1) Let $f(x) = (x^7 - 15x^2 + 3) \cdot (2x^4 + 2x)$. Find $f'(x)$.

(6 points)

$$f'(x) = (7x^6 - 30x)(2x^4 + 2x) + (x^7 - 15x^2 + 3)(8x^3 + 2)$$
2) Let \( f(x) = \frac{(x^7 - 15x^2 + 3)}{(2x^4 + 2x)} \). Find \( f'(x) \).

\( f'(x) = \frac{(7x^6 - 30x)(2x^4 + 2x) - (x^7 - 15x^2 + 3)(8x^3 + 2)}{(2x^4 + 2x)^2} \)
3) Find $\frac{d}{dx}(\sin(x)) = \cos(x)$

(6 points)
4) Find \( \frac{d}{dx}(7^x) = 7^x \ln(7) \)

(6 points)
5) Find the four-hundredth derivative of \( y = x^4 \).

(4 points)

\[ y^{(400)} = \frac{d^{400}}{dx^{400}} x^4 = 0 \]

You can calculate this by finding the first few derivatives and noticing the pattern:

\[ 4x^3, \ 12x^2, \ 24x, \ 24, \ 0, \ 0, \ 0, \ 0, \ ... \]
6) Find the four-hundredth derivative of \( y = \sin(x) \).

(4 points)

\[
y^{(400)} = \frac{d^{400}}{dx^{400}} \sin(x) = \sin(x)
\]

You can calculate this by finding the first few derivatives and noticing the pattern:

\[
y = \sin(x), y' = \cos(x), y'' = - \sin(x), y''' = - \cos(x), y'''' = \sin(x), y^{(5)} = \cos(x), ...
\]
7) Find the four-hundredth derivative of $y = 7^x$.
(4 points)

$$y^{(400)} = \frac{d^{400}}{dx^{400}} 7^x = 7^x (\ln(7))^{400}$$

You can calculate this by finding the first few derivatives and noticing the pattern:
$7^x \ln(7), \ 7^x (\ln(7))^2, \ 7^x (\ln(7))^3, \ldots$
A table of values is given below for the function $f(x) = \frac{4x^2 - 13x - 12}{x-4}$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>17</td>
<td>4.5</td>
<td>21</td>
</tr>
<tr>
<td>3.9</td>
<td>18.6</td>
<td>4.1</td>
<td>19.4</td>
</tr>
<tr>
<td>3.99</td>
<td>18.96</td>
<td>4.01</td>
<td>19.04</td>
</tr>
<tr>
<td>3.999</td>
<td>18.996</td>
<td>4.001</td>
<td>19.004</td>
</tr>
</tbody>
</table>

8) What would you guess the value of the limit is?
(4 points)

19, obviously, or so I thought, because the value of the function is going to 19.

But apparently it wasn’t that obvious, because a lot of people put 4. That’s what $x$ is going to, but that’s the argument, not the value of the limit.
9) In the previous question you guessed the value of a limit. What limit did you guess?
(Your answer should be an equation with proper limit notation on one side and your answer to #8 on the other)
(4 points)

\[
\lim_{{x \to 4}} \frac{4x^2 - 13x - 12}{x - 4} = 19
\]
10) The graph to the right is the graph of $y = f(x)$. On the same graph, sketch the derivative $y = f'(x)$. (6 points)

Anything close to this that illustrates the key features was given full credit.

Some key features to look for:

- Where is the derivative zero?
- Where does the derivative keep growing?
- Where does the derivative not exist?
11) Estimate the derivative of \( y = f(x) \) at \( x = -2 \).

(4 points)

Maybe \(-2\)?

(Full credit for anything between \(-1\) and \(-10\)).

Only 1 point was awarded for any positive number.
Because, at least you knew it was a positive number?
Seriously, folks! The function is decreasing so obviously the derivative (rate of change) of the function is negative!!!
*This is me being very annoyed*
12) Sketch the tangent line to $f$ at $x = -2$.

(4 points)
13) Why is $f$ not continuous at $x = 2$?

(4 points)

It has a jump discontinuity there.
14) Why is \( f \) not differentiable at \( x = 2 \)?

(4 points)

It isn’t continuous, so it can’t be differentiable.

OR

There is no one tangent line to the function: if you try to construct one, we see that there are many ways a line can approximate the function.
15) Calculate the limits below.
(6 points)
\[
\lim_{x \to 2^-} f(x) = 3
\]
\[
\lim_{x \to 2^+} f(x) = 2
\]
\[
\lim_{x \to 2} f(x) \text{ DNE}
\]
16) State the formal definition of the derivative.
(4 points)

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]
17) Complete ONE of the following problems. (6 points)

A) Use your formal definition to find \( f'(x) \) for \( f(x) = 3x^2 \).

B) Explain, using the formal definition, why it calculates the slope of the tangent line.

A)

\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x + h)^2 - 3x^2}{h} = \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h} \\
= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} = \lim_{h \to 0} \frac{6xh + 3h^2}{h} \\
= \lim_{h \to 0} 6x + 3h = 6x + 3 \cdot 0 = 6x
\]

B) The derivative is the instantaneous rate of change, which we can calculate by finding the average rate of change between two points \((x, x + h)\), and taking the limit as one goes to the other.

Additional information not required nor expected for full credit but that I think is interesting:

Also, if you understand what that means, you should understand why the formula below also works:

\[
f'(x) = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}
\]
Calculate the following limits. (6 points each)

\[ 18) \lim_{x \to 2^+} \frac{2x^2 - 8x + 8}{(x-2)^2(x+3)(x-4)^2} = \lim_{x \to 2^+} \frac{2(x-2)^2}{(x-2)^2(x+3)(x-4)^2} = \lim_{x \to 2^+} \frac{2}{(x+3)(x-4)^2} = \frac{2}{5 \cdot 4} = \frac{1}{10} \]
19) \( \lim_{x \to \infty} \frac{2x^2 - 8x + 8}{(x-2)^2(x+3)(x-4)^2} = 0 \)

(Denominator degree is larger than numerator)
\[
\lim_{{x \to 3^-}} \frac{{(x-4)^2}}{x-3} = -\infty
\]