Name ____ Test 3, Fall 2017

1) Find the integral below. (8 points)

$$\int 4x\cos(4x^2+3)\,dx = \frac{1}{2}\int \cos(u)\,du = \frac{1}{2}\sin(u) + C = \frac{1}{2}\sin(4x^2+3) + C$$

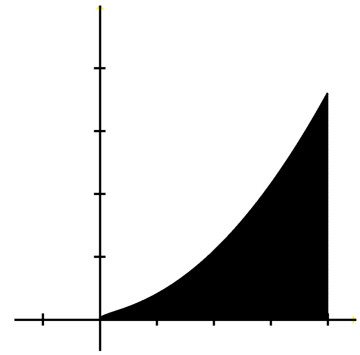
$$u = 4x^2 + 3$$
$$du = 8xdx$$

2) Find the integral below. (8 points)

$$\int \frac{1}{10x-3}dx = \frac{1}{10}\int \frac{1}{u}du = \frac{1}{10}\ln(|u|) + C = \frac{1}{10}\ln(|10x-3|) + C$$

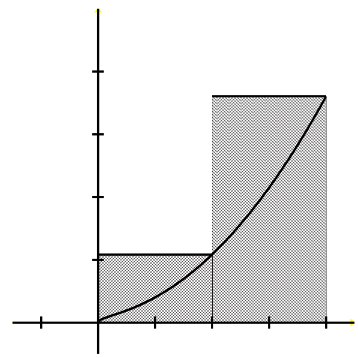
$$u = 10x - 3$$
$$du = 10dx$$

3) Illustrate (do not calculate) the area under the curve given below. (2 points)



4) Illustrate (do not calculate) an approximation to the area under the curve given below. (2 points)

(There are multiple answers)



5) Calculate the approximation you illustrated in the previous question. (2 points)

(There are multiple answers, but each is unique and based on your answer above)

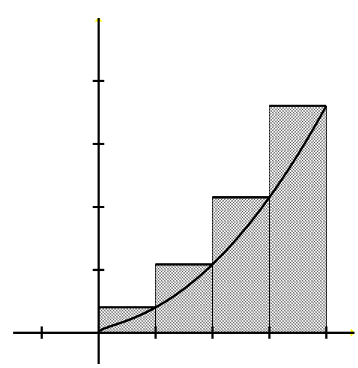
$1.1 \cdot 2 + 3.5 \cdot 2$

6) Is your approximation above an overestimate or an underestimate? (1 point)

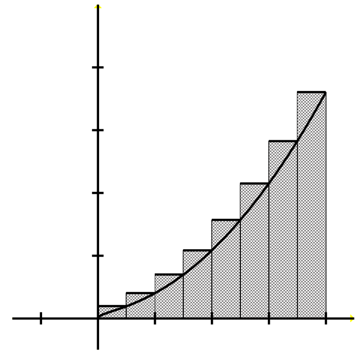
(There are multiple answers, but each is unique and based on your answer above)

$1.1 \cdot 2 + 3.5 \cdot 2$

7) Illustrate a better approximation than you came up with. (2 points)



8) Illustrate an even better approximation than you came up with in the previous question. (1 points)



9) Find the integral below. (8 points)

$$\int_0^1 2x(4-x^2)dx = \int_4^3 -udu = -\frac{u^2}{2}\bigg|_4^3 = -\frac{3^2}{2} - \left(-\frac{4^2}{2}\right) = -\frac{9}{2} + 8$$

$u = 4 - x^2$
$du = -2xdx$

OR

$$\int_0^1 2x(4-x^2)dx = \int_{x=0}^{x=1} -udu = -\frac{u^2}{2}\bigg|_{x=0}^{x=1} = -\frac{(4-x^2)^2}{2}\bigg|_0^1 = -\frac{3^2}{2} - \left(-\frac{4^2}{2}\right) = -\frac{9}{2} + 8$$

$u = 4 - x^2$
$du = -2xdx$

10) Find the integral below. (8 points)

$$\int_0^{\ln(4)} \frac{e^x}{3+2e^x}\,dx = \frac{1}{2}\int_5^{11} \frac{du}{u} = \frac{1}{2}\ln|u|\Big|_5^{11} = \frac{1}{2}\ln(11) - \frac{1}{2}\ln(5)$$

$u = 3 + 2e^x$
$du = 2e^x dx$

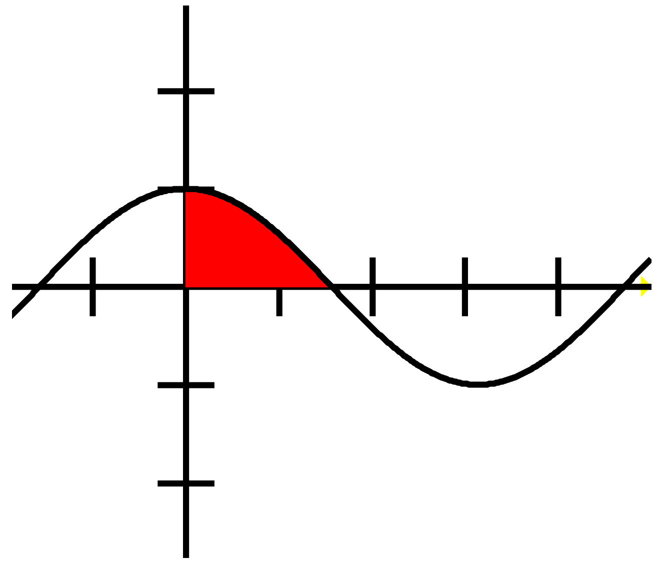
When $x = 0, u = 5$
When $x = \ln(4), u = 11$

11) The curve $y = \cos(x)$ is shown to the right. The region bounded by this curve, $x = 0$, and $y = 0$ is to be rotated around the x-axis.

(a) Shade the region described. (1 point)

(b) Describe the 3-dimensional shape in words. (3 points)

It looks like a sideways Hershey's kiss, or some kind of misshaped cone.



(c) Set up, but do not evaluate, the integral for the volume of the solid created. Indicate your method. (6 points)

(Disk/washer) (Cylindrical Shell)

Disk method is easier in this case:

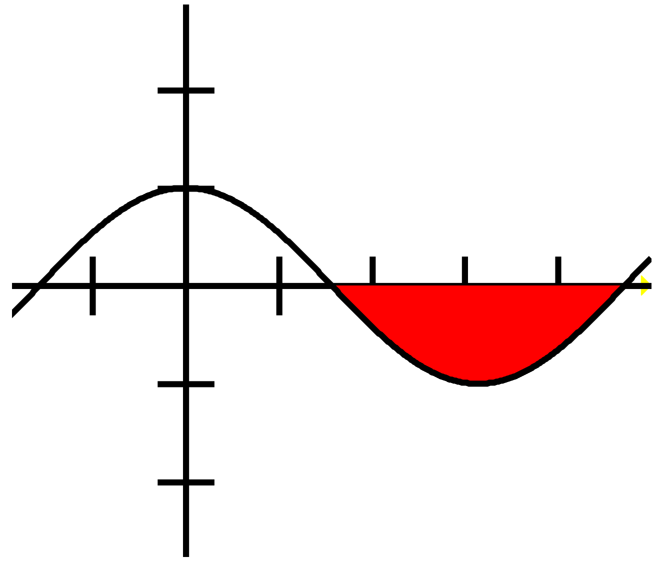
$$\int_0^{\frac{\pi}{2}} \pi r^2 dx = \int_0^{\frac{\pi}{2}} \pi \cos^2(x)\, dx$$

12) The curve $y = \cos(x)$ is shown to the right. The region bounded by this curve, $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, and $y = 0$ is to be rotated around the y-axis.

(a) Shade the region described. (2 points)

(b) Describe the 3-dimensional region in words. (3 points)

It's like half a doughnut.



(c) Set up, but do not evaluate, the integral for the volume of the solid created. Indicate your method. (6 points)
(Disk/washer) (Cylindrical Shell)

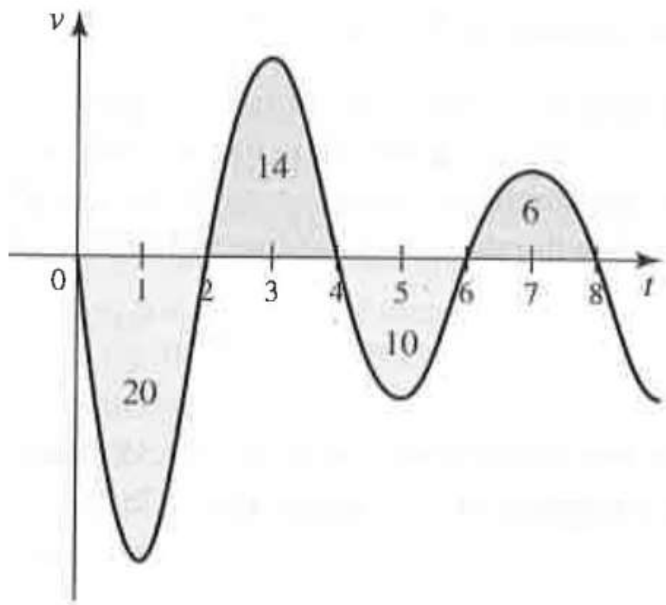
The shell method is easier in this case:

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\pi r h dx = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\pi x(-\cos(x))dx$$

Consider the velocity function shown below of an object moving along a line. Assume time is measured in seconds and distance is measured in meters. The area of four regions bounded by the velocity curve and the t-axis are also given.

13) On what intervals is the objet moving in the negative direction? (4 points)

$(0,2)$ and $(4,6)$



14) What is the displacement of the object over the interval $[2,6]$? (4 points)

4 meters

15) How far does the object travel over the interval $[0,6]$? (4 points)

44 meters

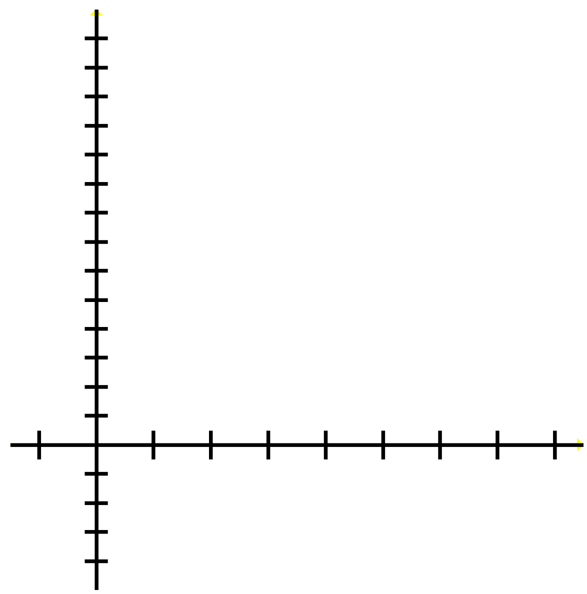
16) Describe the position of the object relative to its initial position after 8 seconds. (4 points)

10 meters in the negative direction.

An automobile is moving along a straight highway over a two-hour period. A table of velocities at certain times is given below. Below is a table of values of a function.

$t\ (hr)$	0	0.5	1	1.5	2
$v\ (mi/hr)$	20	25	30	35	35

17) Sketch a smooth curve passing through the data points. (3 points)



18) Approximate, as closely as you can, the total distance travelled. If you cannot find the exact distance, find an overestimate. (6 points)

19) Find each of the integrals below. (4 points each)

$$\int 3x^2 + 2x + 1dx$$

$$\int \sin(2x)\, dx$$

$$\int 3^5 dx$$