Name _____ Calculus I, Test 2, Spring 2017

Please show all your work and circle your answer when appropriate. You do not need to simplify answers unless the problem specifies to do so.

1) A circle has an initial radius of 50 feet when the radius begins to decrease at a rate of 2 feet per minute. What is the range of change of the area at the instant the radius is 10 feet? (10 points)

2) A cone-shaped funnel has a height of 6 inches tall and base 4 inches across. That's a diameter, not a radius. It is draining such that the depth decreases a constant rate of $\frac{1}{4}$ in/s. At what rate is the liquid coming out of the bottom when the height of the liquid is $\sqrt{3}$ inches? Give your answer as volume per time. (10 points)

(The volume of a cone is $V = \frac{1}{3}\pi r^2 h$)

Use the function $f(x) = \frac{4x^5}{5} - 3x^3 + 5$ for the next **TWO** pages. Also note that not every number on this page will be an integer. It'll work out fairly nicely, but will involve square roots

3) Find x-values that could possibly be a minimum or maximum (critical values) of the function. (4 points)

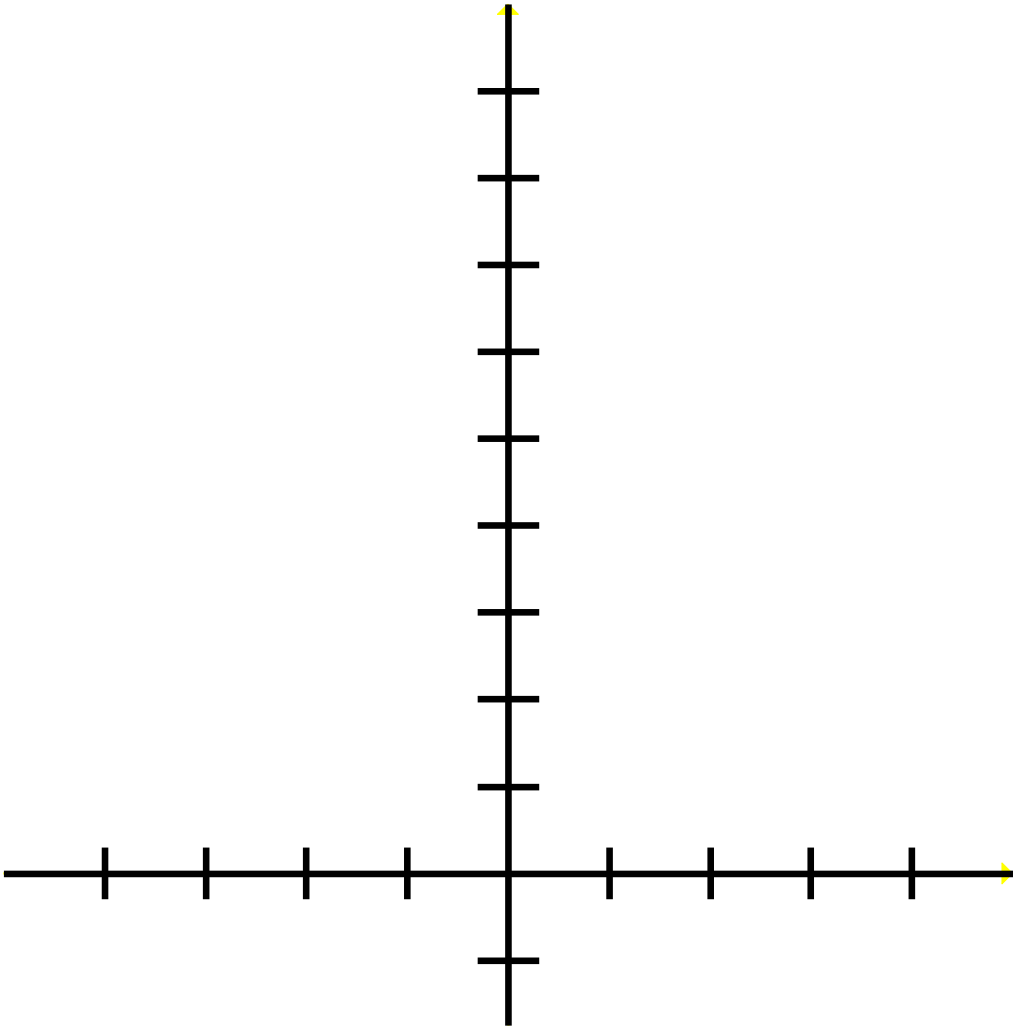
4) For each critical value, determine whether it is a maximizer, minimizer, or neither. (This question has multiple answers. One for each CV) (4 points)

5) At least one of your CVs should have turned out to be a maximizer. Find the local maximum corresponding to each maximizer. Do not simplify your answer. (4 points)

6) Find the x-value for one of the inflection points. Use a sign chart to justify why it is an inflection point. (4 points)

Graph the function. (10 points)

(Accuracy is important. I'll be looking to see that it aligns with all the information above)



On this page we're going to use the same function: $f(x) = \frac{4x^5}{5} - 3x^3 + 5$. However, now we're going to restrict the domain to $[-2, 3]$

7) Find all local maximizers. (4 points)

8) Find all local minimizers. (4 points)

9) Based on the graph you drew, where is the absolute maximum? (If you try to do this algebraically, it will be tedious without a calculator. That's why I'm saying to go back to your graph, and answer this question based on however you drew your function) (4 points)

10) Based on the graph you drew, where is the absolute minimum? (4 points)

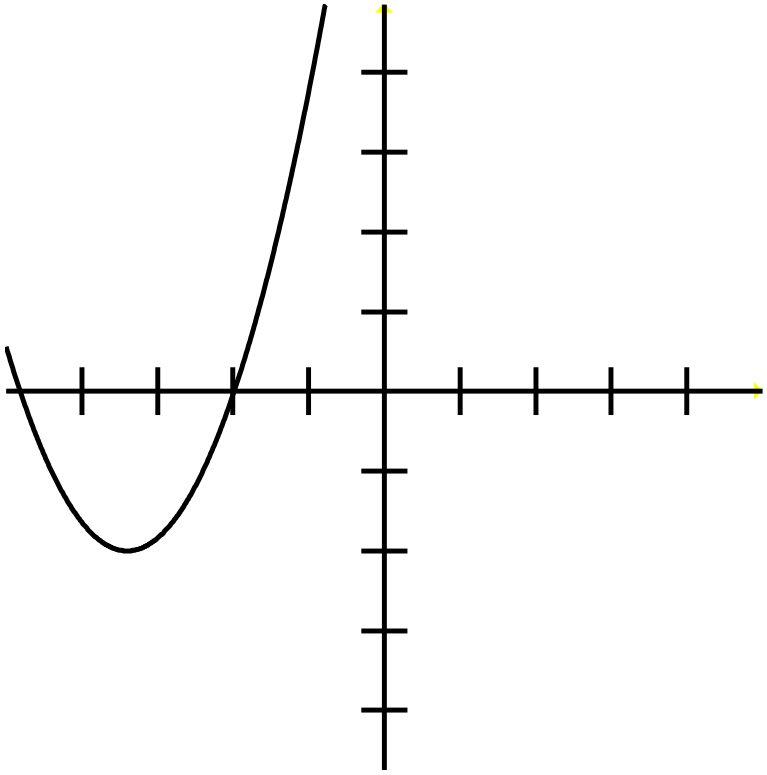
Quick check: Every question on this page was asking for an x-value. Did you answer with x-values, or y-values?

11) Find numbers x and y Satisfying the equation $3x + y = 12$ such that the product of x and y is as large as possible. (8 points)

12) An entrepreneur rents batteries at Central Park to Pokémon Go players and has a profit function as given below. How many batteries should they rent to maximize their profit? The profit P is measured in dollars, while the variable b is measured in hundreds of batteries. $0 \leq b \leq 40$. (8 points)

$$P(b) = 32b - b^2$$

13) Below is a graph of a function. If Dr. Beyerl guesses that a root of the function is $x = -3$, use Newton's Method to improve his guess. Illustrate what you do on the graph, and circle the new guess. (6 points)



14) Compute each of the limits below. (4 points each)

$$\lim_{x \to 3} \frac{x^{15} - 3^{15}}{x^2 - 9}$$

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x}$$

15) Compute each of the limits below. (4 points each)

$$\lim_{x \to 7} \frac{\ln(x) - \ln(7)}{x - 7}$$

$$\lim_{x \to 0^+} x \ln(x)$$