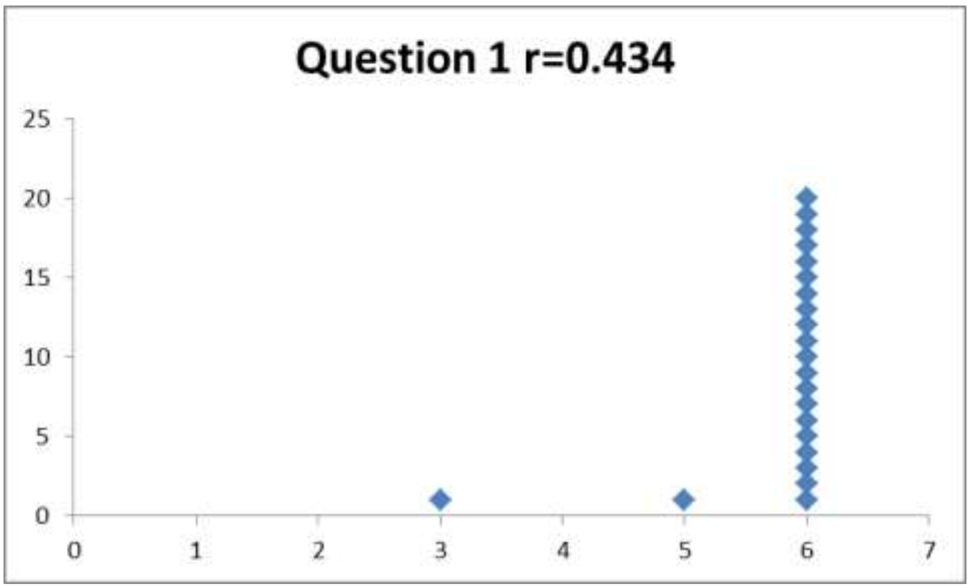
Name ____ Test 1, Fall 2017

1) Let $f(x) = (x^7 - 15x^2 + 3) \cdot (2x^4 + 2x)$. Find $f'(x)$.
(6 points)

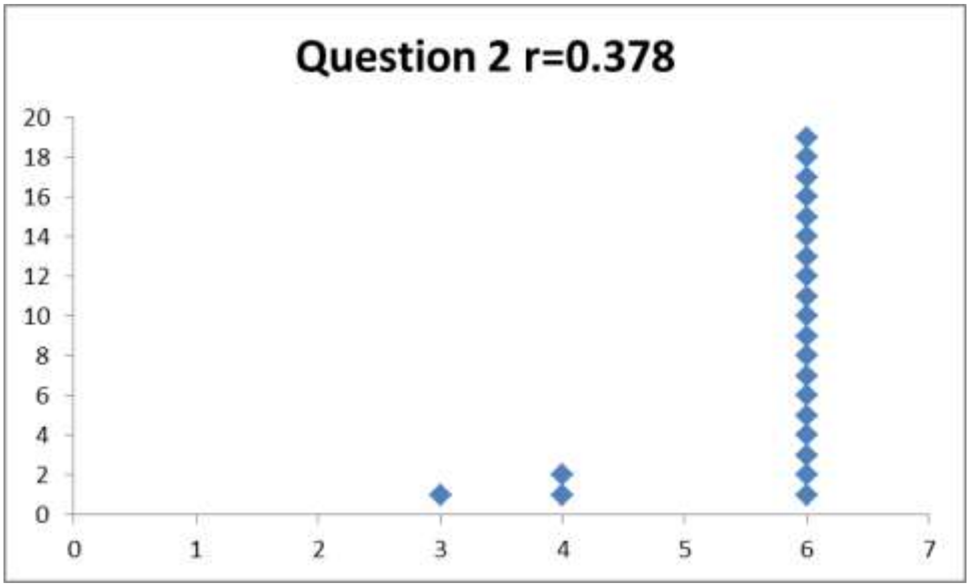
$$f'(x) = (7x^6 - 30x)(2x^4 + 2x) + (x^7 - 15x^2 + 3)(8x^3 + 2)$$



2) Let $(x)=\frac{(x^7-15x^2+3)}{(2x^4+2x)}$. Find $f'(x)$.

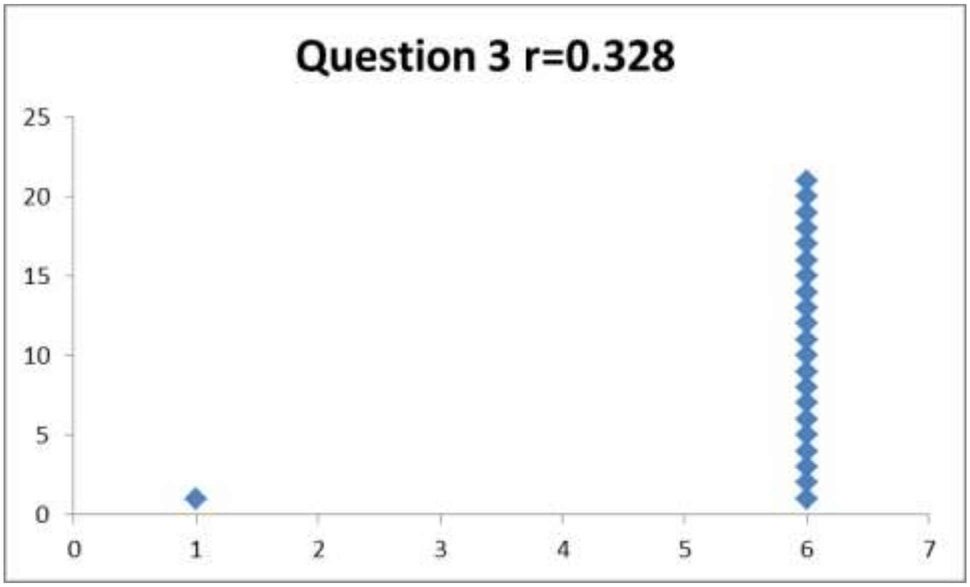
(6 points)

$$f'(x)=\frac{(7x^6-30x)(2x^4+2x)-(x^7-15x^2+3)(8x^3+2)}{(2x^4+2x)^2}$$



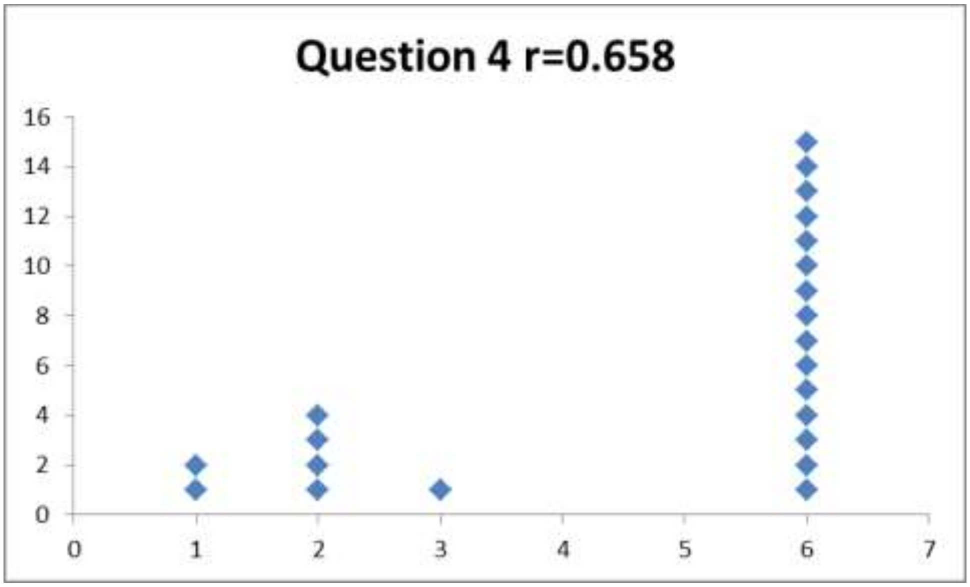
3) Find $\frac{d}{dx}(\sin(x)) = \cos(x)$

(6 points)



4) Find $\frac{d}{dx}(7^x) = 7^x \ln(7)$

(6 points)

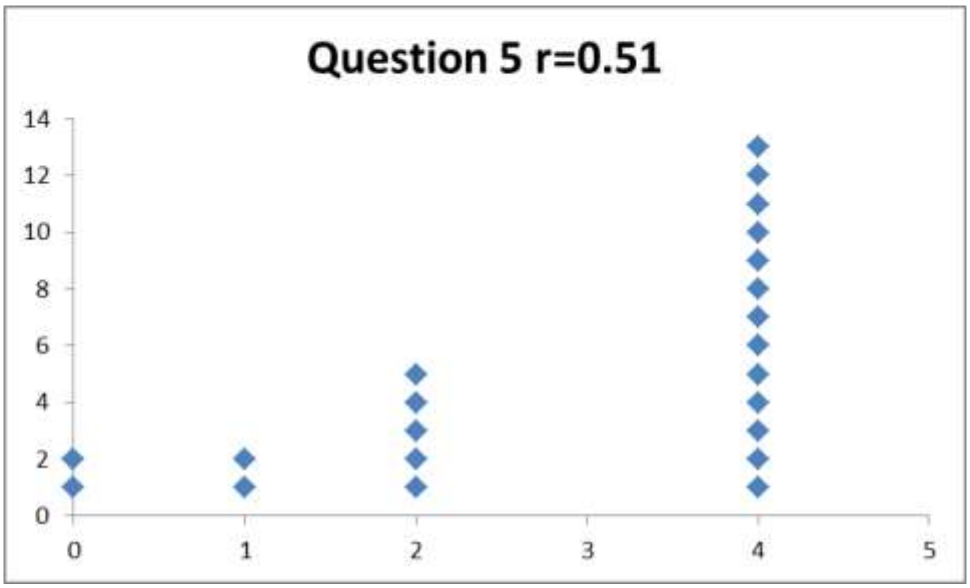


5) Find the four-hundredth derivative of $y = x^4$.
(4 points)

$$y^{(400)} = \frac{d^{400}}{dx^{400}} x^4 = 0$$

You can calculate this by finding the first few derivatives and noticing the pattern:

$$4x^3,\ \ 12x^2,\ \ 24x,\ \ 24,\ \ 0,\ \ 0,\ \ 0,\ \ 0,\ \ 0, \ldots$$

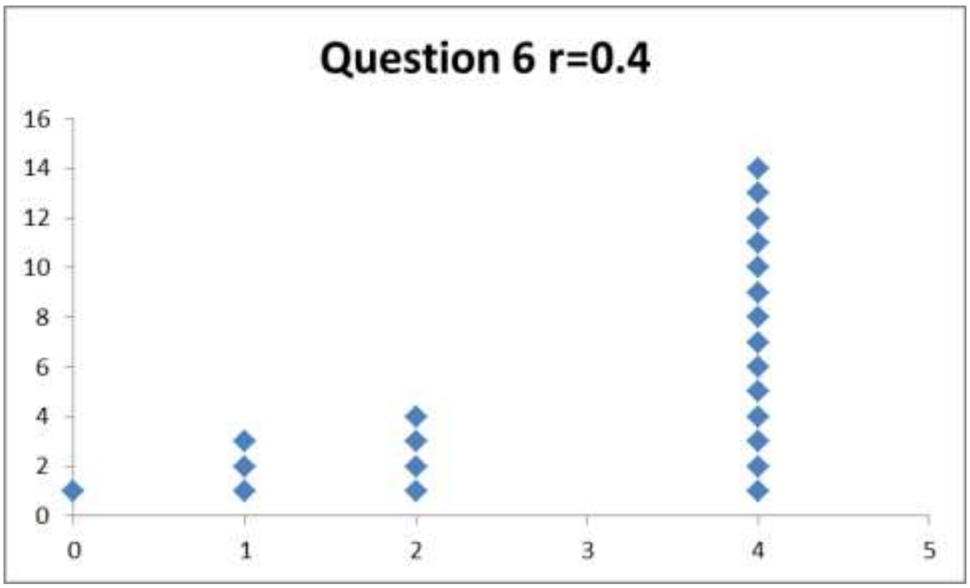


6) Find the four-hundredth derivative of $y = \sin(x)$.
(4 points)

$$y^{(400)} = \frac{d^{400}}{dx^{400}}\sin(x) = \sin(x)$$

You can calculate this by finding the first few derivatives and noticing the pattern:

$$y = \sin(x)\,, y' = \cos(x)\,, y'' = -\sin(x)\,, y''' = -\cos(x)\,, y'''' = \sin(x)\,, y^{(5)} = \cos(x)\,, \ldots$$

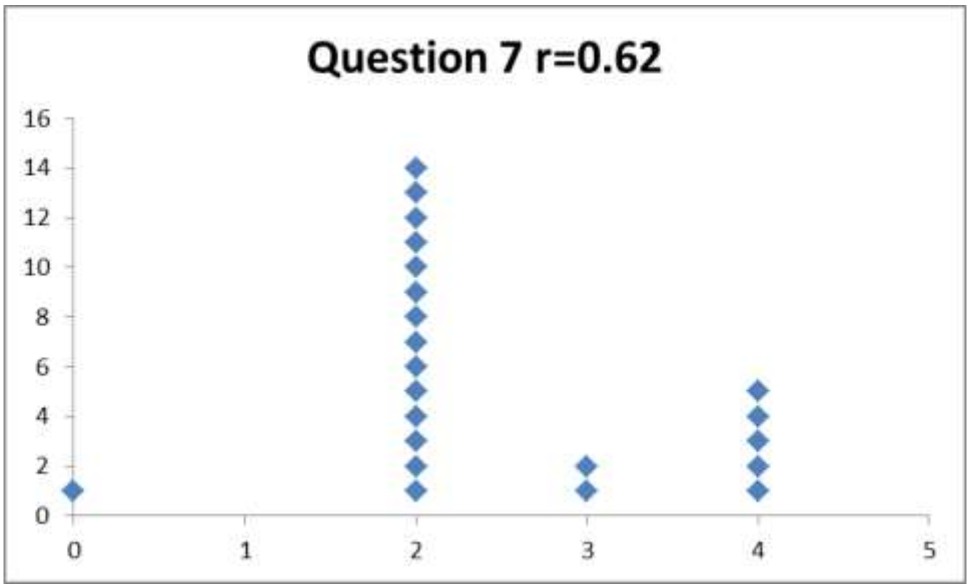


7) Find the four-hundredth derivative of $y = 7^x$.
(4 points)

$$y^{(400)} = \frac{d^{400}}{dx^{400}} 7^x = 7^x (\ln(7))^{400}$$

You can calculate this by finding the first few derivatives and noticing the pattern:

$$7^x \ln(7), \quad 7^x (\ln(7))^2, \quad 7^x (\ln(7))^3, \ldots$$



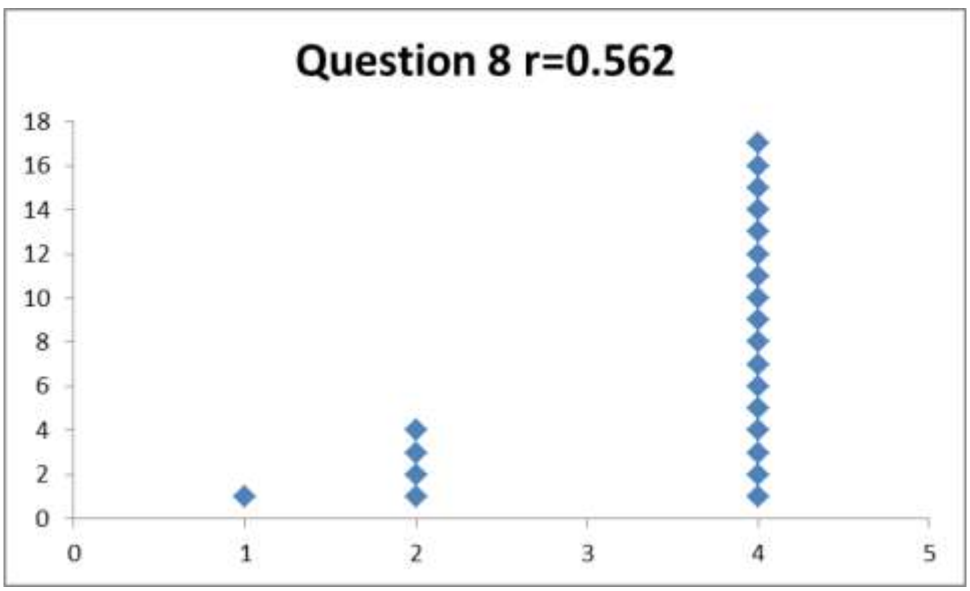
A table of values is given below for the function $f(x) = \frac{4x^2-13x-12}{x-4}$

x	$f(x)$	x	$f(x)$
3.5	17	4.5	21
3.9	18.6	4.1	19.4
3.99	18.96	4.01	19.04
3.999	18.996	4.001	19.004

8) What would you guess the value of the limit is?
(4 points)

19, obviously, or so I thought, because the value of the function is going to 19.

But apparently it wasn't that obvious, because a lot of people put 4. That's what x is going to, but that's the argument, not the value of the limit.

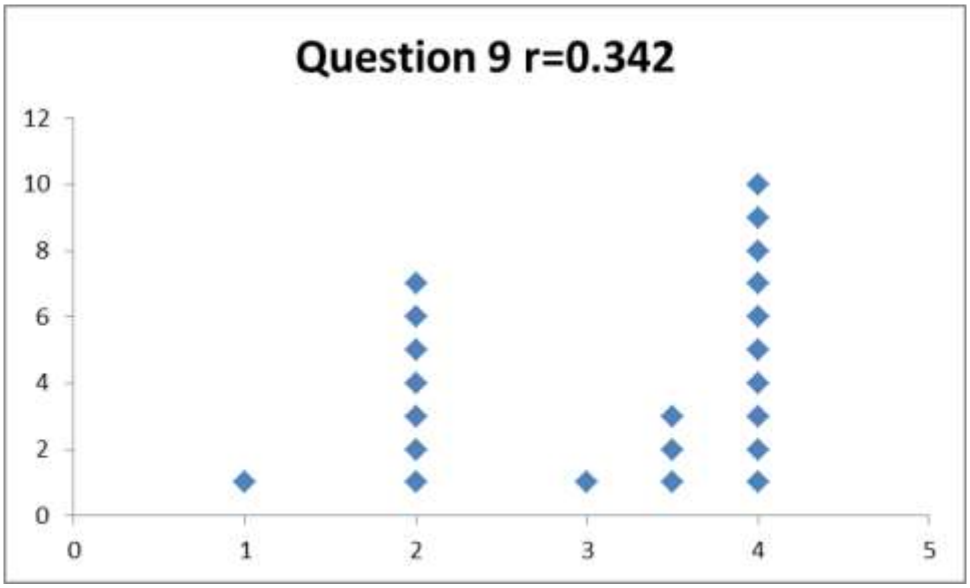


9) In the previous question you guessed the value of a limit. What limit did you guess?

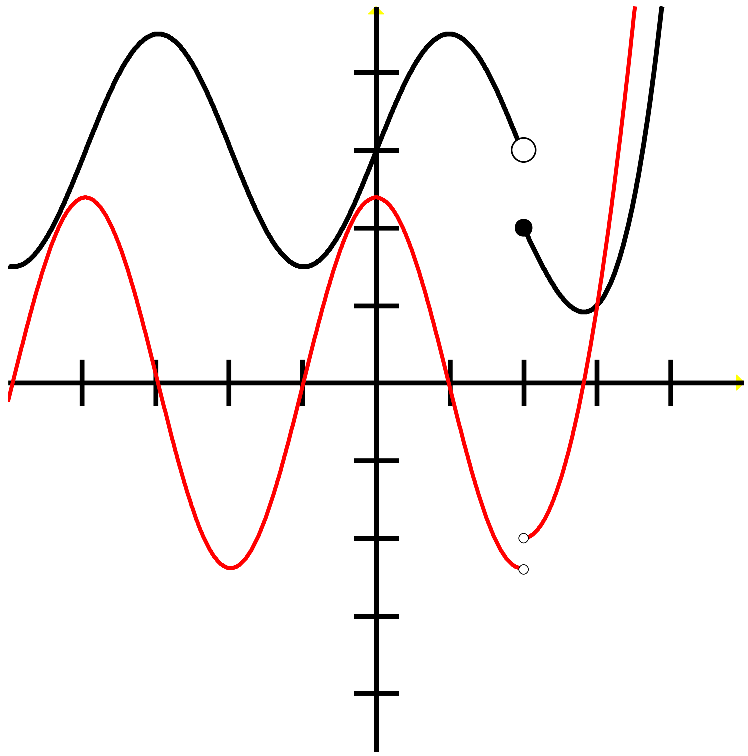
(Your answer should be an equation with proper limit notation on one side and your answer to #8 on the other)

(4 points)

$$\lim_{x \to 4} \frac{4x^2 - 13x - 12}{x - 4} = 19$$



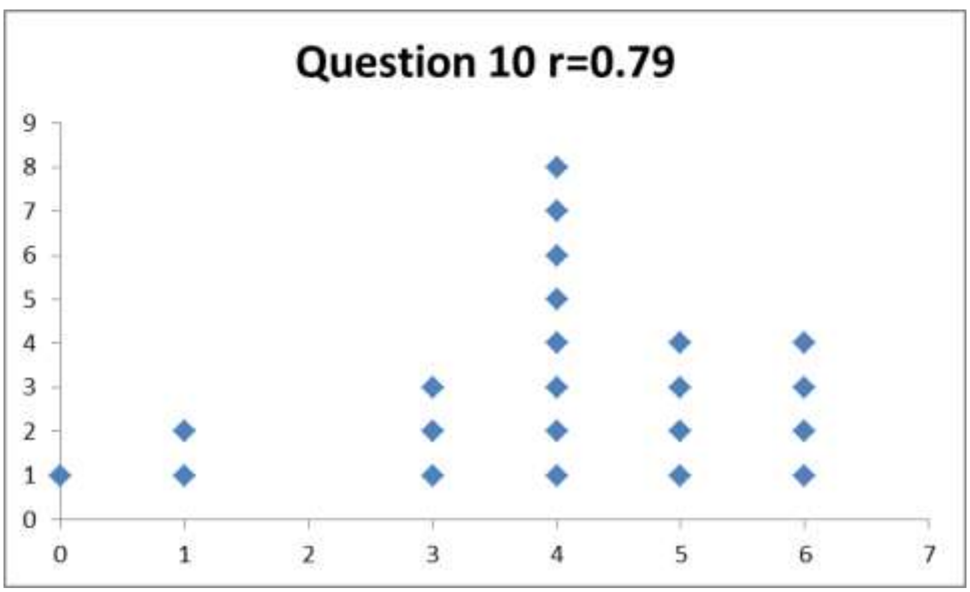
10) The graph to the right is the graph of $y = f(x)$. On the same graph, sketch the derivative $y = f'(x)$. (6 points)



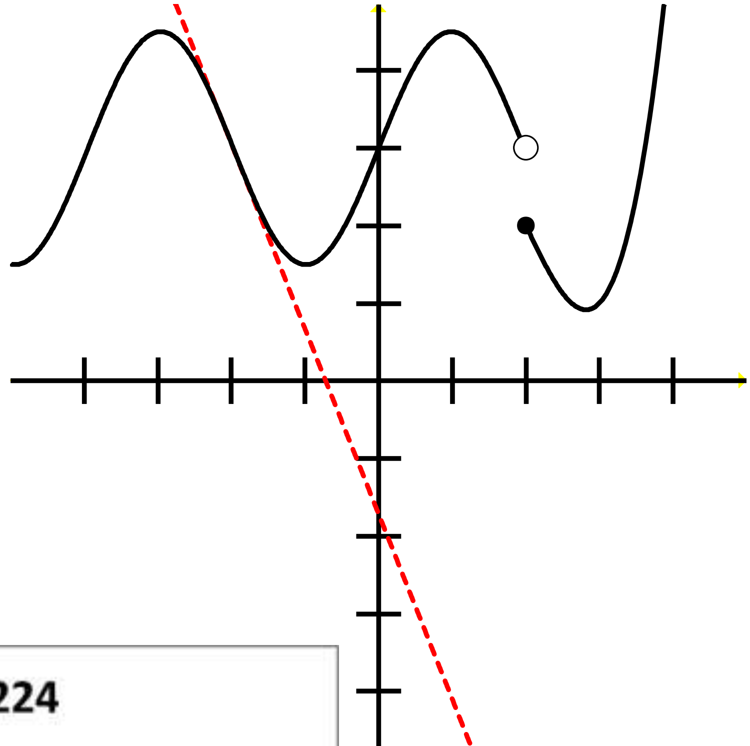
Anything close to this that illustrates the key features was given full credit.

Some key features to look for:

- Where is the derivative zero?
- Where does the derivative keep growing?
- Where does the derivative not exist?



Use the graph to the right to complete the following FIVE questions.

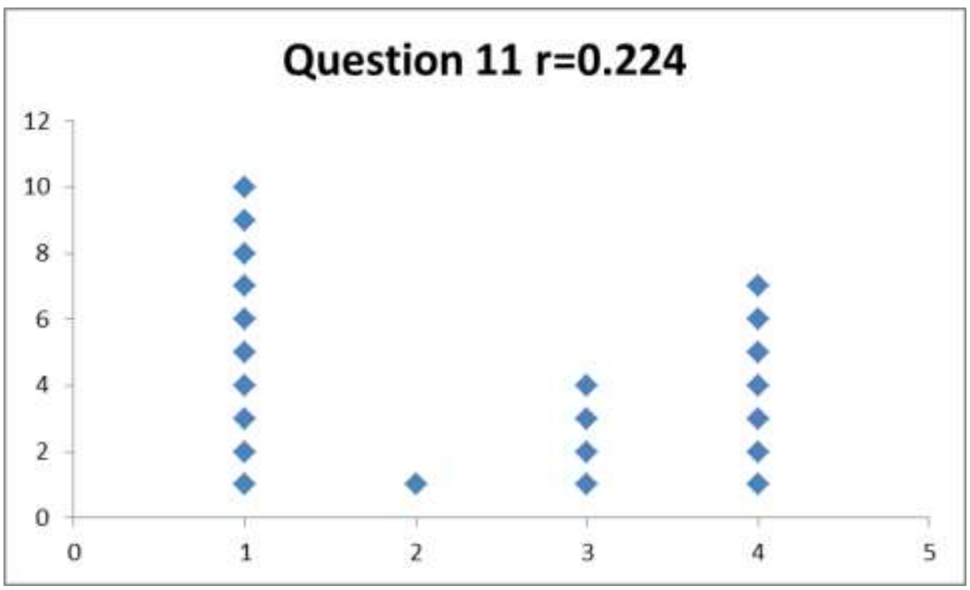


11) Estimate the derivative of $y = f(x)$ at $x = -2$.
(4 points)

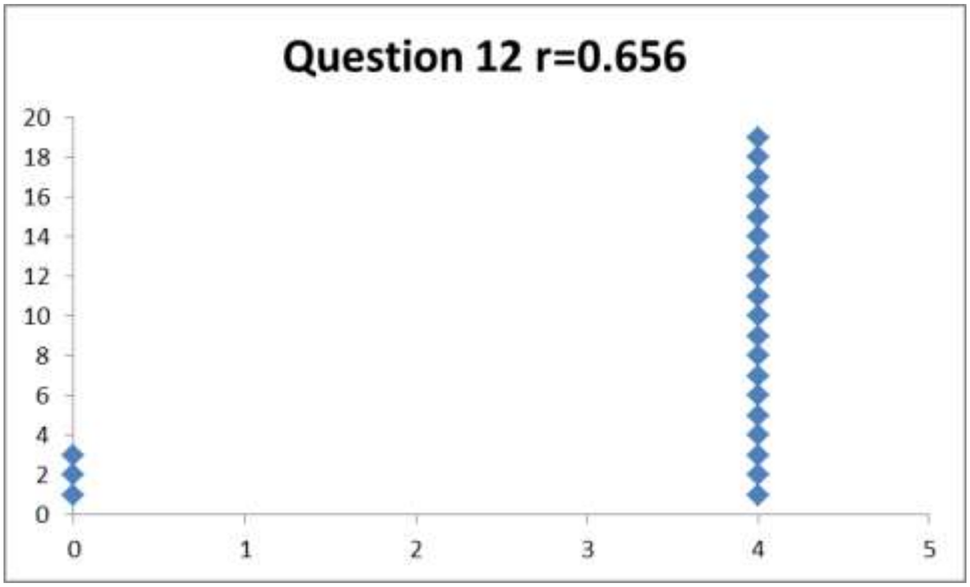
Maybe -2?

(Full credit for anything between -1 and -10).

Only 1 point was awarded for any positive number.
Because, at least you knew it was a positive number?
Seriously, folks! The function is decreasing so obviously the derivative (rate of change) of the function is negative!!!
This is me being very annoyed

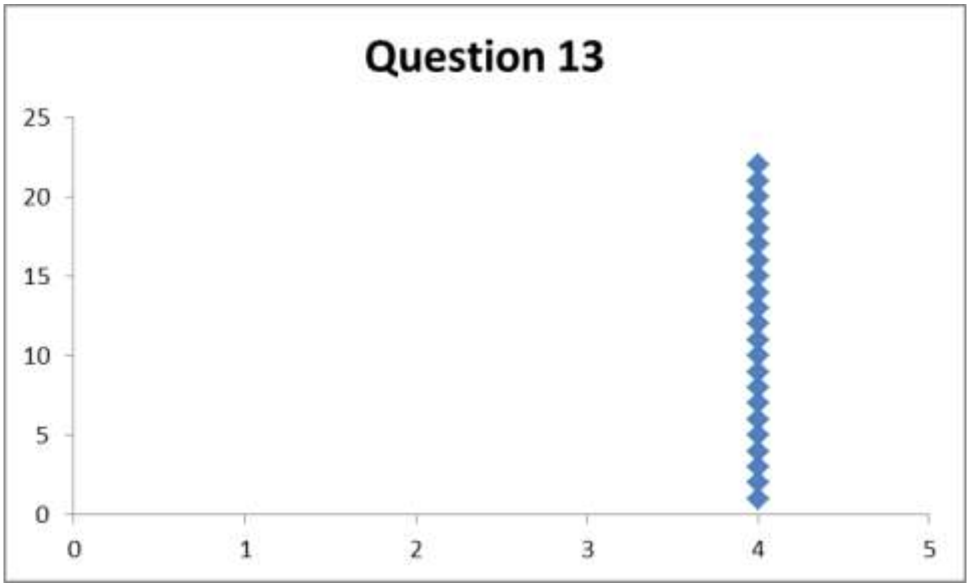


12) Sketch the tangent line to f at $x = -2$.
(4 points)



13) Why is f not continuous at $x = 2$?
(4 points)

It has a jump discontinuity there.

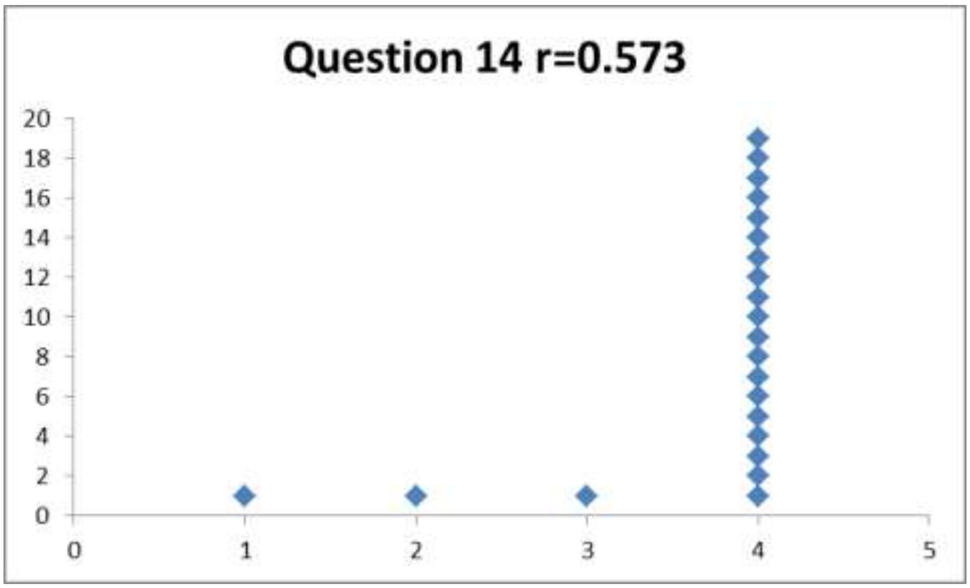


14) Why is f not differentiable at $x = 2$?
(4 points)

It isn't continuous, so it can't be differentiable.

OR

There is no one tangent line to the function: if you try to construct one, we see that there are many ways a line can approximate the function.

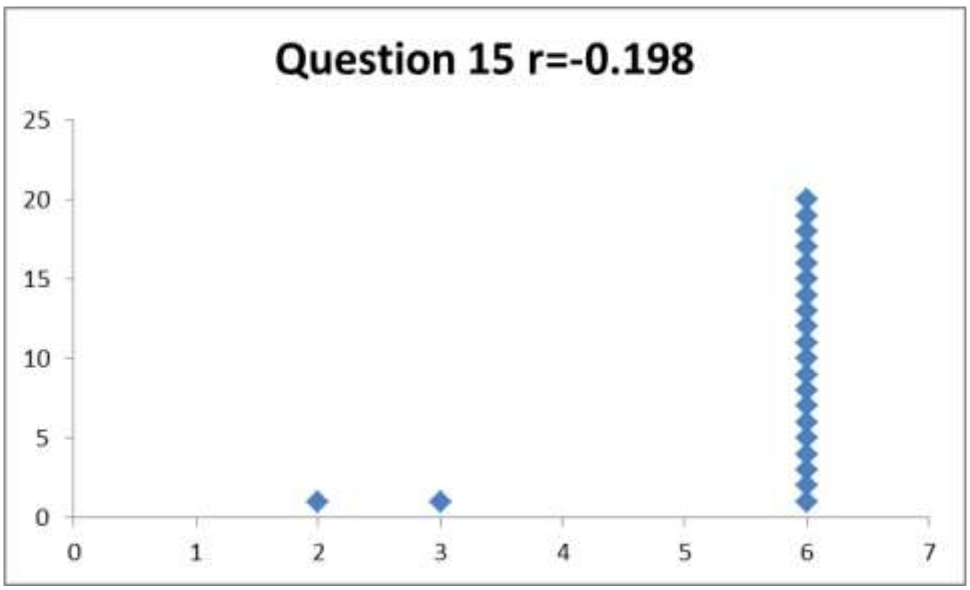


15) Calculate the limits below.
(6 points)

$$\lim_{x \to 2^-} f(x) = 3$$

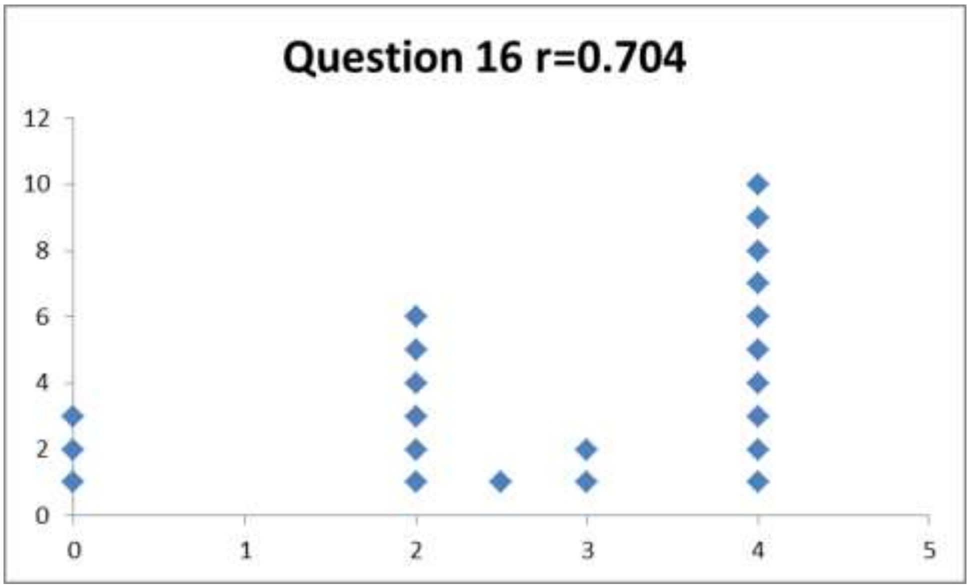
$$\lim_{x \to 2^+} f(x) = 2$$

$$\lim_{x \to 2} f(x) \quad \text{DNE}$$



16) State the formal definition of the derivative.
(4 points)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



17) Complete ONE of the following problems. (6 points)

A) Use your formal definition to find $f'(x)$ for $f(x) = 3x^2$.

B) Explain, using the formal definition, why it calculates the slope of the tangent line.

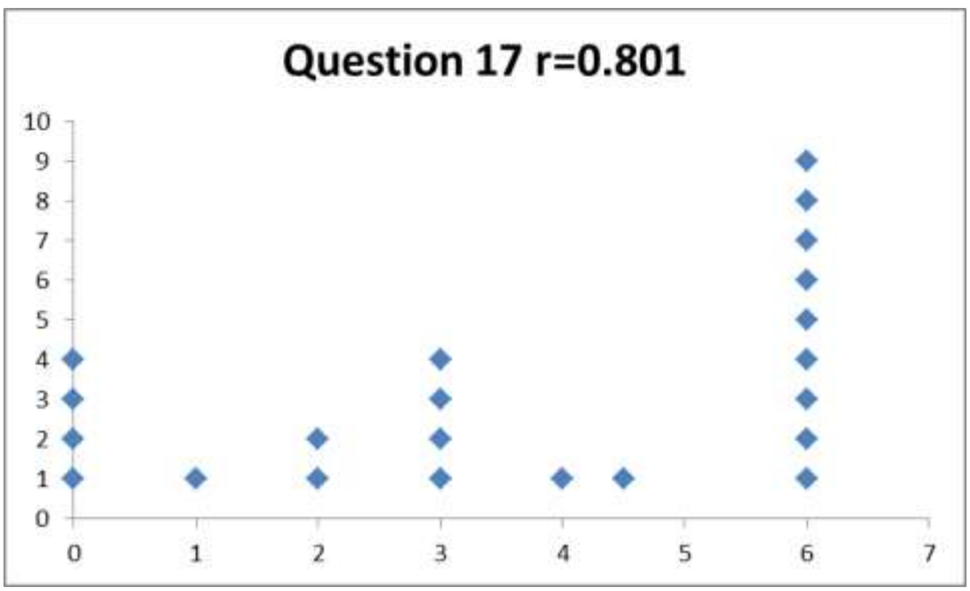
A)

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h} = \lim_{h\to 0}\frac{3(x^2+2xh+h^2)-3x^2}{h}$$
$$= \lim_{h\to 0}\frac{3x^2+6xh+3h^2-3x^2}{h} = \lim_{h\to 0}\frac{6xh+3h^2}{h}$$
$$= \lim_{h\to 0} 6x+3h = 6x+3\cdot 0 = 6x$$

B) The derivative is the instantaneous rate of change, which we can calculate by finding the average rate of change between two points (x and $x+h$), and taking the limit as one goes to the other.

Additional information not required nor expected for full credit but that I think is interesting:
Also, if you understand what that means, you should understand why the formula below also works:

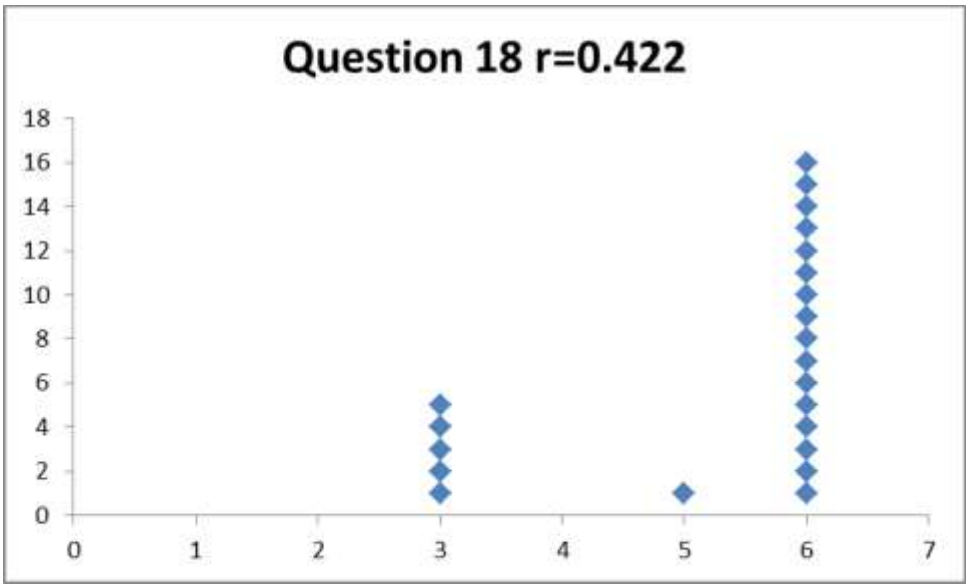
$$f'(x) = \lim_{x_2\to x_1}\frac{f(x_2)-f(x_1)}{x_2-x_1}$$



Calculate the following limits. (6 points each)

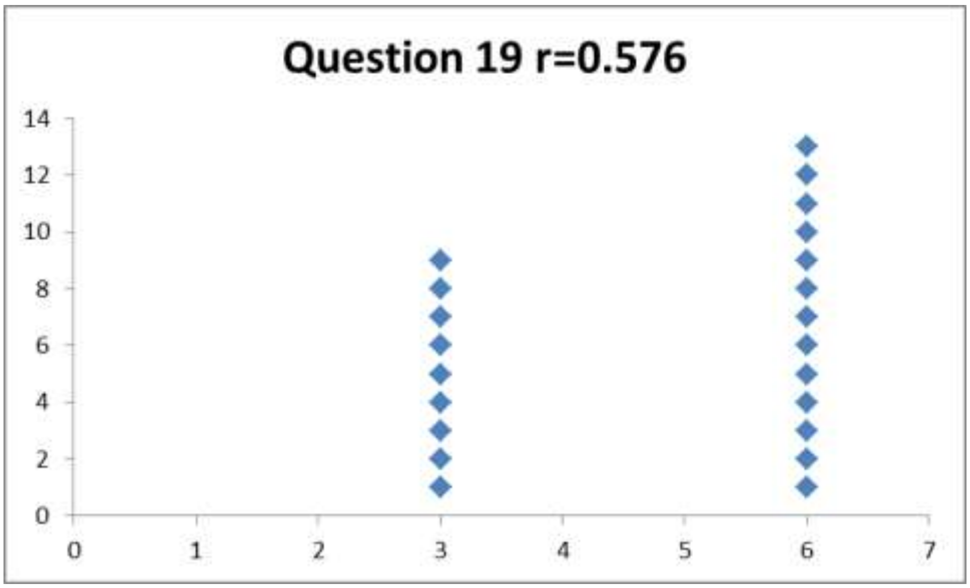
18) $\lim_{x\to 2^+} \frac{2x^2-8x+8}{(x-2)^2(x+3)(x-4)^2} = \lim_{x\to 2^+} \frac{2(x-2)^2}{(x-2)^2(x+3)(x-4)^2} = \lim_{x\to 2^+} \frac{2}{(x+3)(x-4)^2}$

$= \frac{2}{(2+3)(2-4)^2} = \frac{2}{5\cdot 4} = \frac{1}{10}$

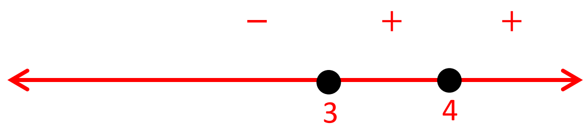


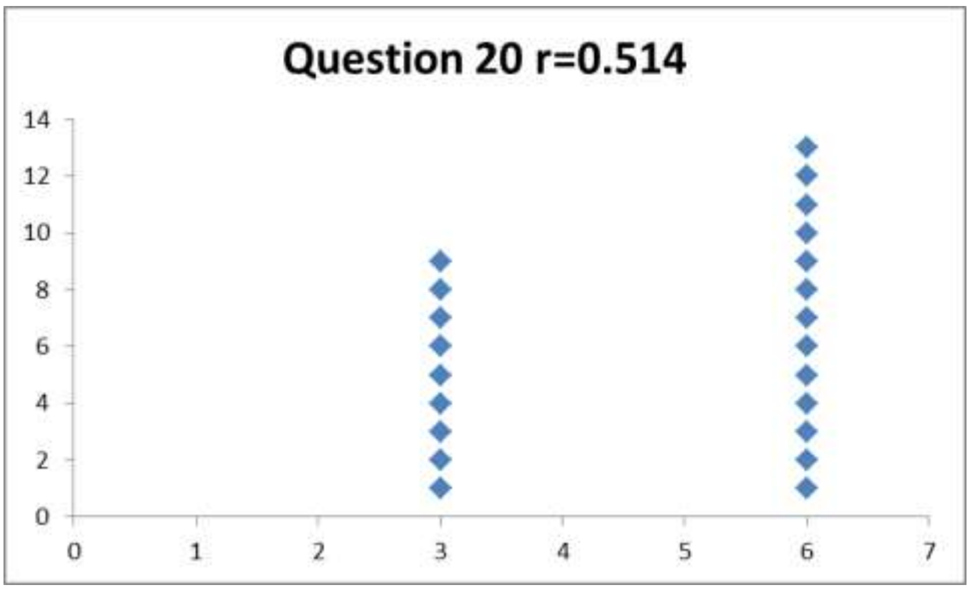
19) $\lim_{x \to \infty} \frac{2x^2-8x+8}{(x-2)^2(x+3)(x-4)^2} = 0$

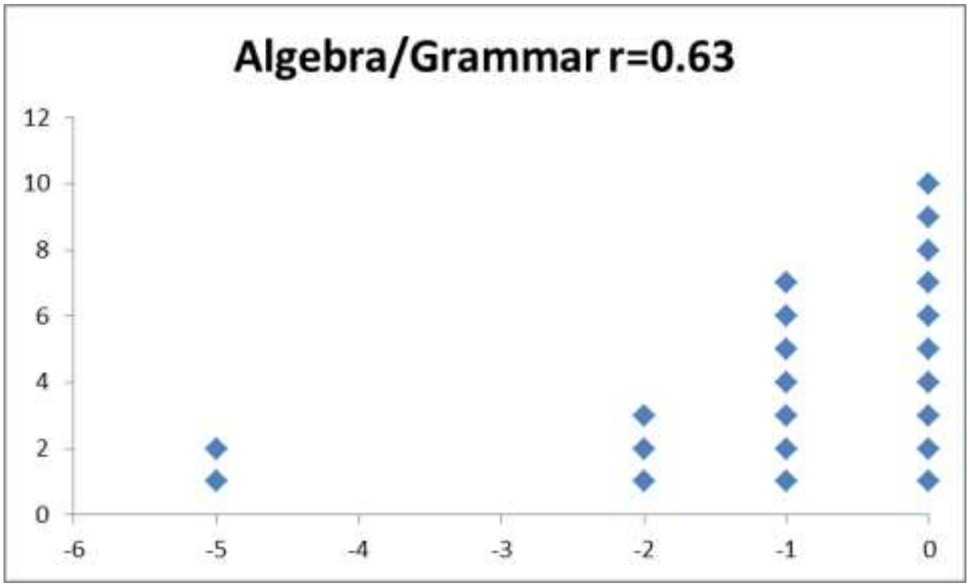
(Denominator degree is larger than numerator)



20) $\lim_{x \to 3^-} \frac{(x-4)^2}{(x-3)} = -\infty$







Algebra/Grammar r=0.63
12
10
8
6
4
2
0
-6
-5
-4
-3
-2
-1
0