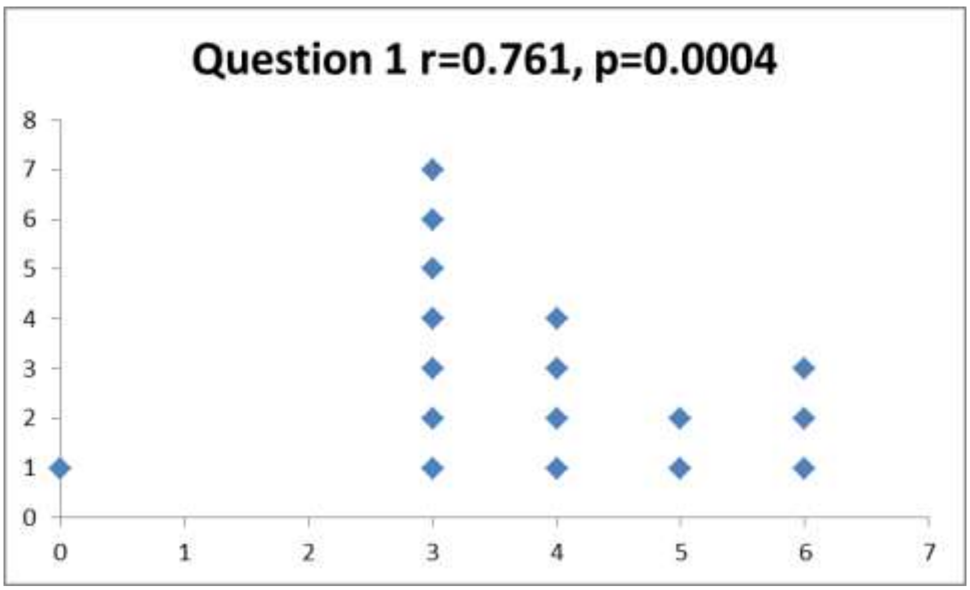
Name __Solutions____ Calculus I, Test 1, Spring 2017

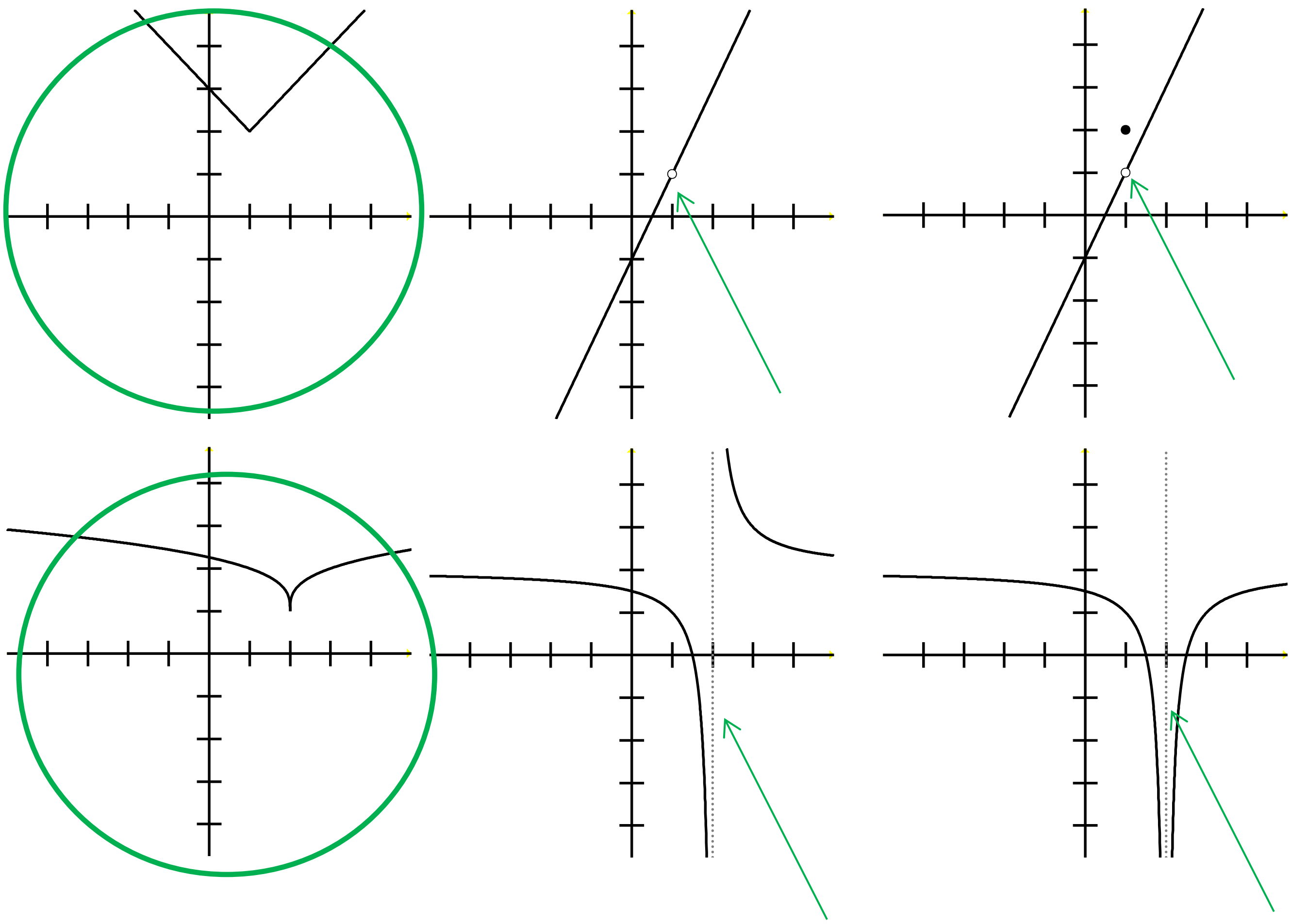
Please show all your work and circle your answer when appropriate. You do not need to simplify answers unless the problem specifies to do so.

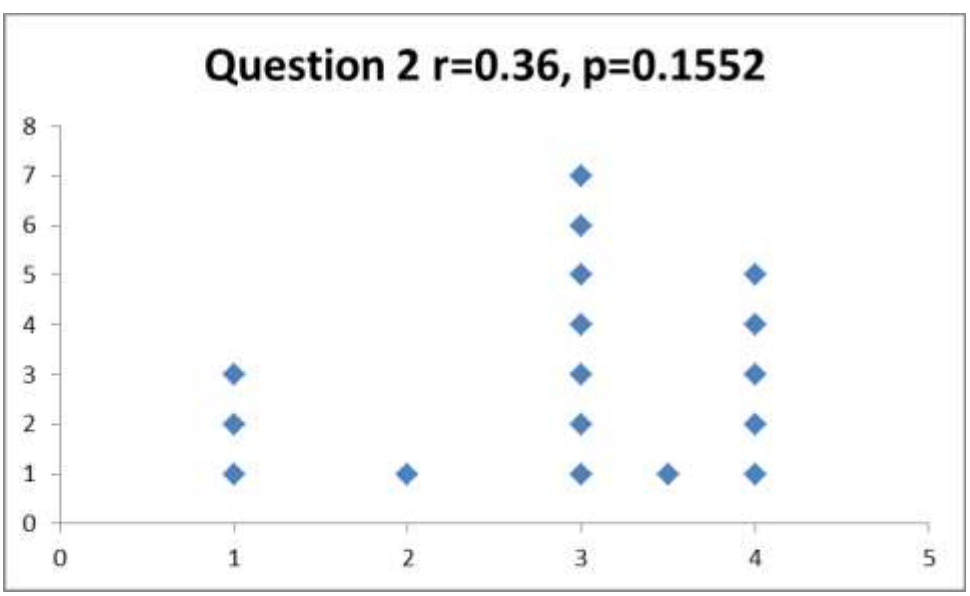
1) Find $\frac{1}{\sqrt{x}} \cdot \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$. (6 points)

$$\frac{1}{\sqrt{x}} \cdot \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\sqrt{x}} \cdot \frac{d}{dx}\left(x^{-\frac{1}{2}}\right) = \frac{1}{\sqrt{x}} \cdot \left(-\frac{1}{2}x^{-\frac{3}{2}}\right) = -\frac{1}{2x^2}$$



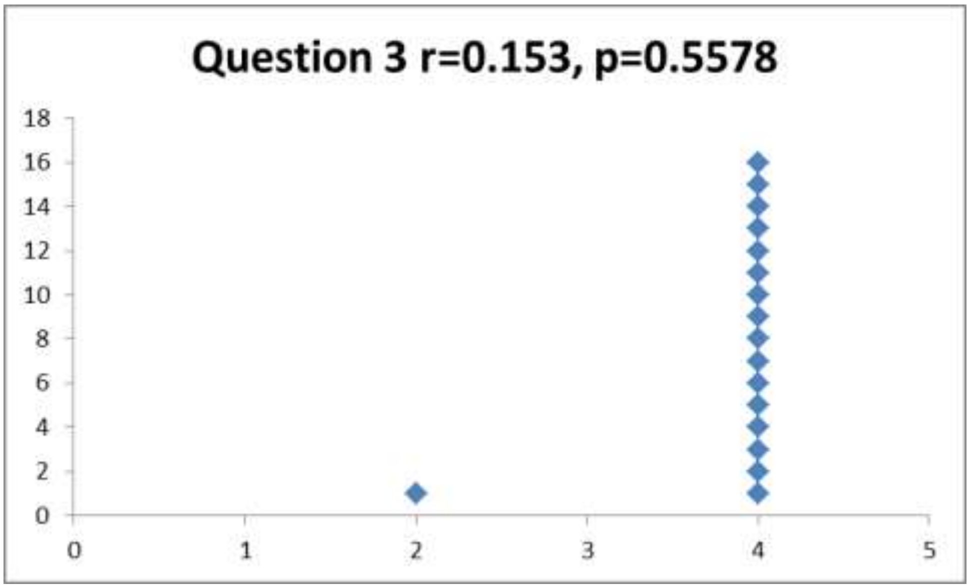
2) Below are graphs of 6 functions. Some of them are continuous functions on the real line. Some of them are not. Circle those that are continuous. For those that are not continuous, draw a single arrow pointing to a feature of the graph that makes you think it's not continuous. (4 points)





3) Calculate $\lim_{x \to 1} \frac{5x^2+6x+1}{8x-4}$ (4 points)

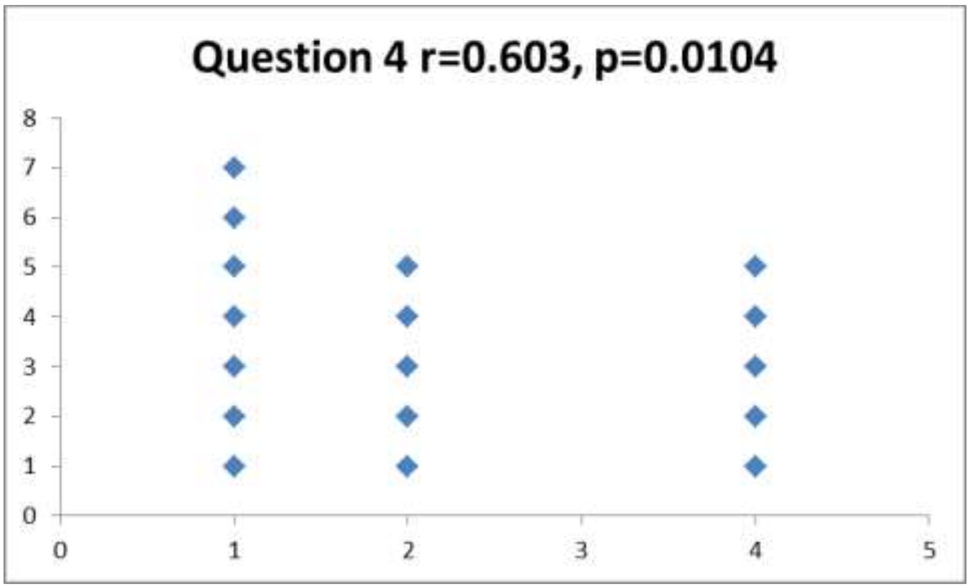
$$\lim_{x \to 1} \frac{5x^2+6x+1}{8x-4} = \frac{5+6+1}{8-4} = \frac{12}{4} = 3$$



4) Calculate $\lim_{x \to 1^+} \frac{x-2}{(x-1)^3}$ (4 points)

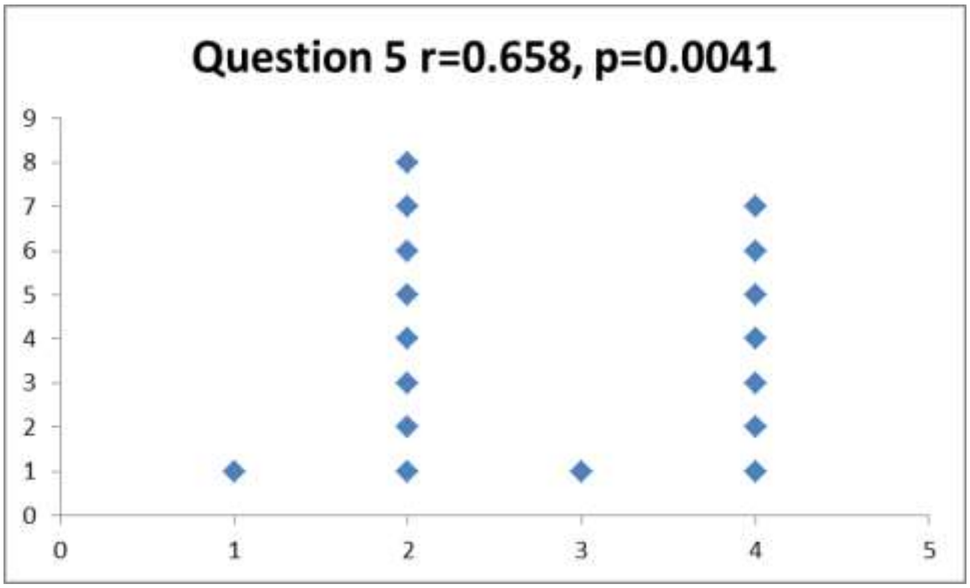
$$\lim_{x \to 1^+} \frac{x-2}{(x-1)^3} = -\infty$$

(Note that the numerator is negative!!)



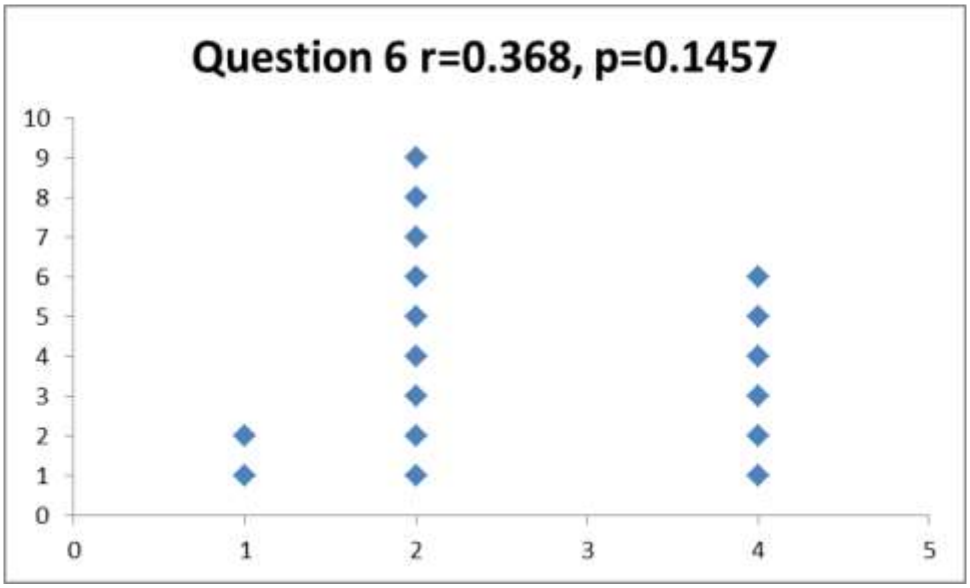
5) Calculate $\lim_{x \to -2^-} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2}$ (4 points)

$$\lim_{x \to -2^-} \frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2} = \lim_{x \to -2^-} \frac{x(x^2 - 5x + 6)}{x^2(x^2 - 4)} = \lim_{x \to -2^-} \frac{x(x-2)(x-3)}{x^2(x-2)(x+2)} = \lim_{x \to -2^-} \frac{x(x-3)}{x^2(x+2)} = -\infty$$

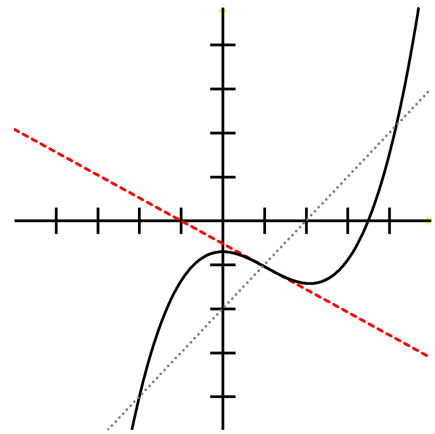


6) Calculate $\lim_{x \to -b} \frac{(x+b)^7 + (x+b)^{10}}{4(x+b)}$ (4 points)

$$\lim_{x \to -b} \frac{(x+b)^7 + (x+b)^{10}}{4(x+b)} = \lim_{x \to -b} \frac{(x+b)^6 + (x+b)^9}{4} = 0$$



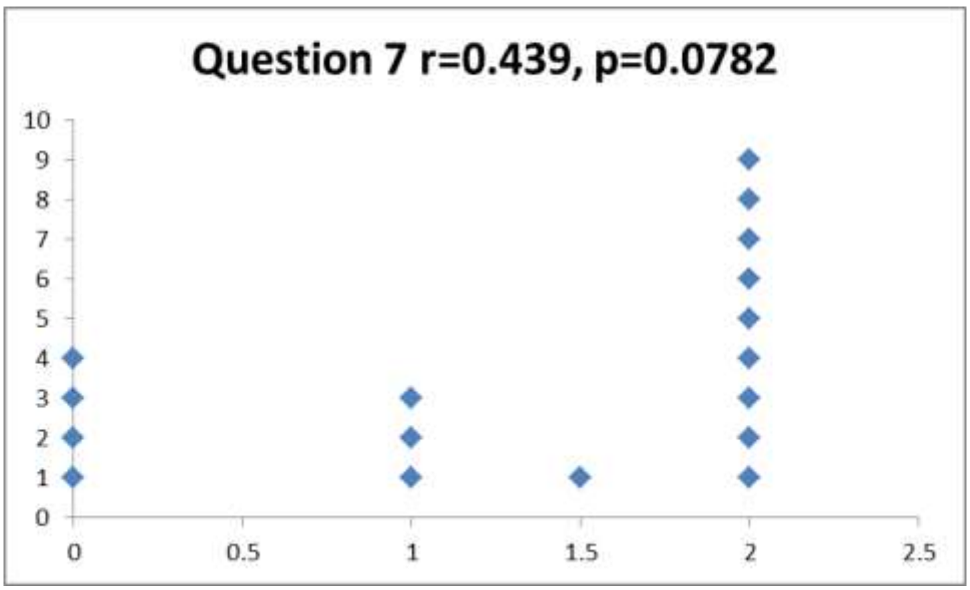
7) Given the graph to the right, estimate the average rate of change between the points given by $x = 1$ and $x = 4$. (2 points)



The grey line represents the average rate of change.

$$\frac{\Delta y}{\Delta x} = \frac{2-(-1)}{4-1} = \frac{3}{3} = 1$$

It's an estimate, so anything on the interval $\left[\frac{1}{2}, 3\right]$ was given full credit.

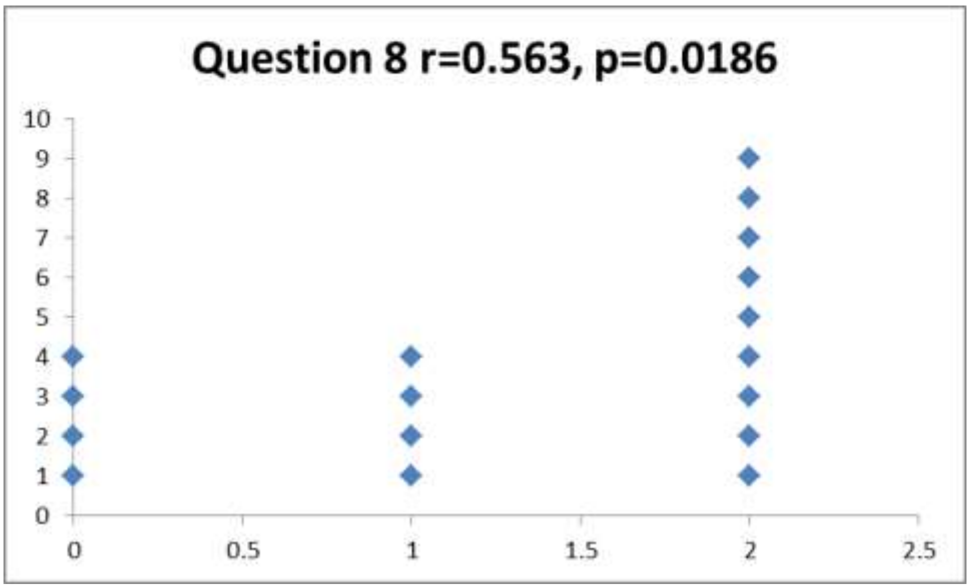


8) Given the graph to the right, estimate the instantaneous rate of change at point given by $x = 1$. (2 points)

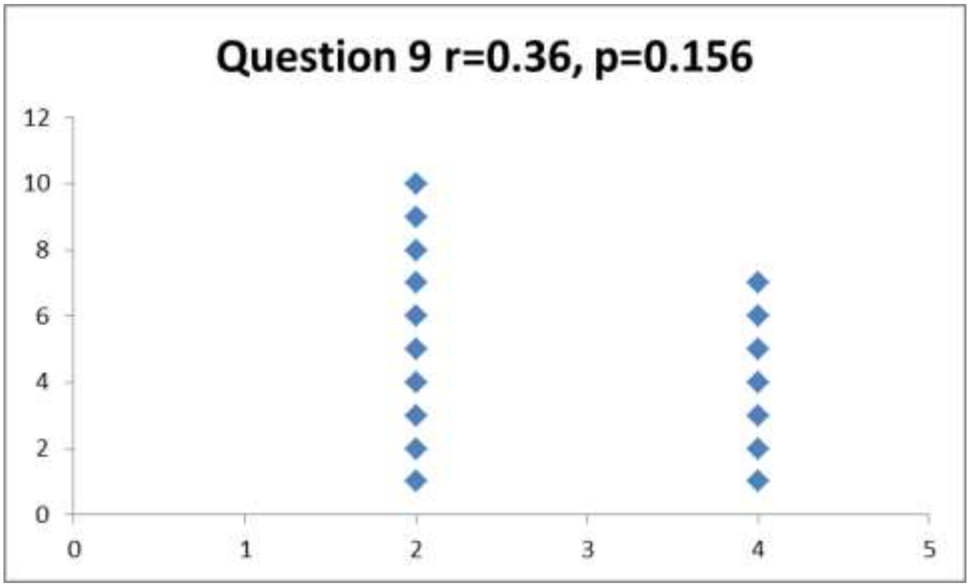
This is the red line.

$$\frac{\Delta y}{\Delta x} = \frac{-1}{2} = -\frac{1}{2}$$

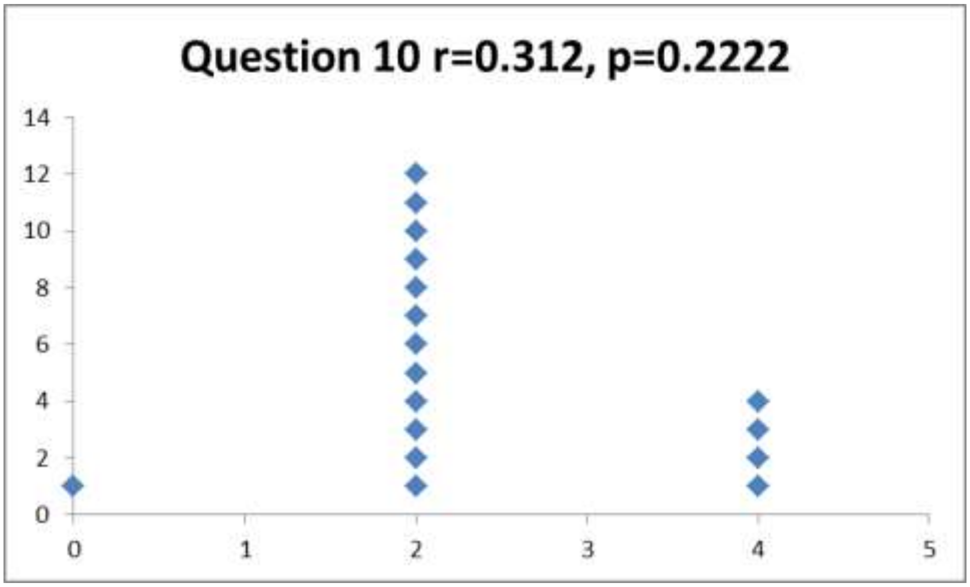
It's an estimate, so anything on the interval $[-0.25, -1.5]$ was given full credit.



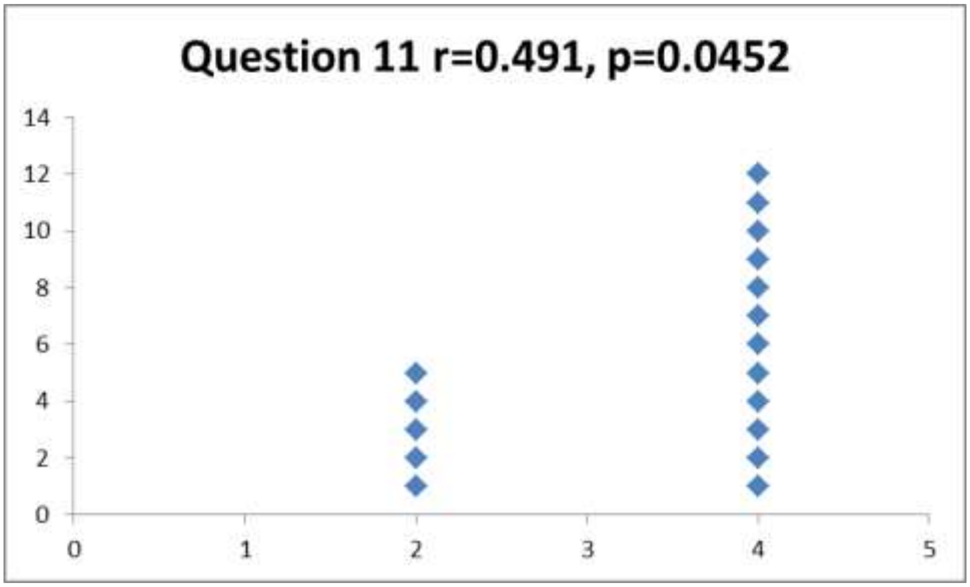
9) Find $\lim_{x \to \infty} x^{-6} = 0$



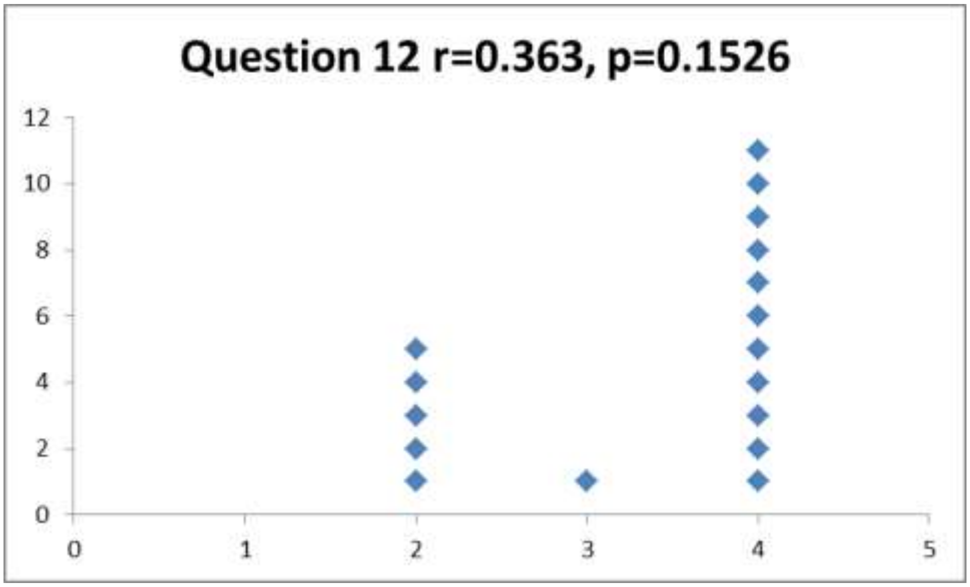
10) Find $\lim_{x \to -\infty} 2^x = 0$



11) Find $\lim_{x \to \infty} \frac{x^2+5}{x^2-3} = 1$



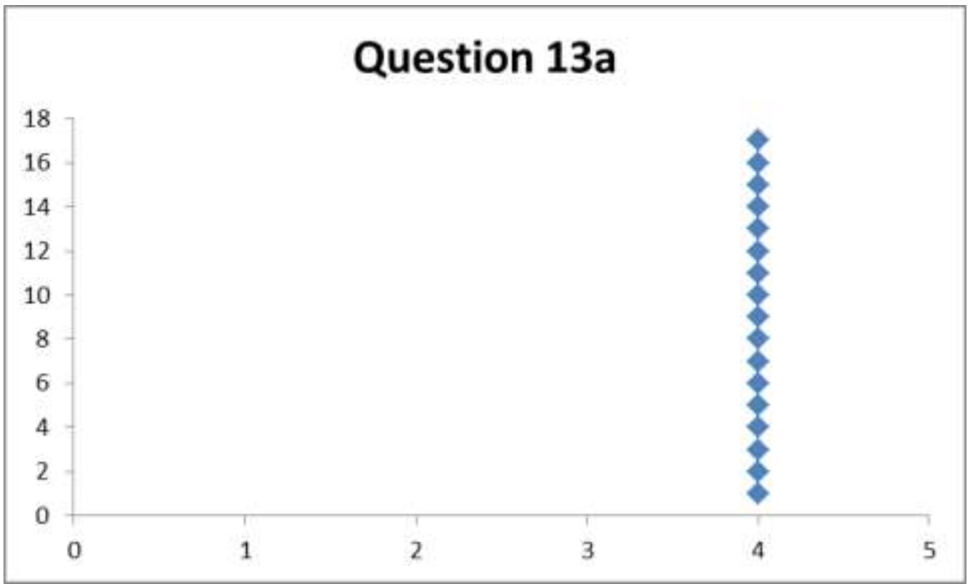
12) Find $\lim_{x \to -\infty} \frac{-x^3+2x+4}{x^2-2x^3} = \frac{1}{2}$



13) Find the derivative of each of the functions below.

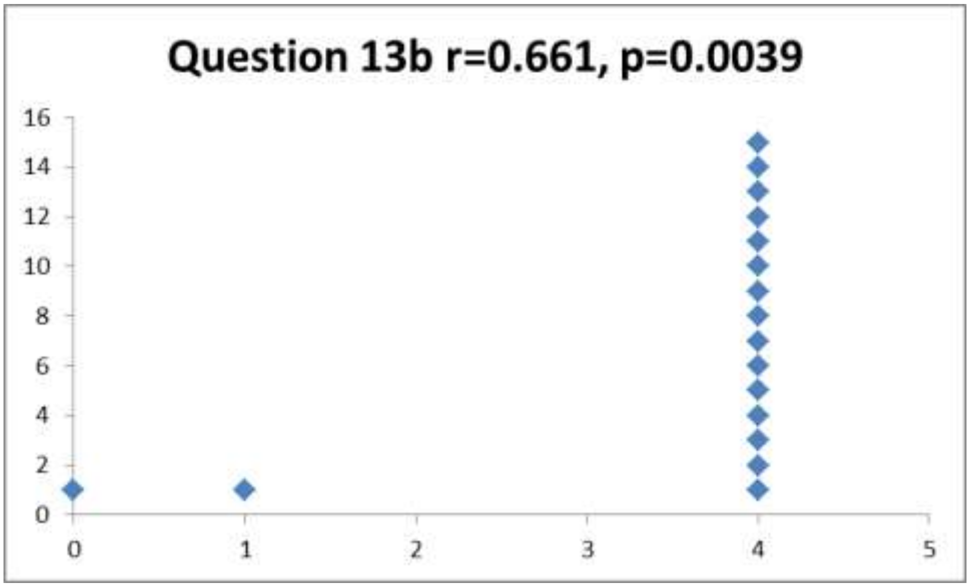
$(a)\ f(x) = 3x^5 - 2x^3$ (4 points)

$$f'(x) = 15x^4 - 6x$$



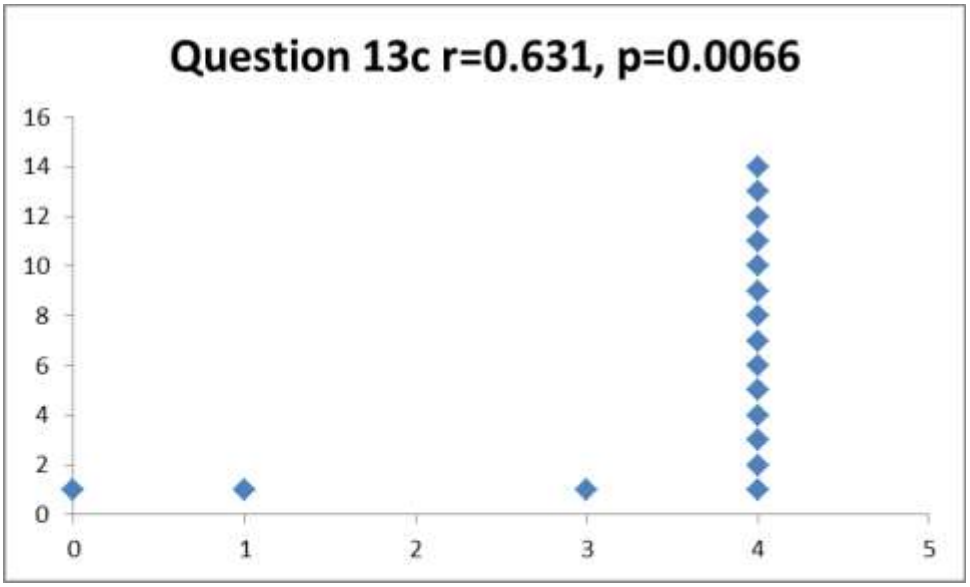
$(b)\ f(x) = 7$ (4 points)

$f'(x) = 0$



$(c)\ f(x) = \frac{x^2}{\sin(x)}$ (4 points)

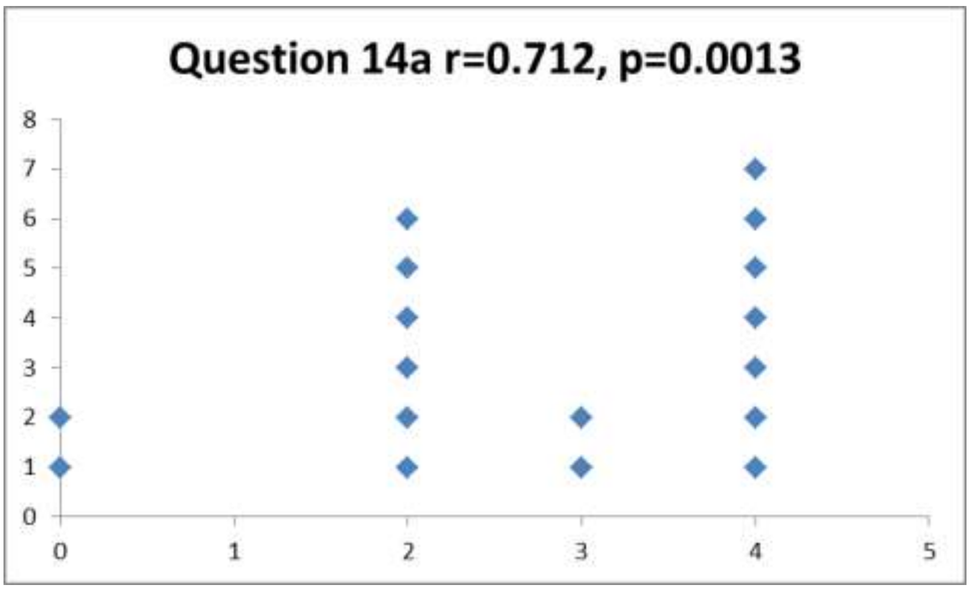
$$f'(x) = \frac{2x\sin(x) - x^2\cos(x)}{\sin^2(x)}$$



14) Find the derivative of each of the functions below.

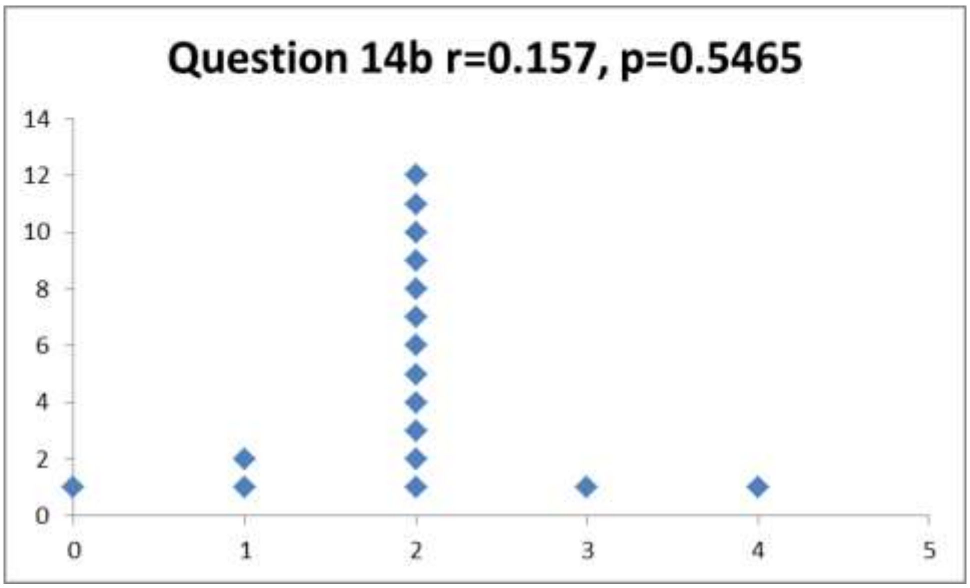
$(d)\ f(x) = 2^x \cdot 3^x$ (4 points)

$$f'(x) = 2^x \ln(2)\, 3^x + 2^x 3^x \ln(3)$$



$(e)\ \ln(x^{34})$ (4 points)

$$f'(x) = \frac{34x^{33}}{x^{34}} = \frac{34}{x}$$



15) Given $f(x) = x^5$, which of the following expressions correctly give $f'(x)$? Circle all that apply. (3.5 points)

$(a)\ f'(x) = 5x^4$

$(b)\ f'(x) = 5x^4 \dfrac{dy}{dx}$

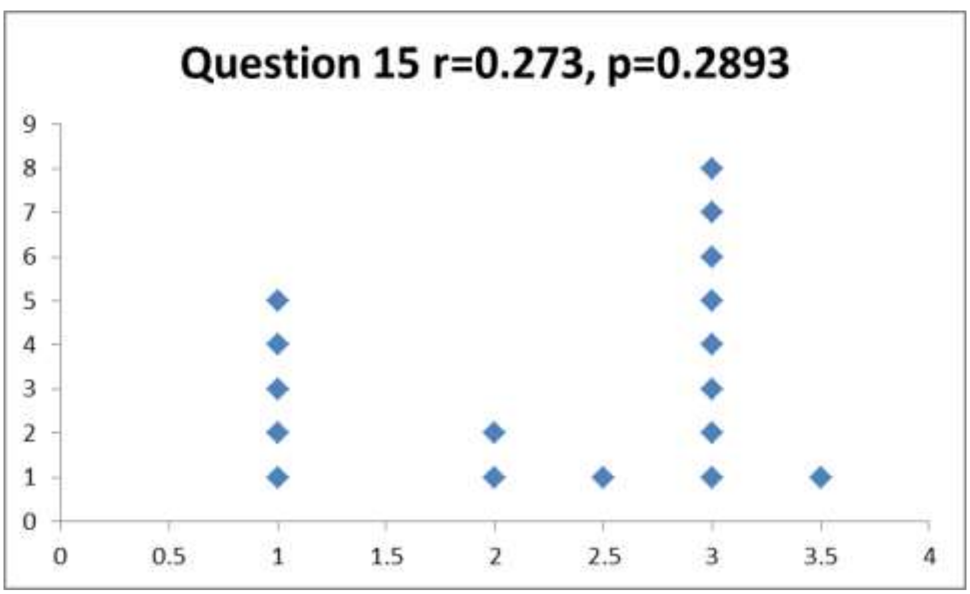
$(c)\ f'(x) = \lim\limits_{h\to 0} \dfrac{(x+h)^5 - x^5}{h}$

$(d)\ f'(x) = \lim\limits_{x\to 0} \dfrac{(x+h)^5 - x^5}{h}$

$(e)\ f'(x) = \lim\limits_{h\to 0} \dfrac{(x+h)^5 - h^5}{h}$

$(f)\ f'(x) = \lim\limits_{x\to 0} \dfrac{(x+h)^5 - h^5}{h}$

$(g)\ f'(x) = \lim\limits_{b\to x} \dfrac{(x)^5 - b^5}{x - b}$



16) Find $\frac{dy}{dx}$, given that $xy^2 + \sin(y) = x^4 + 2$ (6 points)

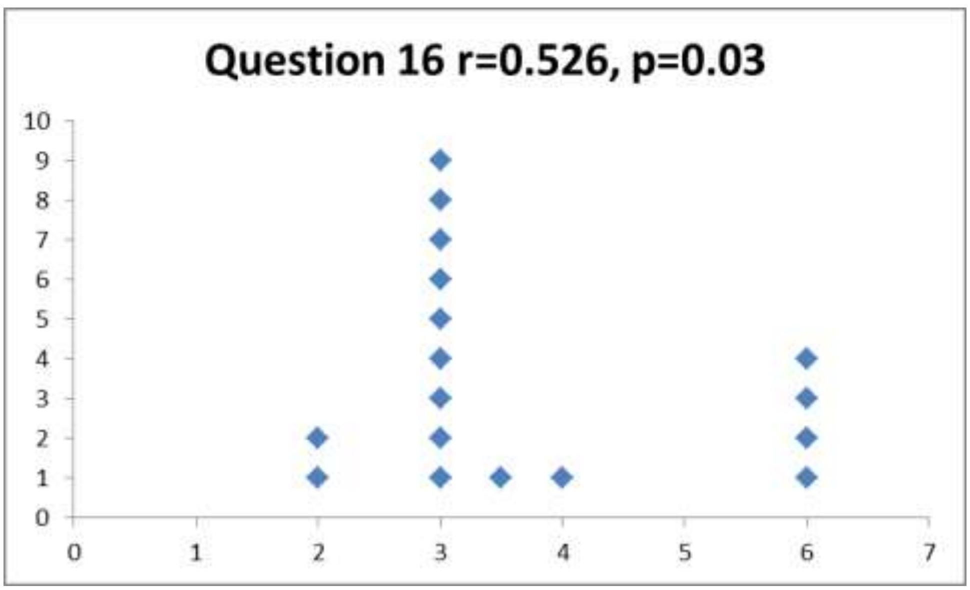
$$\frac{d}{dx}(xy^2 + \sin(y)) = \frac{d}{dx}(x^4 + 2)$$

$$y^2 + 2xy\frac{dy}{dx} + \cos(y)\frac{dy}{dx} = 4x^3$$

$$2xy\frac{dy}{dx} + \cos(y)\frac{dy}{dx} = 4x^3 - y^2$$

$$\frac{dy}{dx}(2xy + \cos(y)) = 4x^3 - y^2$$

$$\frac{dy}{dx} = \frac{4x^3 - y^2}{2xy + \cos(y)}$$

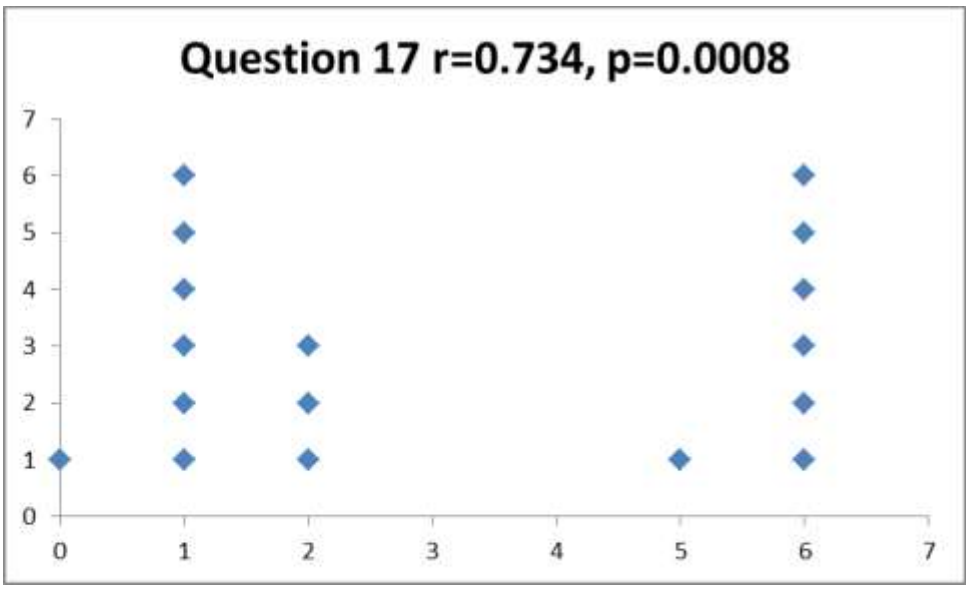


17) The distance of a train from a station is given by $s(t) = \frac{1}{2}t^2$. How fast is the train traveling after 60 seconds? The function s is given in feet ~~per second~~, while t is given in seconds. (6 points)

$$s'(t) = t$$

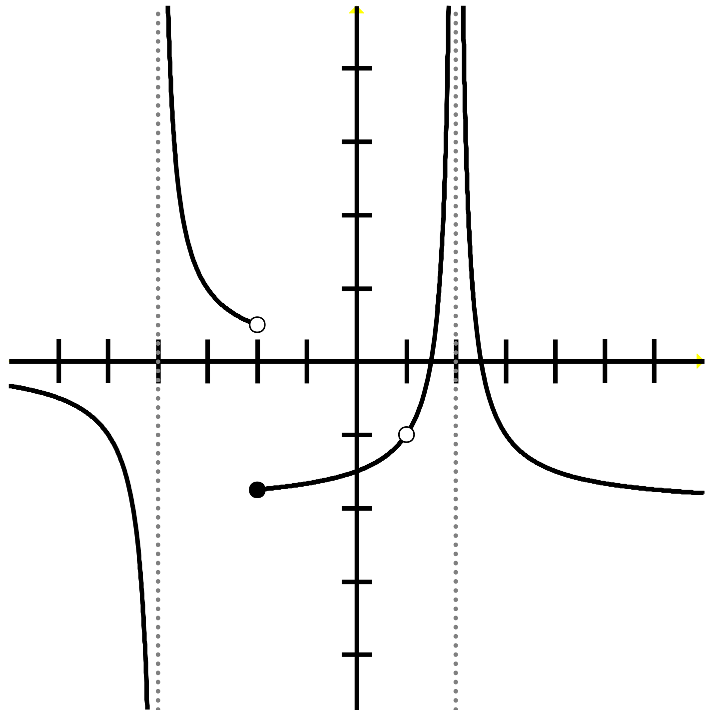
$$s'(60) = 60$$

60 feet per second



18) Use the graph of $y = f(x)$ below to find each of the following. (1 point each)

(a) $\lim_{x \to -4^-} f(x) = -\infty$



(b) $\lim_{x \to -4^+} f(x) = \infty$

(c) $\lim_{x \to -4} f(x)$ DNE

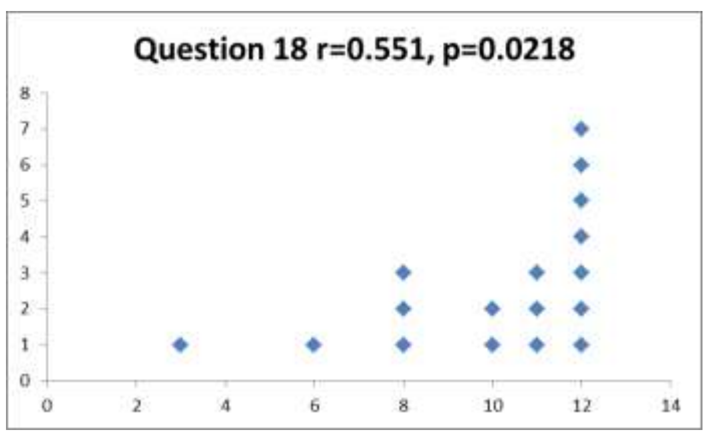
(d) $\lim_{x \to -2^-} f(x) = \frac{1}{2}$

(e) $\lim_{x \to -2^+} f(x) = -1.75$

(f) $\lim_{x \to -2} f(x)$ DNE

(g) $\lim_{x \to 1^-} f(x) = -1$

(h) $\lim_{x \to 1^+} f(x) = -1$



(i) $\lim_{x \to 1} f(x) = -1$

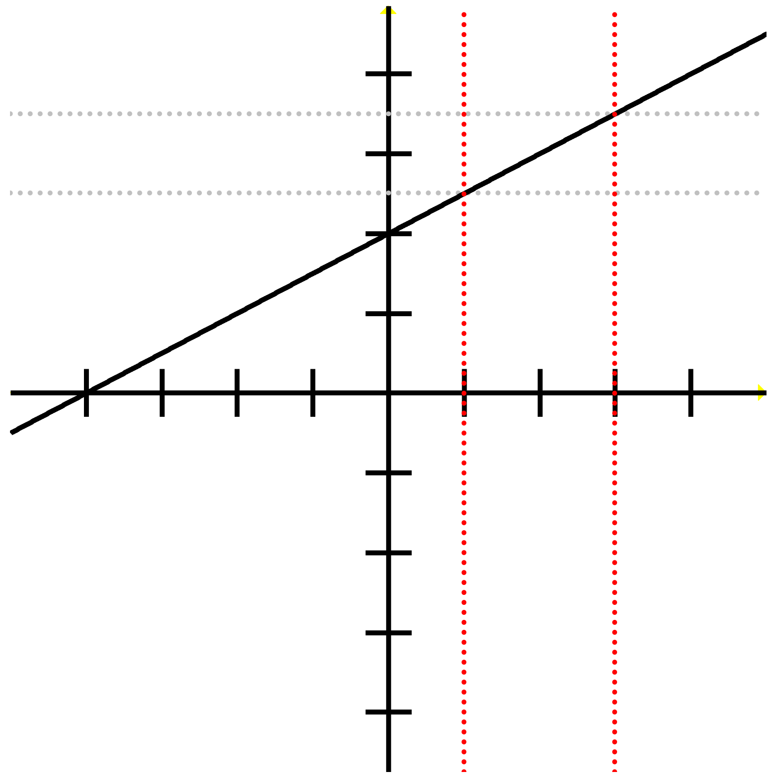
(j) $\lim_{x \to 2^-} f(x) = \infty$

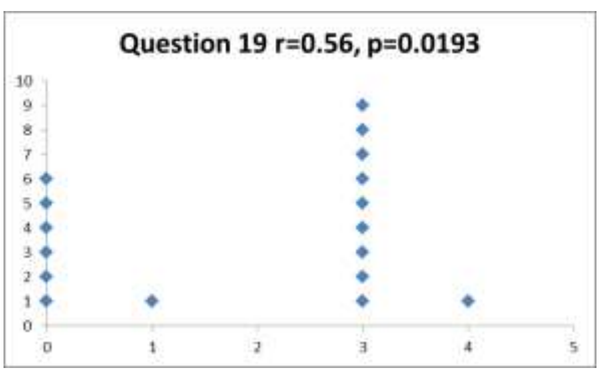
(k) $\lim_{x \to 2^+} f(x) = \infty$

(l) $\lim_{x \to 2} f(x) = \infty$

19) This problem is intended to test your conceptual understanding of the formal definition of a limit. We covered this is class using what we called the "$\varepsilon - \delta$" game. Below is a graph of a function. At $x = 2$, we think that the limit is $\lim_{x \to 2} f(x) = 3$. Today, our notion of closeness on the y-axis will be $\varepsilon = \frac{1}{2}$, so we must force $f(x)$ to be within $\frac{1}{2}$ units of 3. This is illustrated using the dotted lines on the graph below. Find the corresponding notion of closeness on the x-axis that is required, and illustrate it on the graph. (6.5 points)

$\delta = 1$





Mathematical Grammar: Points taken off for notational, poor illustration of work, etc:

These were marked in green
Instead of taking points off of the problem itself, they're pooled together with a cap at -5% – because you might make the same mistake on a dozen problems, such as not writing limit or equating numbers and variables.

However, that cap will increase for subsequent tests:

- 5% cap on test 1
- 10% cap on test 2
- 15% cap on test 3
- 20% cap on the final exam

In particular, if you're adding up your grade to verify the final score, add up the points for all the problems, and then subtract the points deducted in pink – but don't subtract more than 5 even if more are marked.