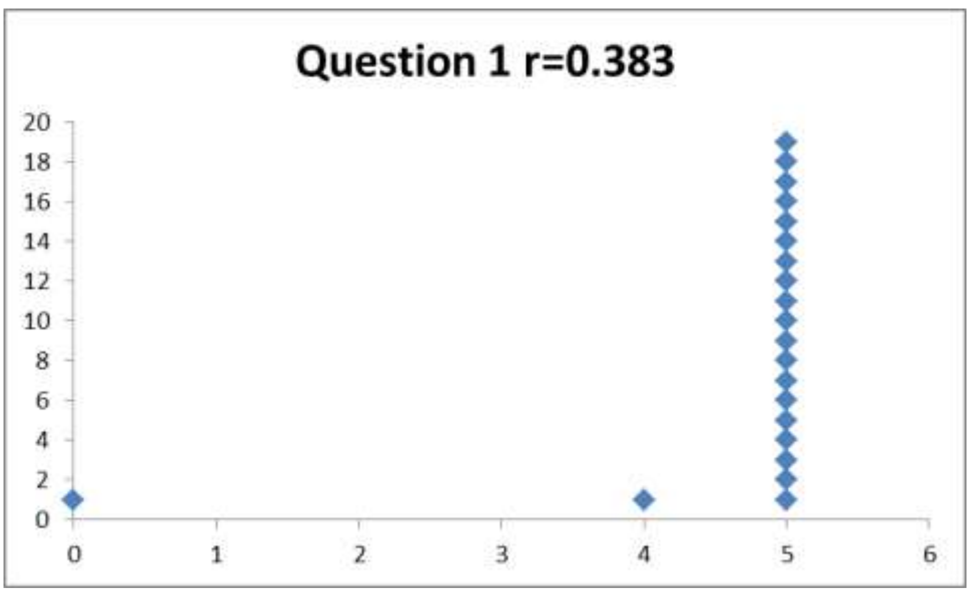
Name ____ Test 2, Fall 2017

1) A mosquito is flying around a yard, and its distance from a gardener in the yard is given by $s(t) = -2t^3 + 15t + 5$ where s is given in inches and t in seconds. How fast is the mosquito moving after 1 second?
(5 points)

$s'(t) = -6t^2 + 15$

$s'(1) = -6 + 15 = 9$

9 inches per second.



2) Find $\frac{dy}{dx}$ when $\sin(xy) = x^2$

(6 points)

$$\frac{d}{dx}\sin(xy) = \frac{d}{dx}x^2$$

$$\cos(xy)\,(y + xy') = 2x$$

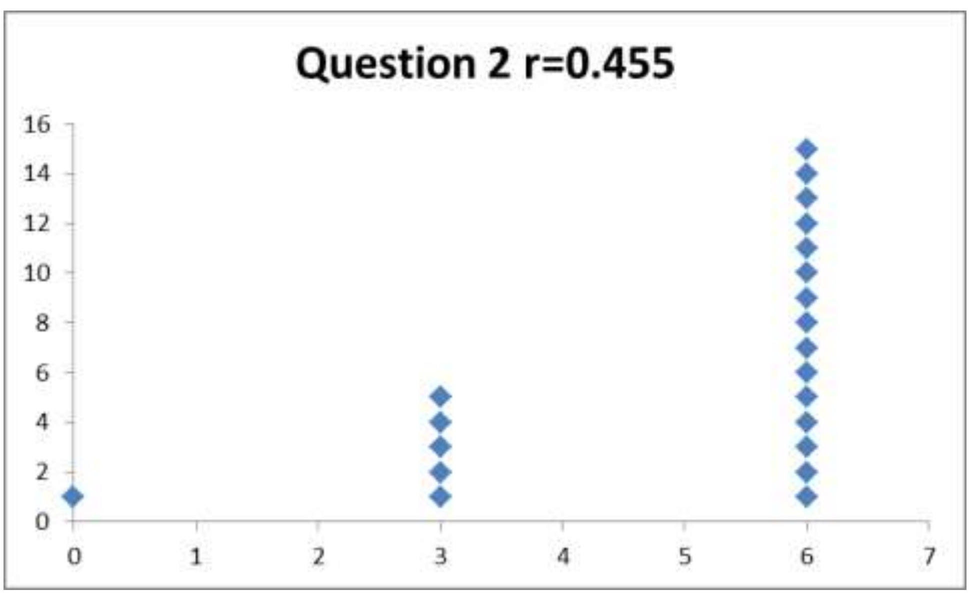
$$\cos(xy)\,y + \cos(xy)\,xy' = 2x$$

$$\cos(xy)\,xy' = 2x - \cos(xy)\,y$$

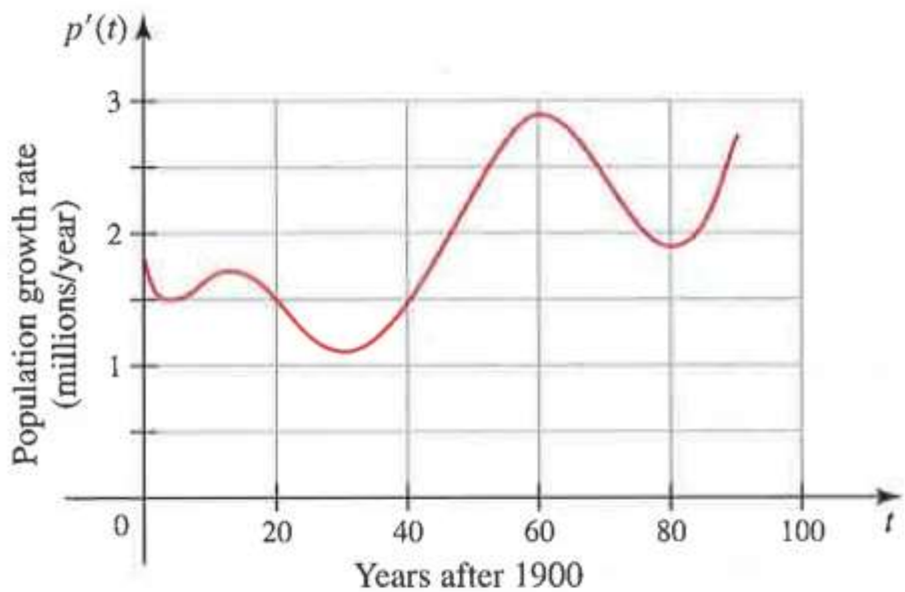
$$y' = \frac{2x - \cos(xy)\,y}{\cos(xy)\,x}$$

While a lot of people got full credit for the calculus problem, a lot of people made algebra mistakes that caused them to have a different answer. Most notably is not using parenthesis when using the chain rule and getting the left side as $\cos(xy) \cdot y + xy'$.

This is a pretty big algebra mistake, and so counted off double. Look for a green "x2" which means that two algebra points were deducted.

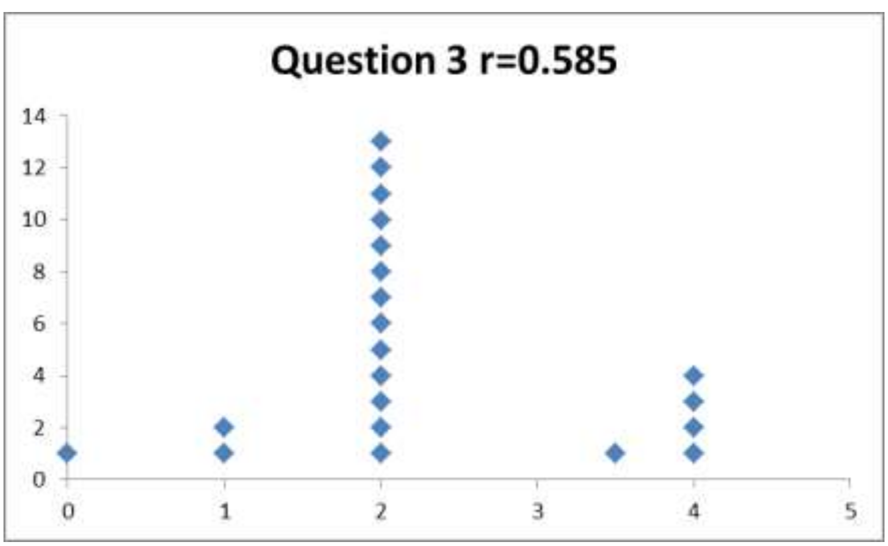


Let $p(t)$ be the population in the United States, in millions, after the year 1900. So for example $t = 32$ represents the year 1932. A graph of $p'(t)$ between 1900 and 1990 is shown below. Use this for the next THREE questions.



3) When was the population increasing?
(4 points)

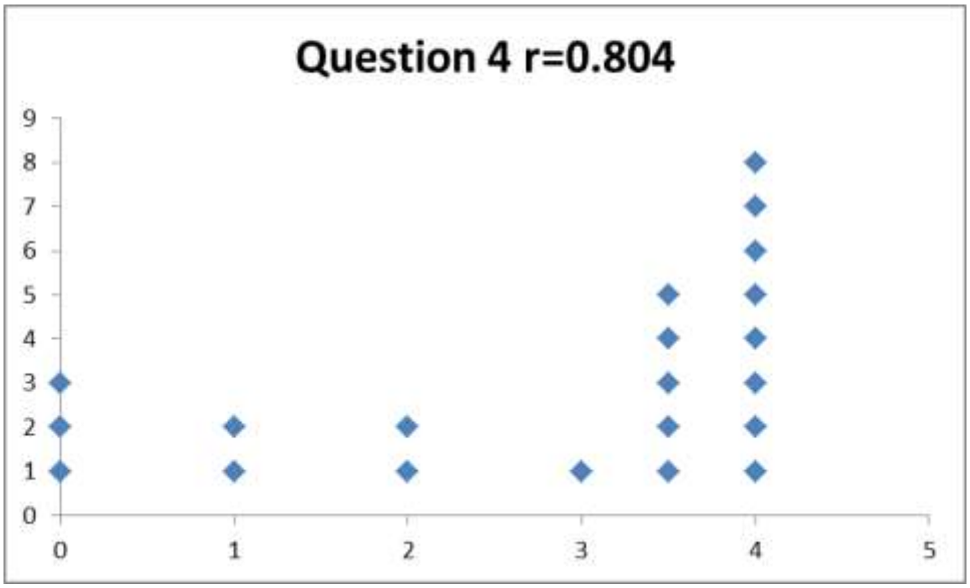
The derivative, $p'(t)$ is positive the whole time, so the population is increasing on entire interval $(1900,1990)$



4) When was the rate of change of the population increasing?
(4 points)

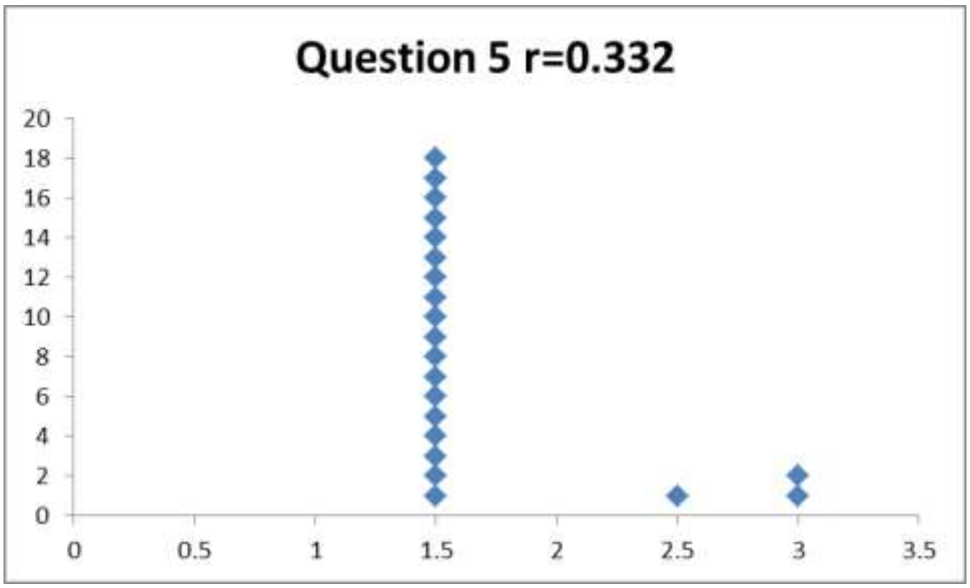
This is the second derivative, we see three intervals that the derivative is increasing:

$$(1905{,}1910), (1930{,}1960), (1980{,}1990)$$



5) When on the graph was the population the largest?
(3 points)

Because it is increasing the whole time, the population is the largest at the end of it: 1990



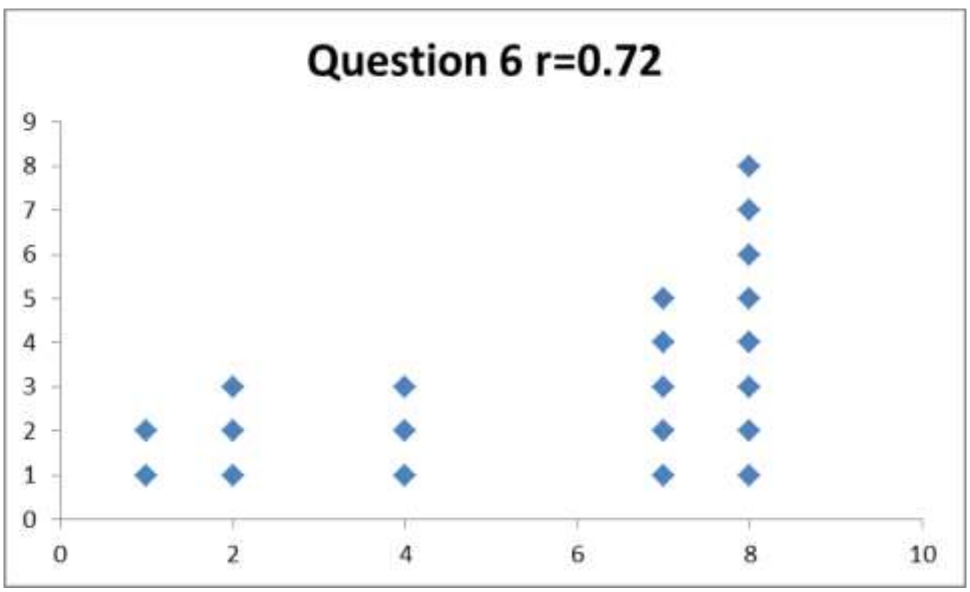
6) The edges of a cube increase at a rate of 2cm/s. How fast is the volume changing when the length of each edge is 5m?
(8 points)

$$V = x^3$$

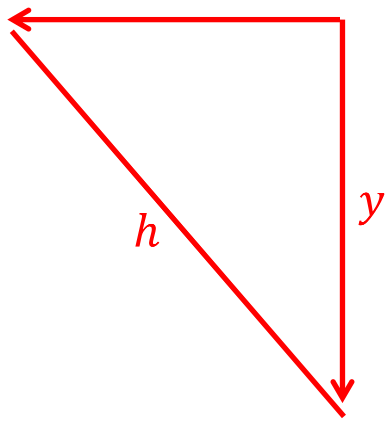
$$x = 500$$
$$\frac{dx}{dt} = 2$$

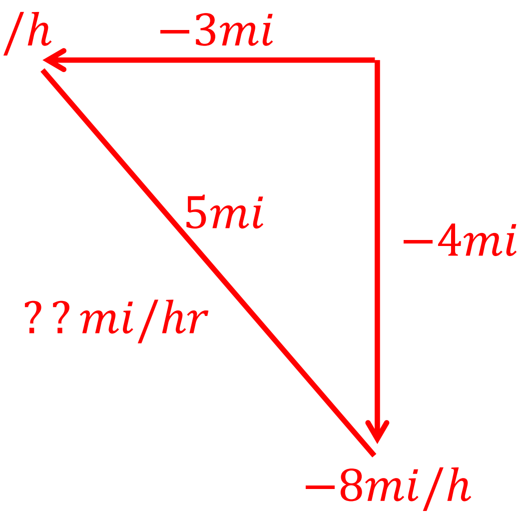
$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$V' = 3 \cdot 500^2 \cdot 2\ cm^3/s$$



7) Two boats leave a port at the same time; one travels west at 6mi/hr and the other travels south at 8mi/hr. At what rate is the distance between them changing 30 minutes after they leave the port? (10 points)



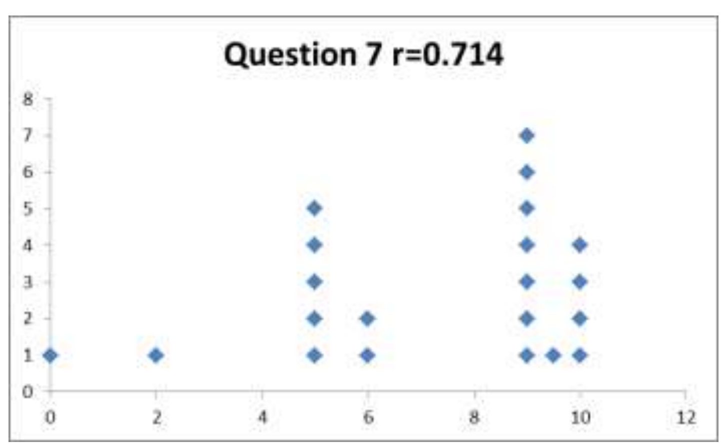


$$h^2 = x^2 + y^2$$

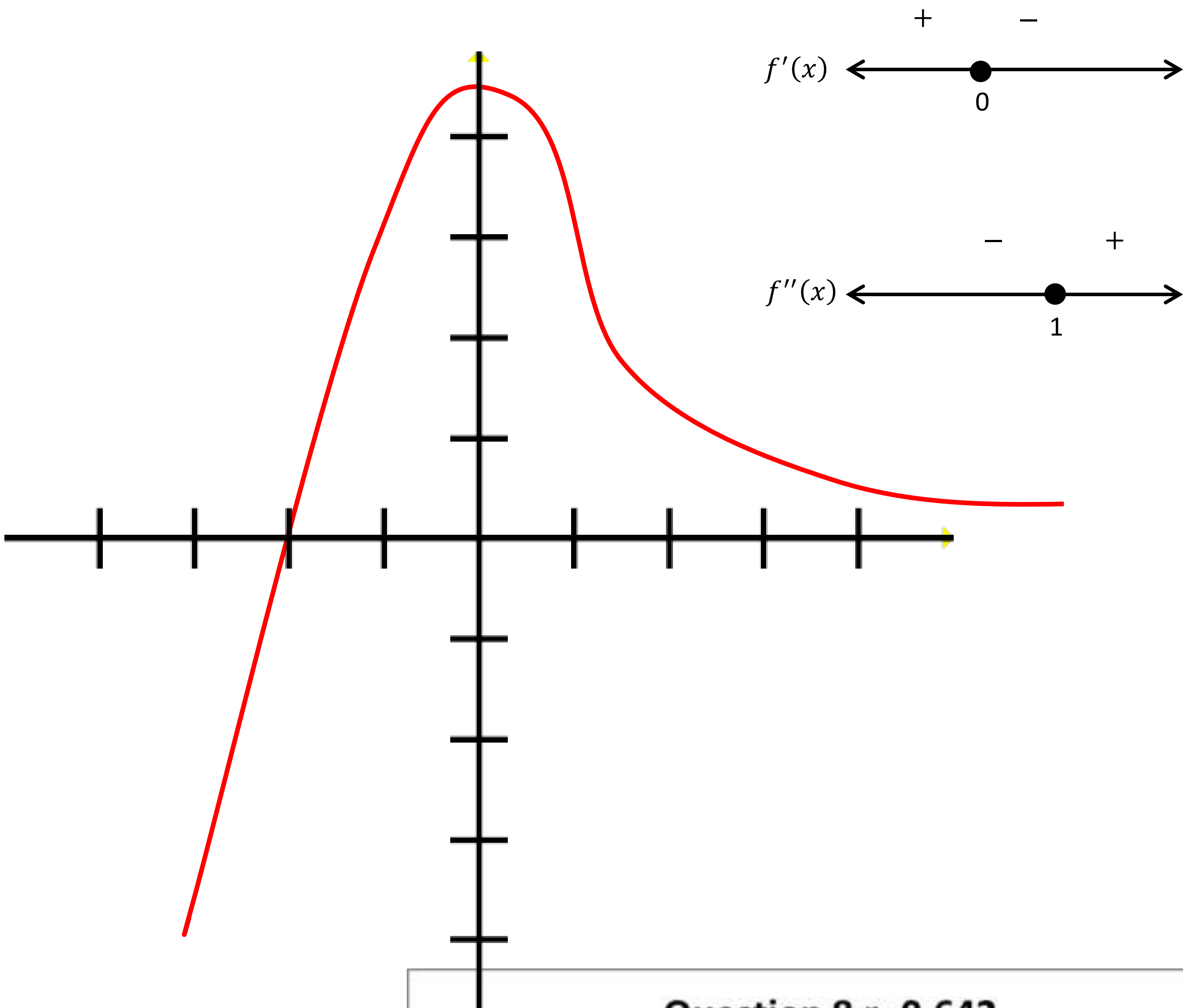
$$x = -3$$
$$y = -4$$
$$\frac{dx}{dt} = -6$$
$$\frac{dy}{dt} = -8$$
$$h = \sqrt{(-3)^2 + (-4)^2} = 5$$
$$\frac{dh}{dt} = ??$$

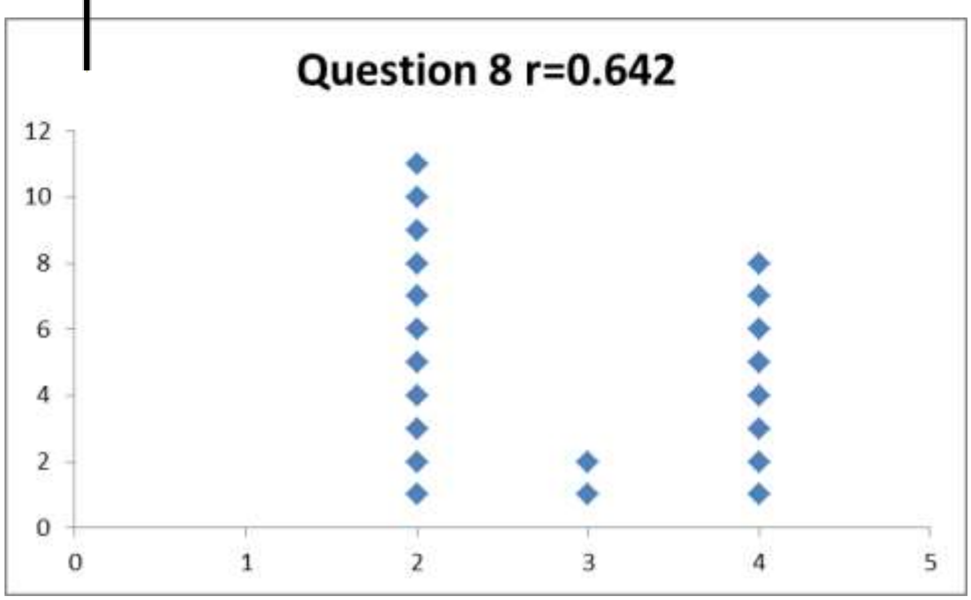
$$2hh' = 2xx' + 2yy'$$
$$h' = \frac{2xx' + 2yy'}{2h} = \frac{2(-3)(-6) + 2(-4)(-8)}{2(5)} mi/h$$

It was common to lose one point on this problem for not using negative signs. Indeed, they canceled out in this problem. But they don't always do so. So if you didn't have negatives I don't know if you *knew* they would cancel out, or just got lucky. Show your work!!

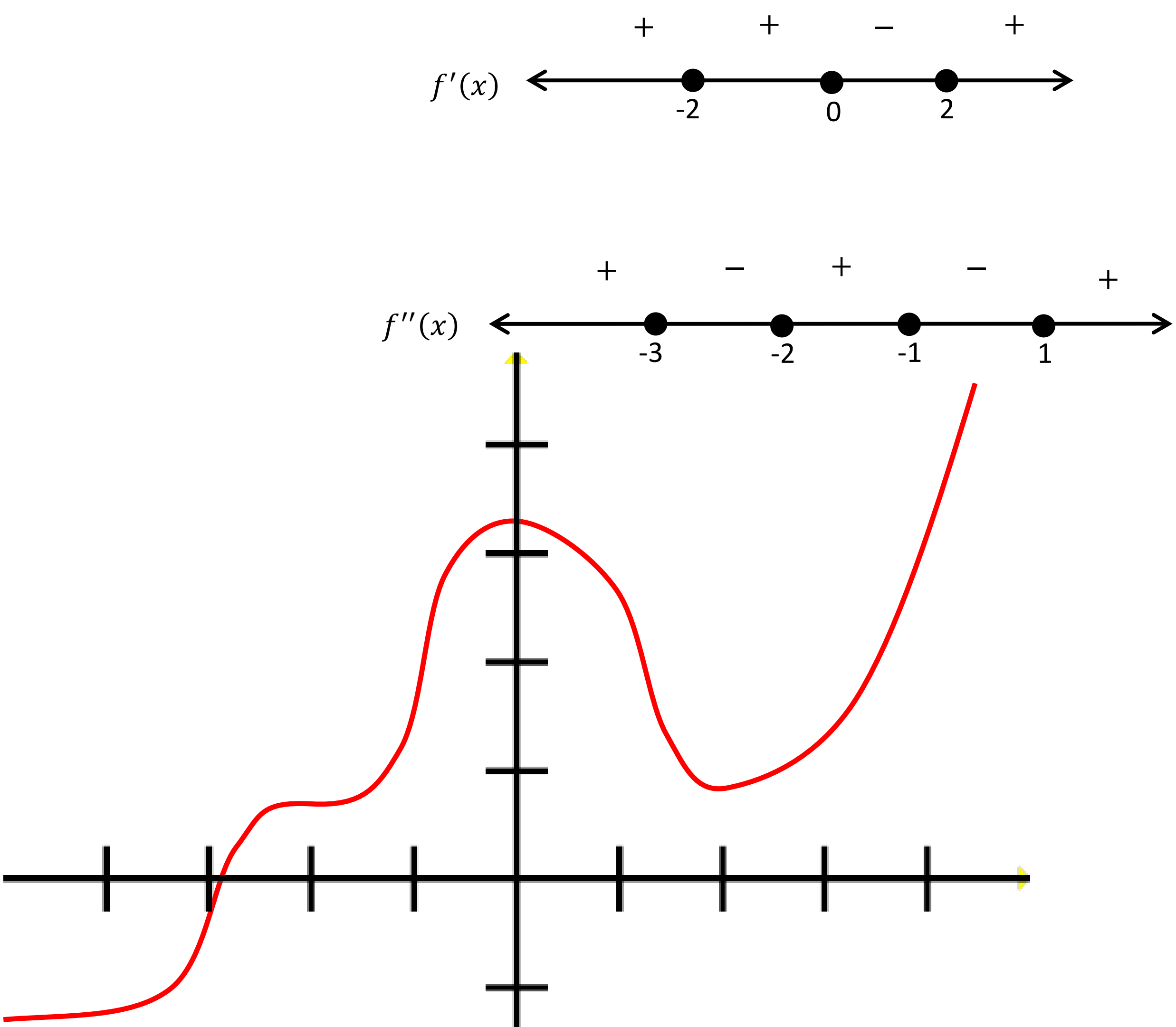


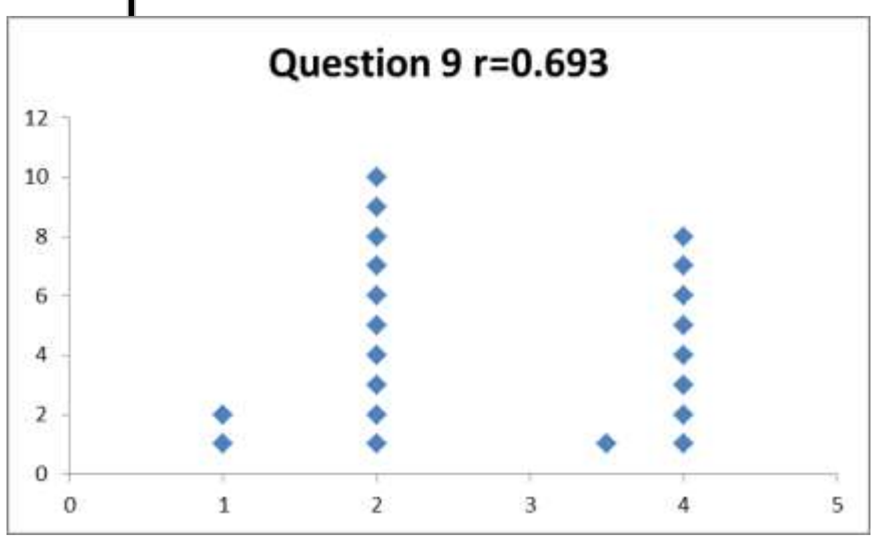
8) Sketch a continuous and differentiable function that satisfies the sign charts below.
(4 points)





9) Sketch a continuous and differentiable function that satisfies the sign charts below.
(4 points)

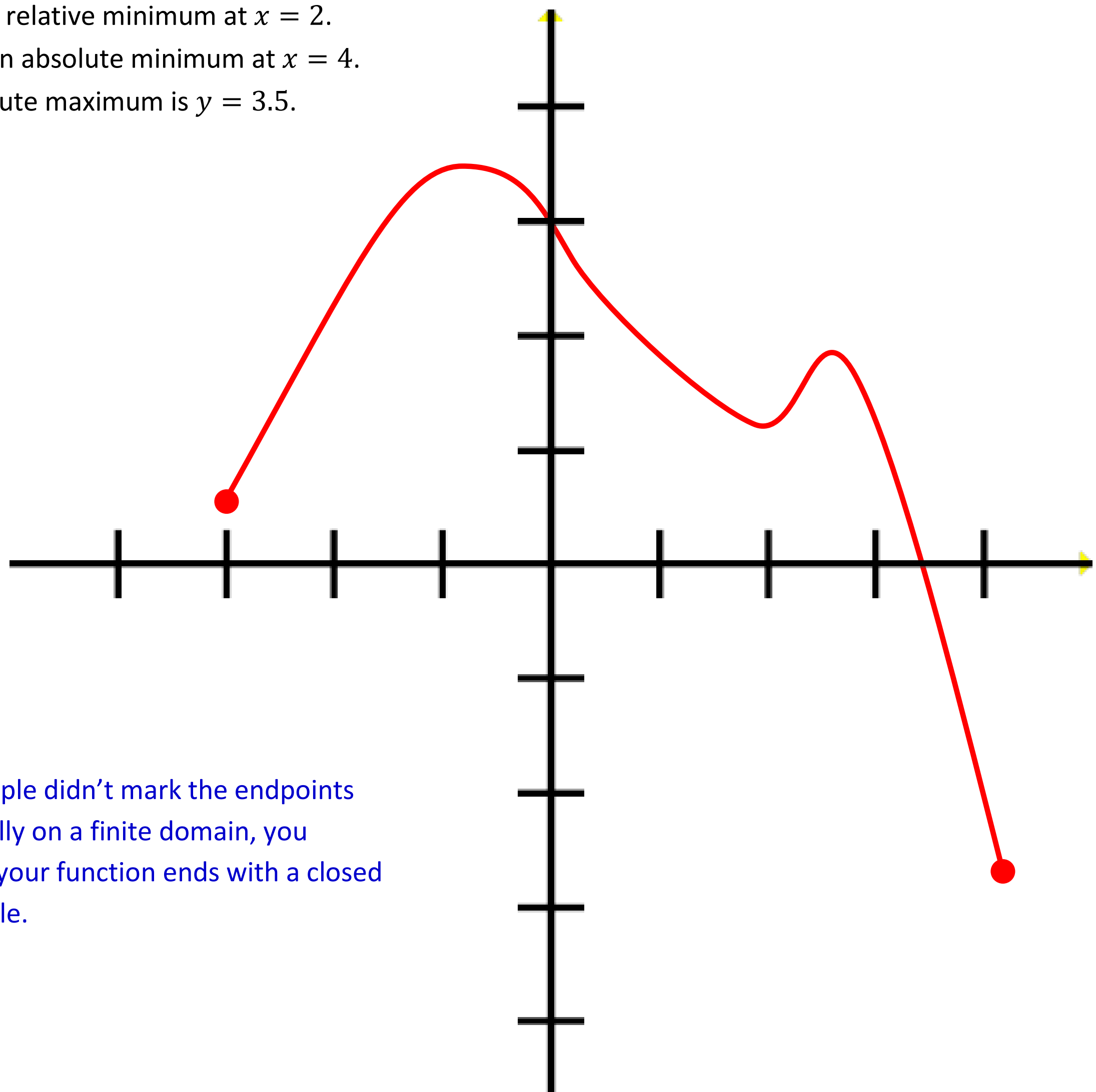




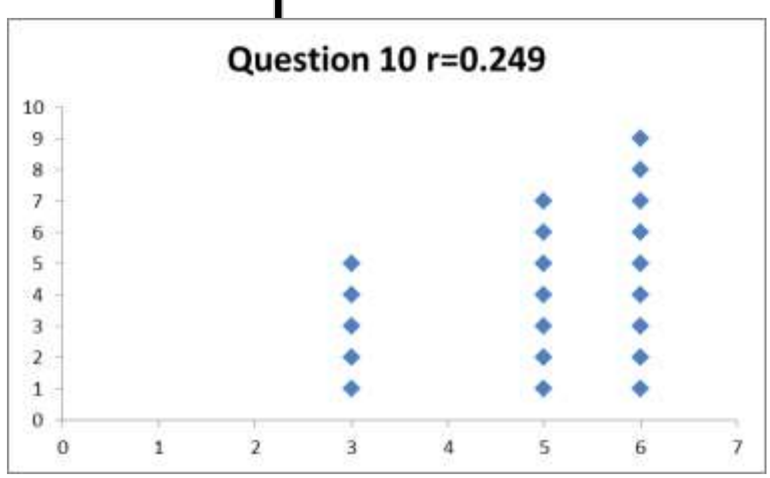
10) Sketch a continuous and differentiable function that has no asymptotes and is defined on $[-3,4]$ that satisfies:

- There is a local minimum at $x = -3$.
- There is an absolute maximum at $x = -1$.
- There is a relative minimum at $x = 2$.
- There is an absolute minimum at $x = 4$.
- The absolute maximum is $y = 3.5$.

(6 points)



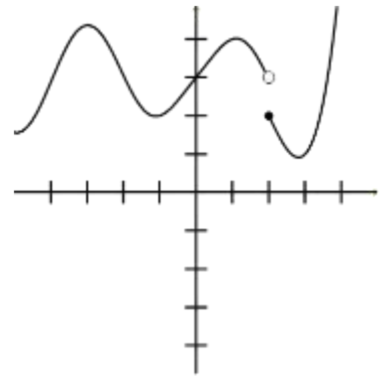
Note: A lot of people didn't mark the endpoints very well. Especially on a finite domain, you should note how your function ends with a closed circle or open circle.

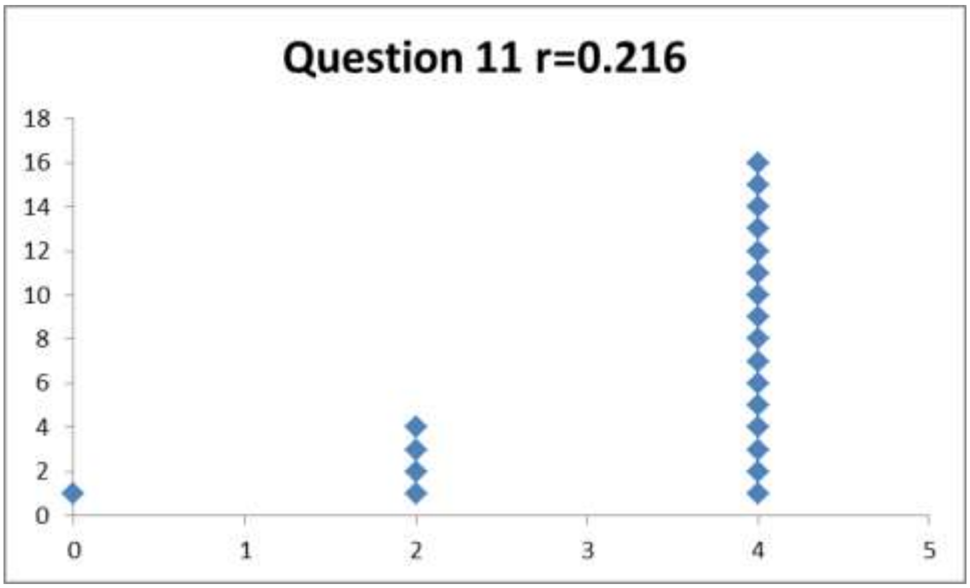


11) Find all minimizers of the function.

(4 points)

$x = -1, 3$

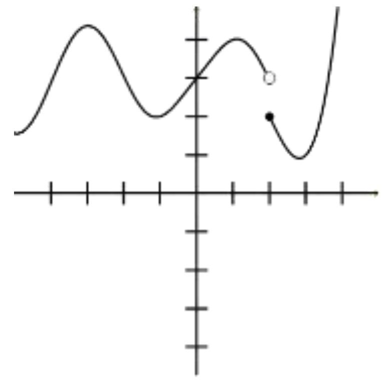


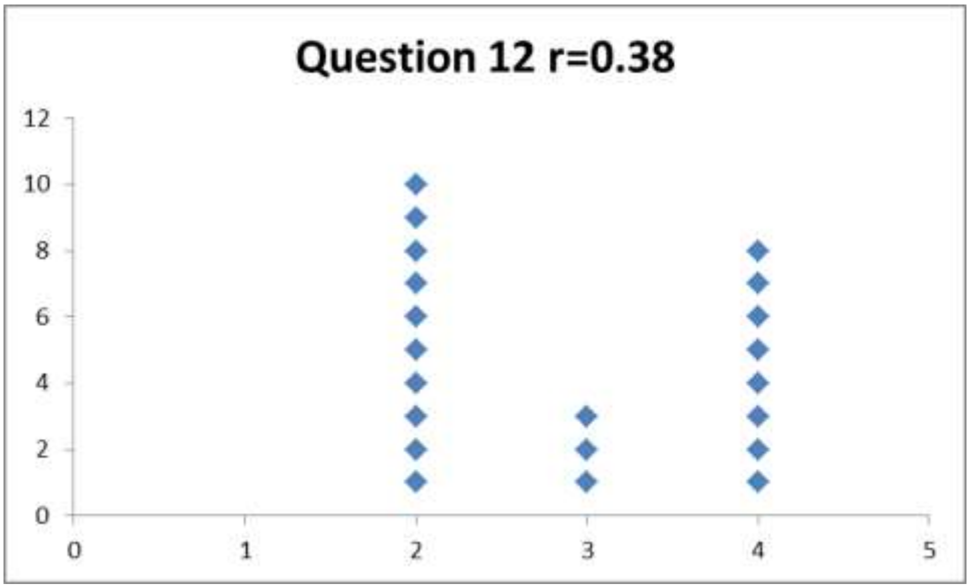


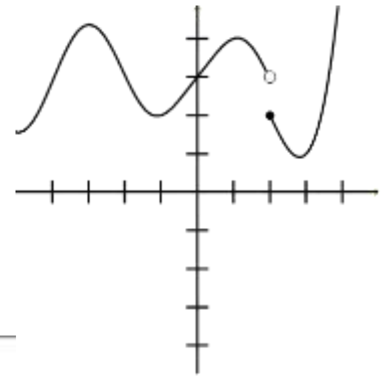
12) Find all relative maximum values of the function.

(4 points)

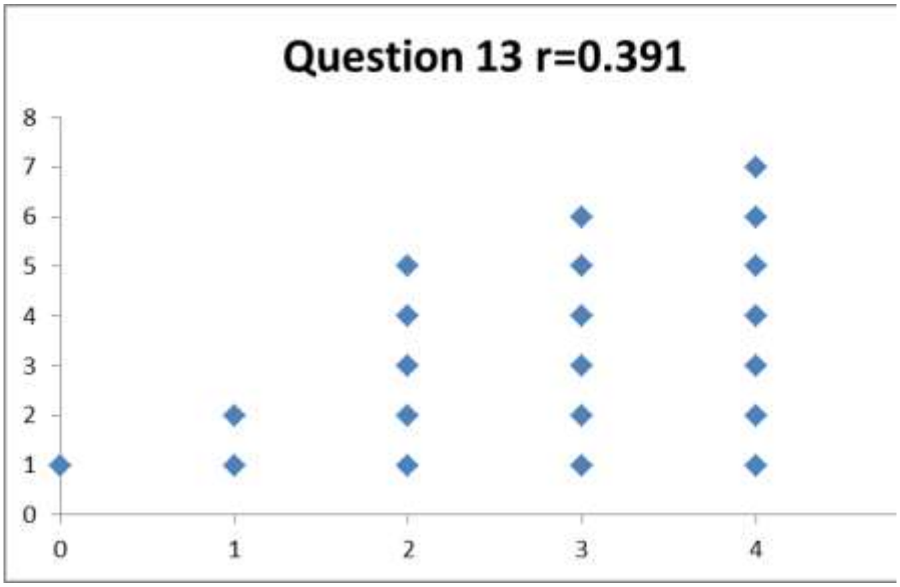
$y = 4.5, 4$







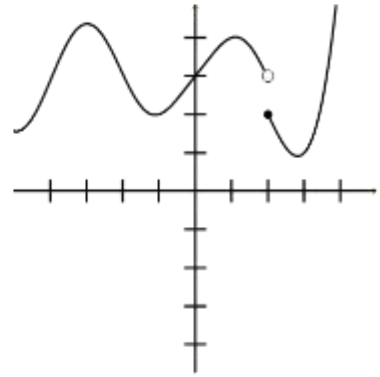
13) Find all x-values where the function changes concavity.
(4 points)
$x = -4, -2, 0, 2$

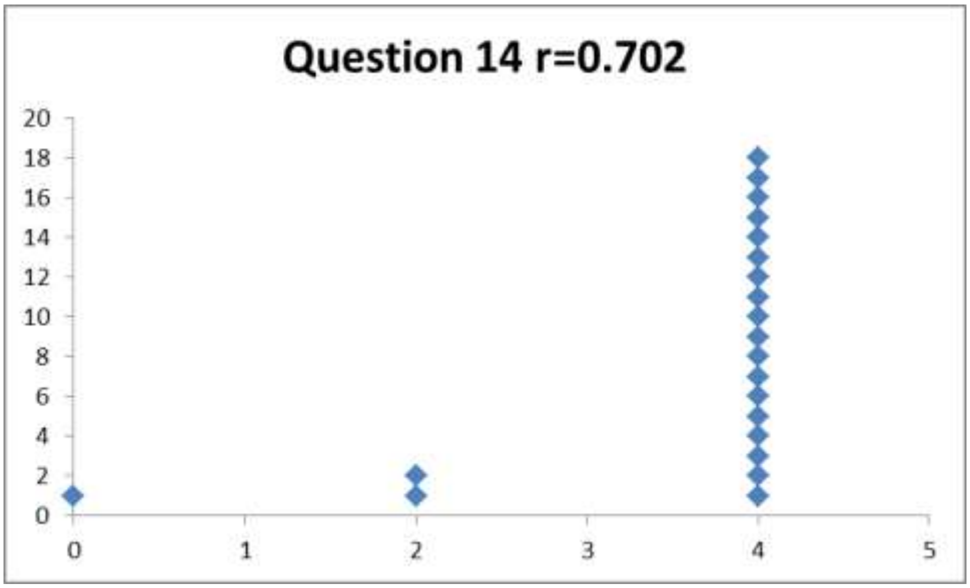


14) Approximate $f(1)$

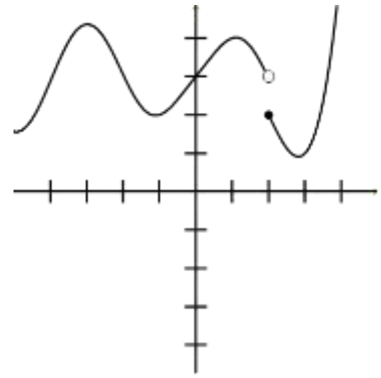
(4 points)

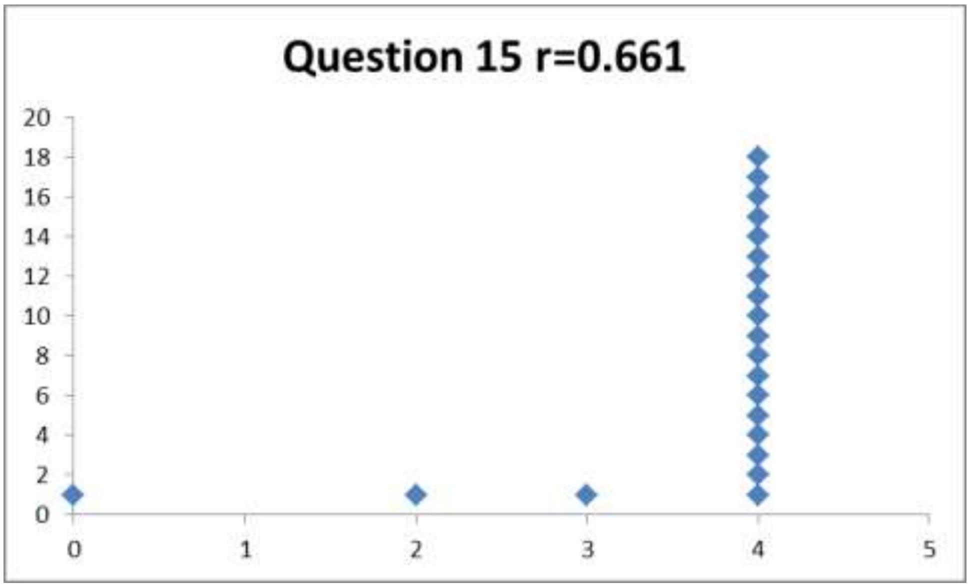
$y = 4$



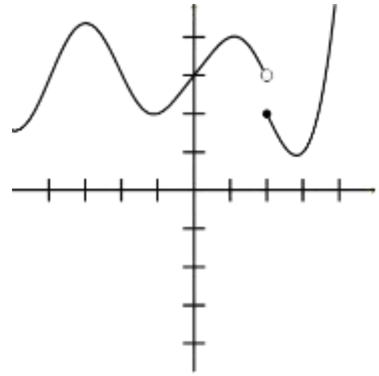


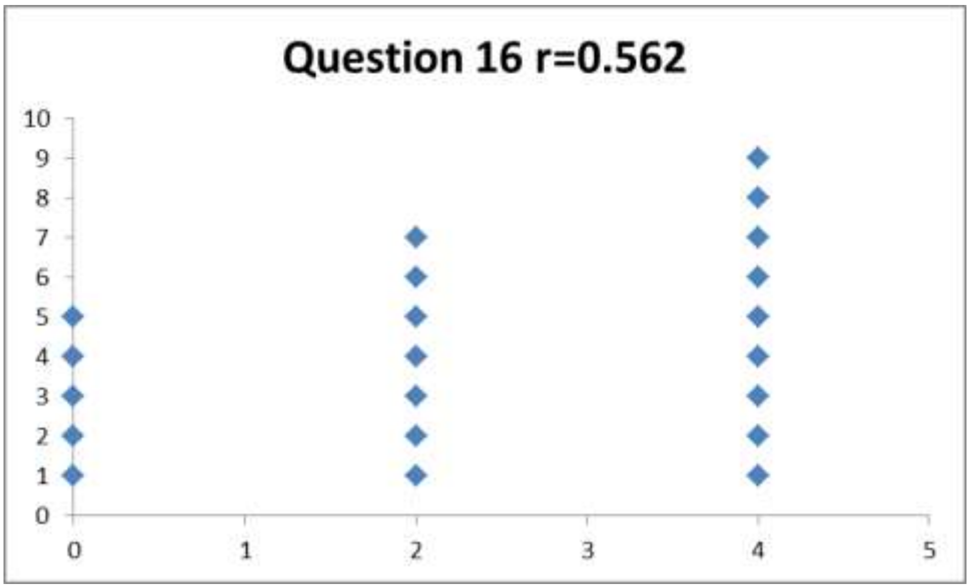
15) Approximate $f'(1)$.
(4 points)
$y' = 0$





16) Approximate $f''(1)$.
(4 points)
$y' = -2$
(This is a real big guess. Maybe it's $-\frac{1}{2}$? Or -10? The important thing is that it's negative)



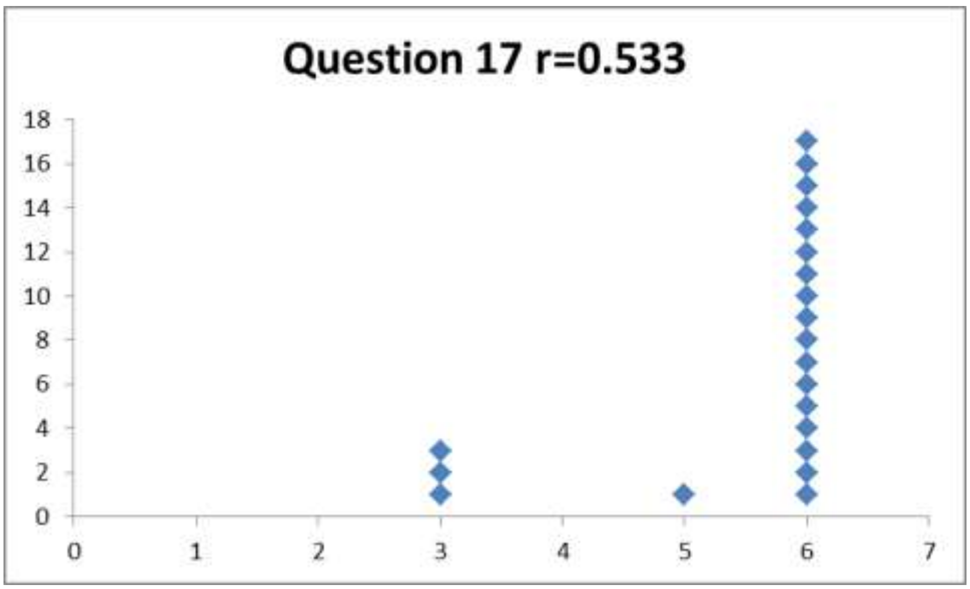


17) Find the limit below.

$$\lim_{x \to 5} \frac{\sqrt{x-4}-1}{3x-15}$$

(6 points)

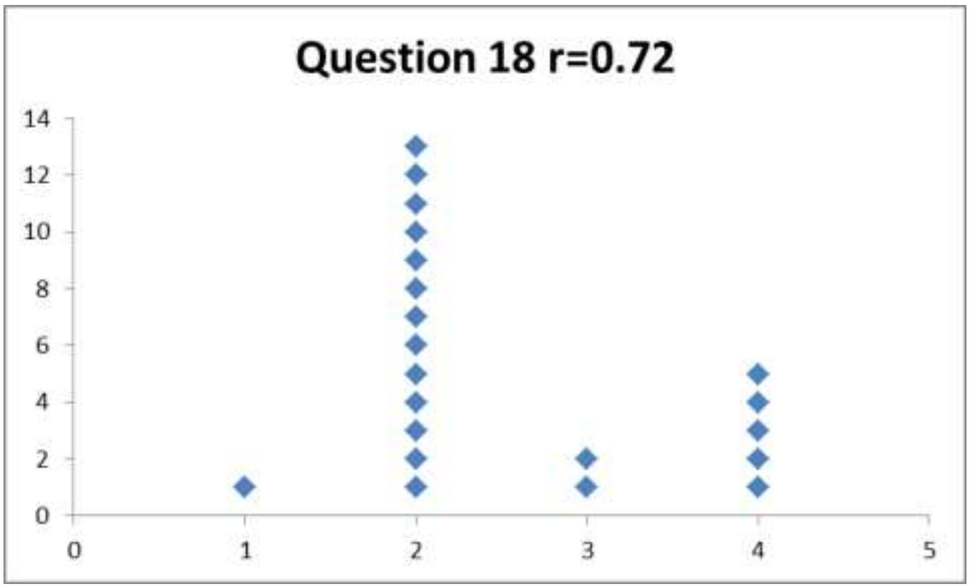
$$\lim_{x \to 5} \frac{\sqrt{x-4}-1}{3x-15} = \lim_{x \to 5} \frac{\frac{1}{2\sqrt{x-4}}}{3} = \frac{\frac{1}{2 \cdot \sqrt{1}}}{3} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$



18) Find $\frac{d}{dx}\cot^2(x^3)$

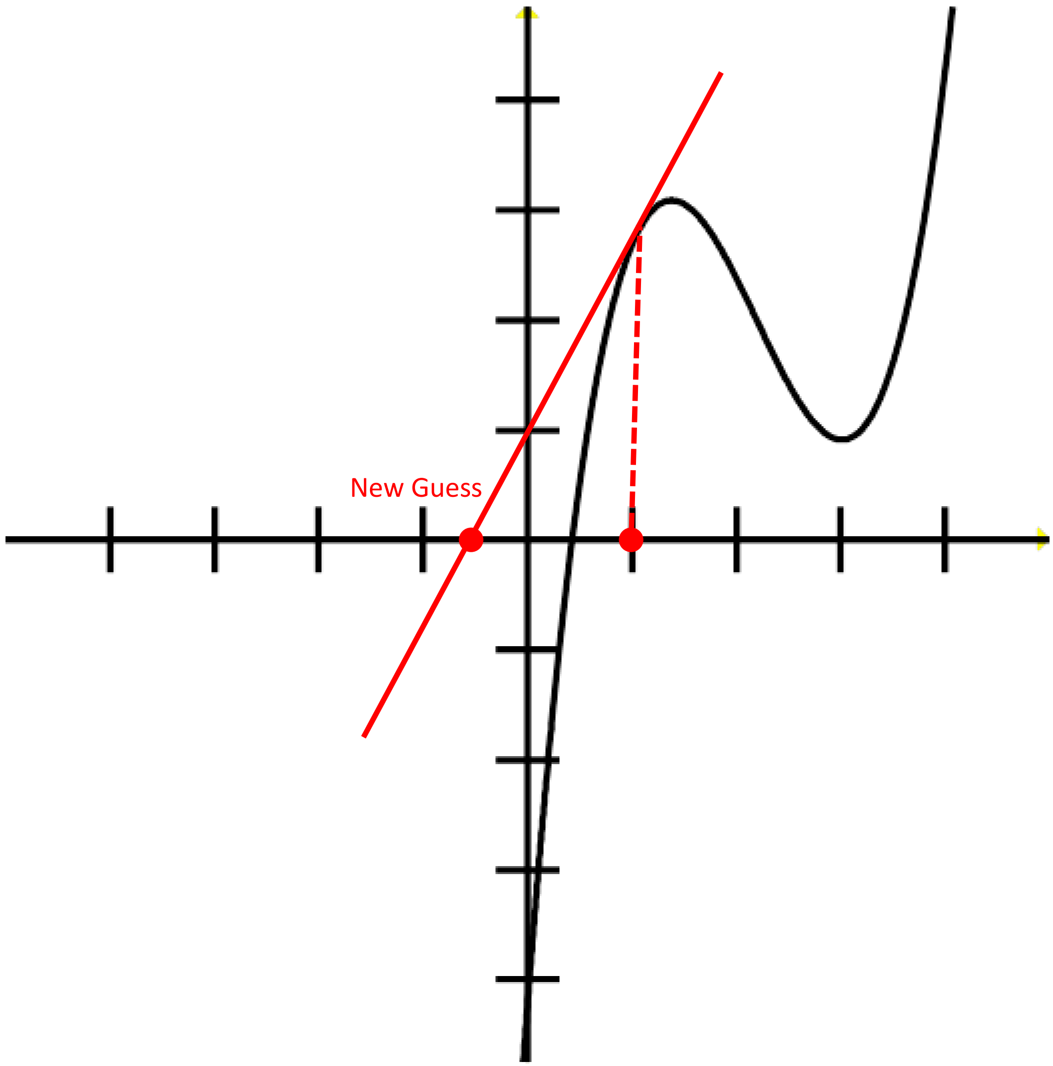
(4 points)

$$\frac{d}{dx}\cot^2(x^3) = 2\cot(x^3)\cdot(-\csc^2(x^3))\cdot 3x$$

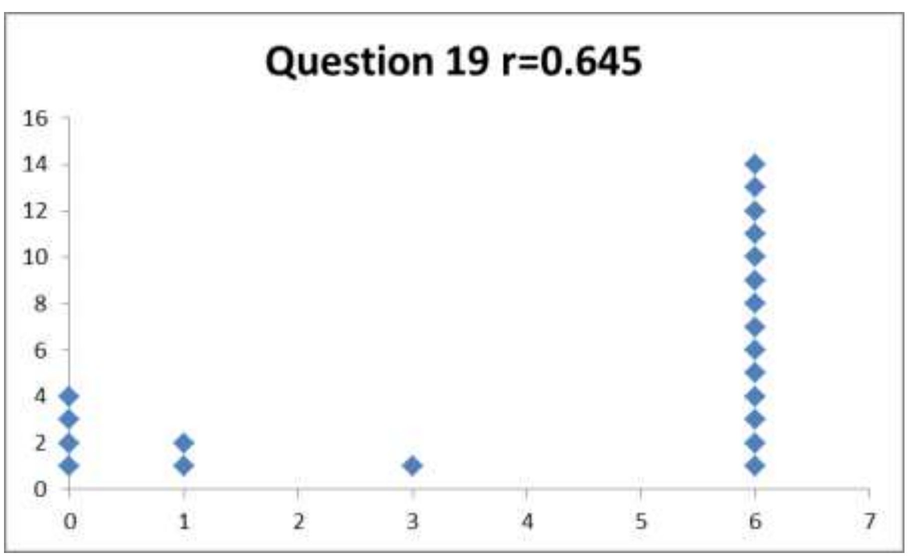


19) On the graph below, suppose we think that a root of the function is at $x = 1$. Use Newton's method to graphically find a better solution.

(6 points)



Will the new guess necessarily be closer to the actual root? In the first few iterations, maybe not. Just like a limit might not get closer to its value in the first few rows of the table. But as we take more and more iterations it will converge to the root. Here I only asked for one iteration.

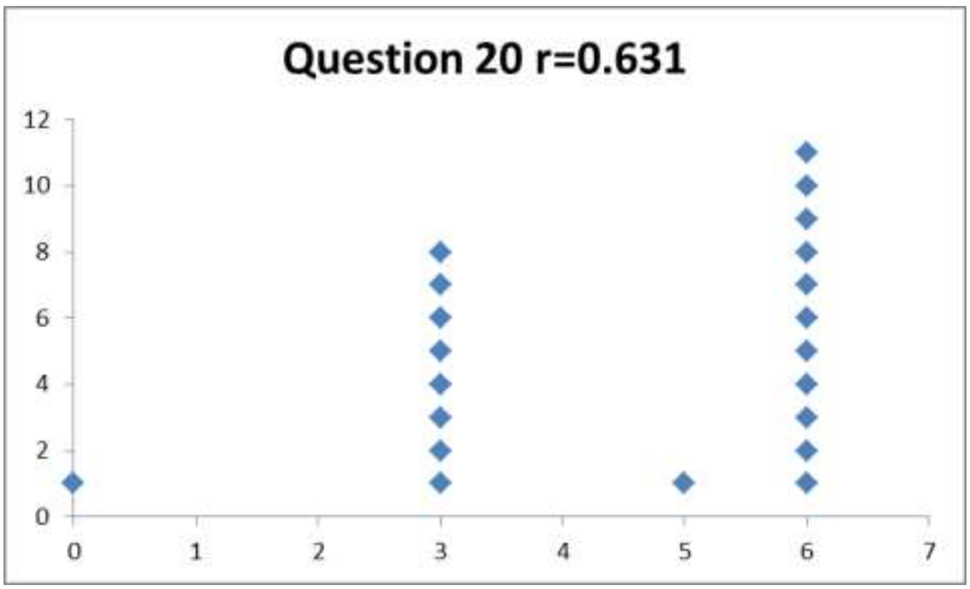


20) Find the limit below.

$$\lim_{x \to e} \frac{\ln(x) - 1}{x - e}$$

(6 points)

$$\lim_{x \to e} \frac{\ln(x) - 1}{x - e} = \lim_{x \to e} \frac{\frac{1}{x}}{1} = \frac{1}{e}$$



Algebra Mistakes

These are points that were deducted for algebra and general mathematical grammar. Look for the green ink.