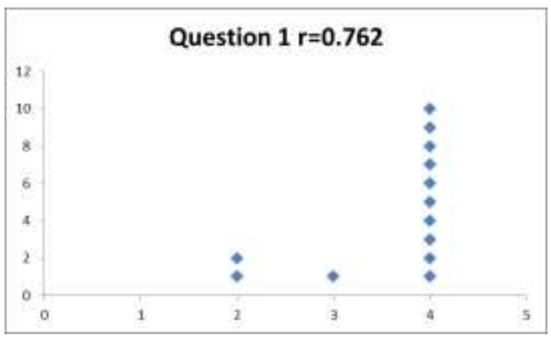
Name __Solutions____ Calculus I, Test 3, Spring 2017

Please show all your work and circle your answer when appropriate. You do not need to simplify answers unless the problem specifies to do so.

1) Find the integral below. (4 points)

$$\int x^{3.6} dx$$

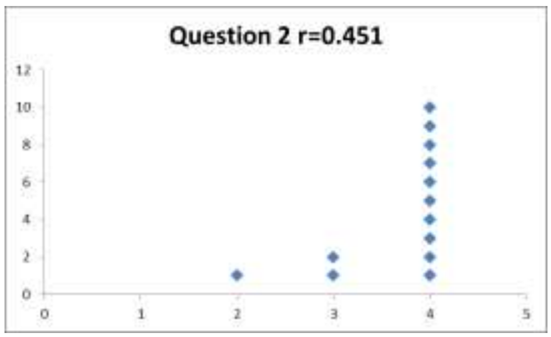
$$\int x^{3.6} dx = \frac{x^{4.6}}{4.6} + C$$



2) Find the integral below. (4 points)

$$\int_2^7 x^{3.6} dx$$

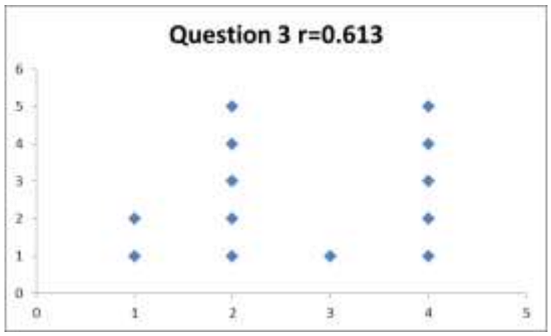
$$\int_2^7 x^{3.6} dx = \frac{x^{4.6}}{4.6}\bigg|_2^7 = \frac{7^{4.6}}{4.6} - \frac{2^{4.6}}{4.6}$$



3) Find the integral below. (4 points)

$$\int \sec^2(x)\,dx$$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

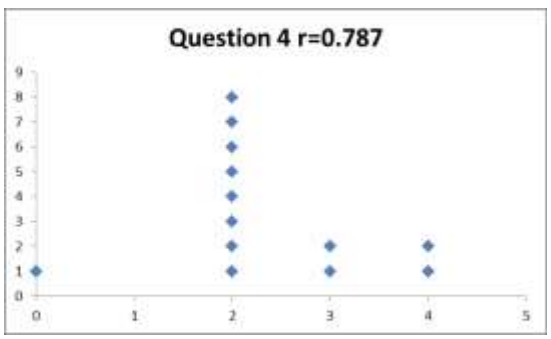


4) Find the integral below. (4 points)

$$\int \frac{3}{1+(2x)^2}\,dx$$

$$u = 2x$$
$$du = 2dx$$

$$\int \frac{3}{1+(2x)^2}\,dx$$
$$= 3\int \frac{1}{1+(2x)^2}\,dx$$
$$= \frac{3}{2}\int \frac{1}{1+(2x)^2}\,2dx$$
$$= \frac{3}{2}\int \frac{1}{1+u^2}\,du$$
$$= \frac{3}{2}\tan^{-1}(u) + C$$
$$= \frac{3}{2}\tan^{-1}(2x) + C$$



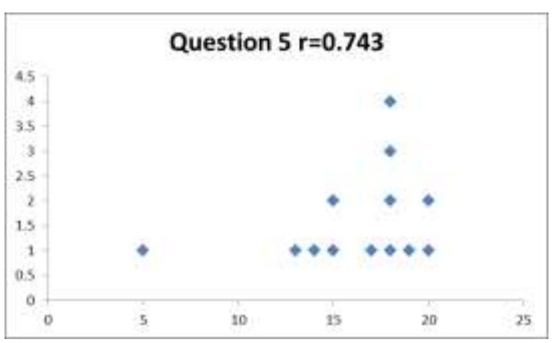
(Note: Problem #5 below counts for 20 points because I'm going to be meticulously looking at the details in your work. For full credit show every step of your work and simplify your answer.)

5) Find the integral below. (20 points)

$$\int_{-1}^{2} x^2 e^{x^3+1} dx$$

$$u = x^3 + 1$$
$$du = 3x^2 dx$$

$$\int_{-1}^{2} x^2 e^{x^3+1} dx$$
$$= \frac{1}{3}\int_{-1}^{2} 3x^2 e^{x^3+1} dx$$
$$= \frac{1}{3}\int_{x=-1}^{x=2} e^u du$$
$$= \frac{1}{3} e^u \Big|_{x=-1}^{x=2}$$
$$= \frac{1}{3} e^{x^3+1} \Big|_{-1}^{2}$$
$$= \frac{1}{3} e^{2^3+1} - \frac{1}{3} e^{(-1)^3+1}$$
$$= \frac{1}{3} e^9 - \frac{1}{3} e^0$$
$$= \frac{1}{3} e^9 - \frac{1}{3}$$

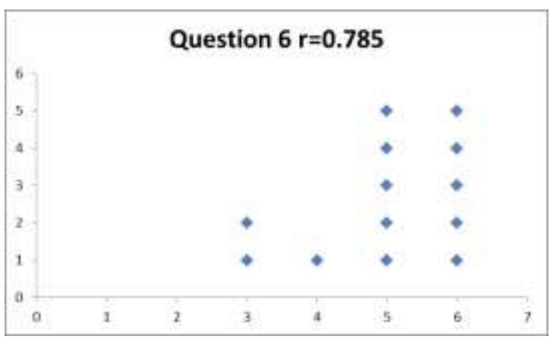


6) Find the integral below. (6 points)

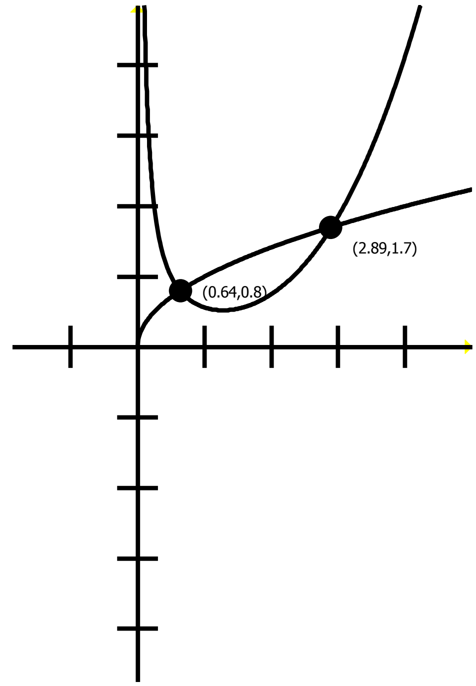
$$\int x^9 \sin(x^{10})\, dx$$

$$u = x^{10}$$
$$du = 10x^9 dx$$

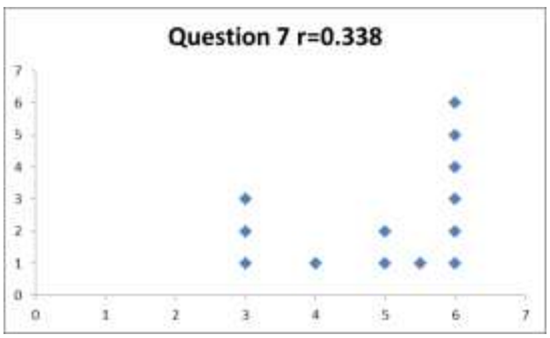
$$\int x^9 \sin(x^{10})\, dx$$
$$= \frac{1}{10}\int 10x^9 \sin(x^{10})\, dx$$
$$= \frac{1}{10}\int \sin(u)\, du$$
$$= \frac{1}{10}(-\cos(u)) + C$$
$$= -\frac{1}{10}\cos(x^{10}) + C$$



7) The curves shown below are given by $y = \frac{x^3}{24} + \frac{1}{2x}$ and $= \sqrt{x}$. Set up the integral for the area bounded between the two curves. Do not evaluate the integral. (6 points)

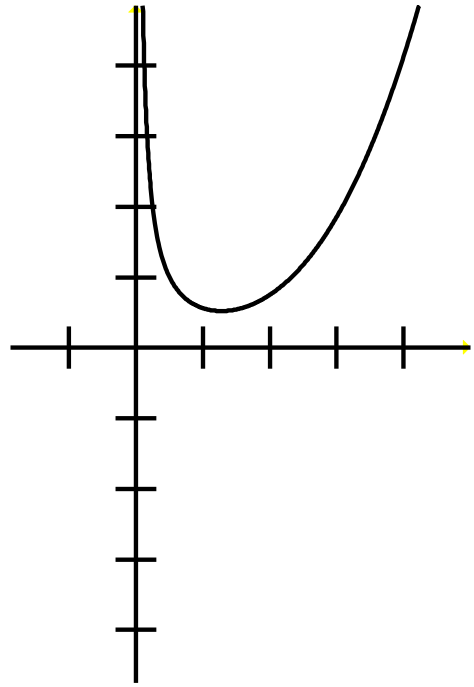


$$\int_{0.64}^{2.89} \sqrt{x} - \left(\frac{x^3}{24} + \frac{1}{2x}\right) dx$$



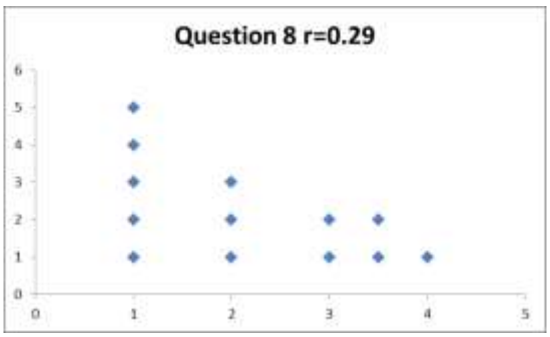
8) The curve shown below is given by $y = \frac{x^3}{24} + \frac{1}{2x}$. Set up the integral for the length of the piece between $x = 1$ and $x = 4$. Do not evaluate the integral.

(4 points)



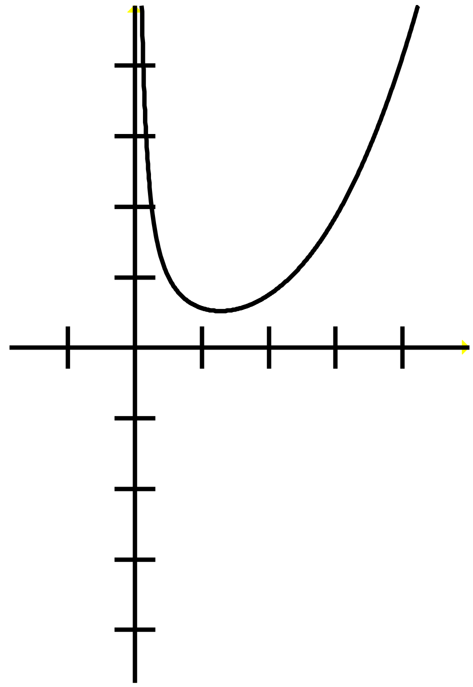
$$y' = \frac{3x^2}{24} - \frac{1}{2x^2}$$

$$L = \int_1^4 \sqrt{1 + \left(\frac{x^2}{8} - \frac{1}{2x^2}\right)^2}\, dx$$



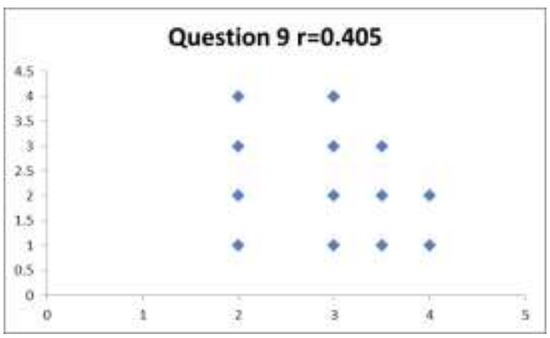
9) The curve shown below is given by $y = \frac{x^3}{24} + \frac{1}{2x}$. Suppose the piece between $x = 1$ and $x = 4$ is rotated around the x-axis. Set up the integral for the surface area of the surface created. Do not evaluate the integral.

(4 points)

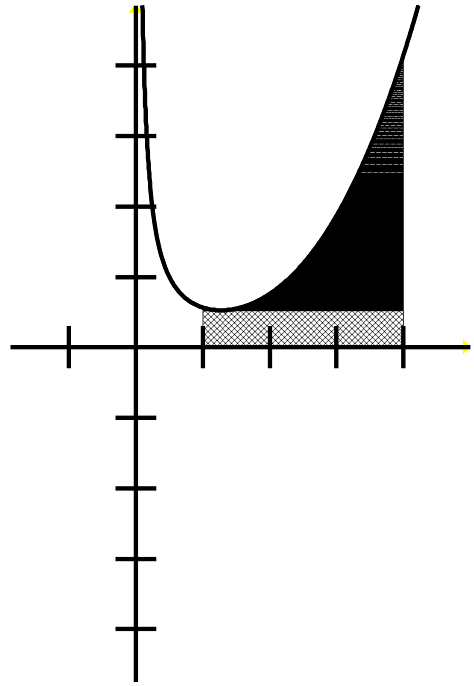


$$y' = \frac{3x^2}{24} - \frac{1}{2x^2}$$

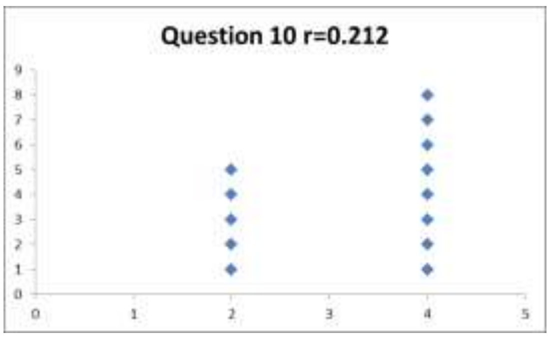
$$A = \int_1^4 2\pi\left(\frac{x^3}{24} + \frac{1}{2x}\right)\sqrt{1 + \left(\frac{x^2}{8} - \frac{1}{2x^2}\right)^2}\, dx$$



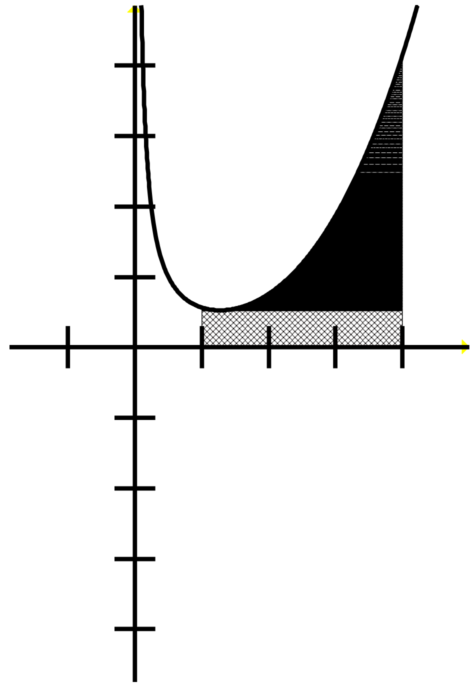
10) The curve shown below is given by $y = \frac{x^3}{24} + \frac{1}{2x}$. Suppose the shaded region is rotated around the x-axis. Set up the integral for the volume of the solid created. Do not evaluate the integral.
(4 points)



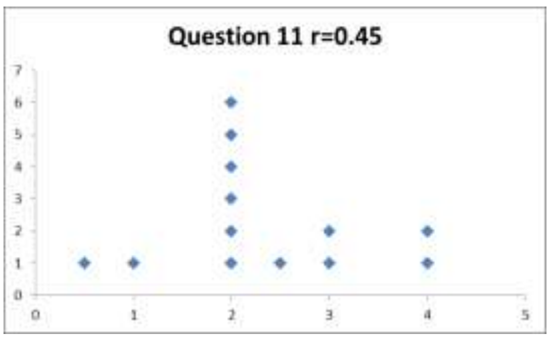
$$V = \int_1^4 \pi \left(\frac{x^3}{24} + \frac{1}{2x}\right)^2 dx$$



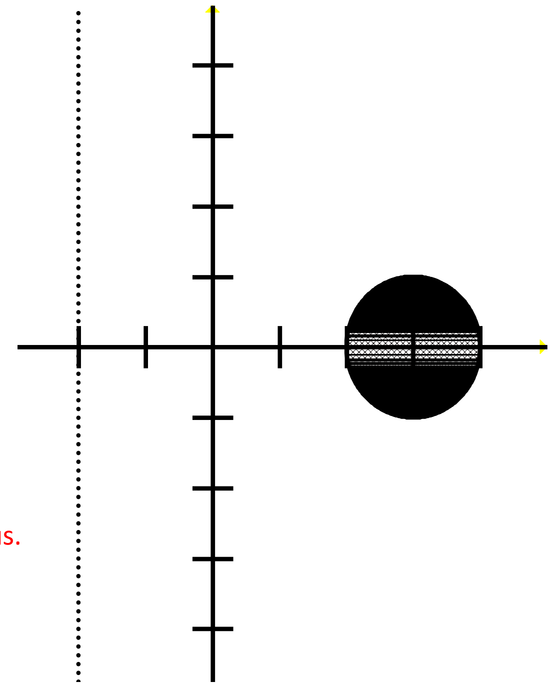
11) The curve shown below is given by $y = \frac{x^3}{24} + \frac{1}{2x}$. Suppose the shaded region is rotated around the y-axis. Set up the integral for the volume of the surface created. Do not evaluate the integral.
(4 points)



$$V = \int_1^4 2\pi x\left(\frac{x^3}{24} + \frac{1}{2x}\right)dx$$



The curve shown below is given by the pair of equations $y = \sqrt{1-(x-3)^2}$ and $y = -\sqrt{1-(x-3)^2}$. Together they bound the shaded region shown here.



12) What is the area of the shaded region? (1 point)

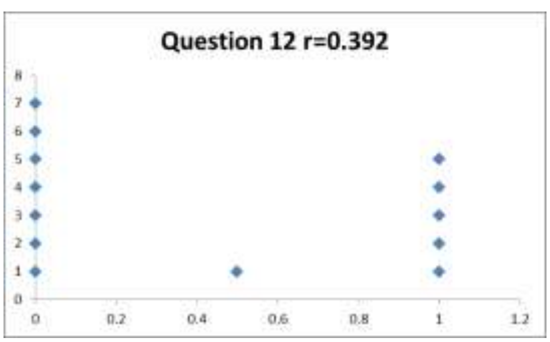
π

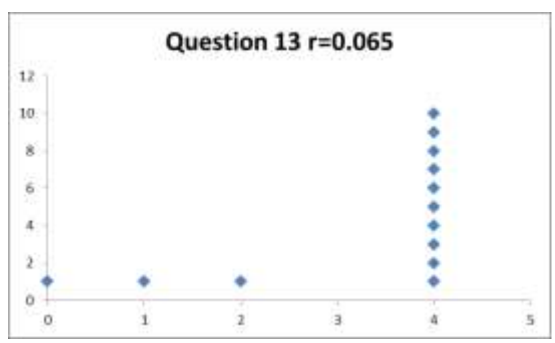
13) If the shaded region is rotated around the line $x = -2$, describe what the solid shape created looks like. (4 points)

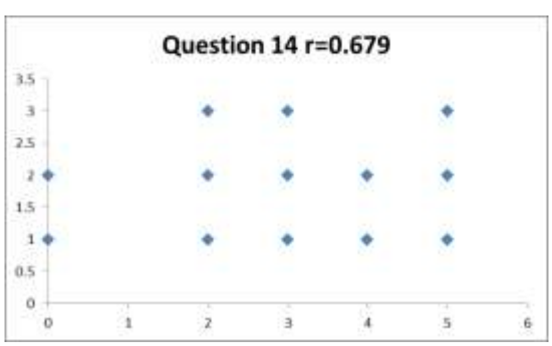
An inner tube, a doughnut, a ring, or the technical term is a torus.

14) Set up, but do not evaluate, the integral for the volume of the solid described in the previous question. (6 points)

$$\int_2^4 2\pi(x+2)\left(\sqrt{1-(x-3)^2} - \left(-\sqrt{1-(x-3)^2}\right)\right)dx = \int_2^4 2\pi(x+2)\left(2\sqrt{1-(x-3)^2}\right)dx$$







15) The velocity of an object moving along a line is given by $v(t) = 9 - t^2$ meters per second on the interval $[0,4]$. The position of the same object is given by $s(t)$. Unfortunately all that is known about $s(t)$ is that the starting position is given by $s(0) = -2$. Find a formula for $s(t)$. (10 points)

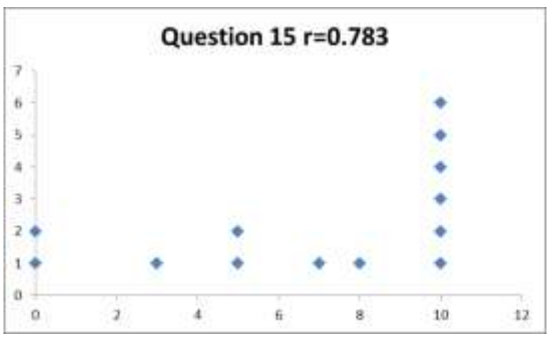
The position is given by the antiderivative of velocity:

$$\int 9 - t^2 dt = 9t - \frac{t^3}{3} + C$$

However, there is only one position function, so we use the initial condition to find C:

$$s(0) = -2$$
$$9 \cdot 0 - \frac{0^3}{3} + C = -2$$
$$C = -2$$

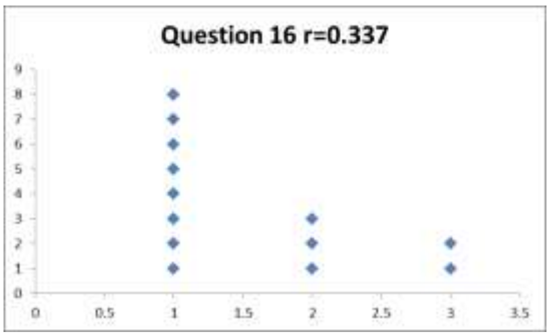
$$s(t) = 9t - \frac{t^3}{3} + C$$



16) Evaluate the expression below. (3 points)

$$\frac{d}{dx}\int_3^x (t^2+t+1)dt = x^2+x+1$$

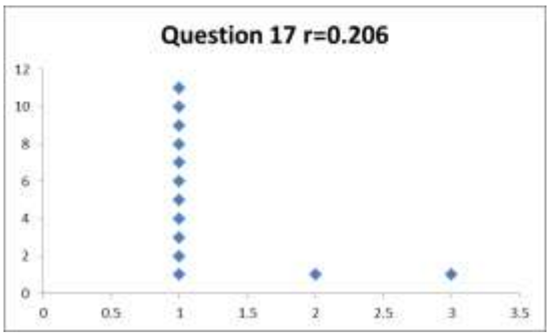
(Use the fundamental theorem of calculus!!!)



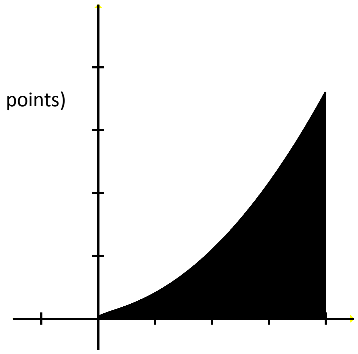
17) Evaluate the expression below. (2 points)

$$\frac{d}{dx}\int_{3}^{x^2}(t^2+t+1)dt = ((x^2)^2+x^2+1)\cdot 2x$$

(Use the fundamental theorem of calculus!!! Oh, and the chain rule.)

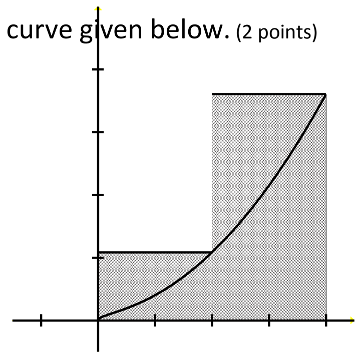


18) Illustrate (do not calculate) the area under the curve given below. (2 points)



19) Illustrate (do not calculate) an approximation to the area under the curve given below. (2 points)

(There are multiple answers)



20) Calculate the approximation you illustrated in #19. (2 points)

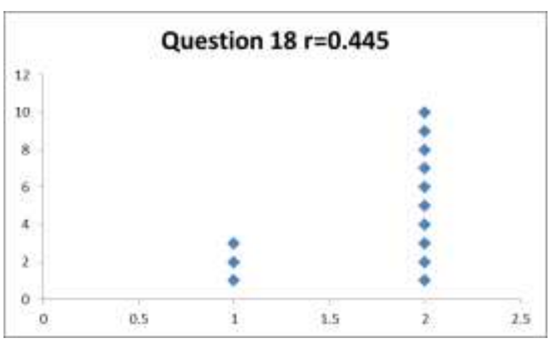
(There are multiple answers, but each is unique and based on your answer to #19)

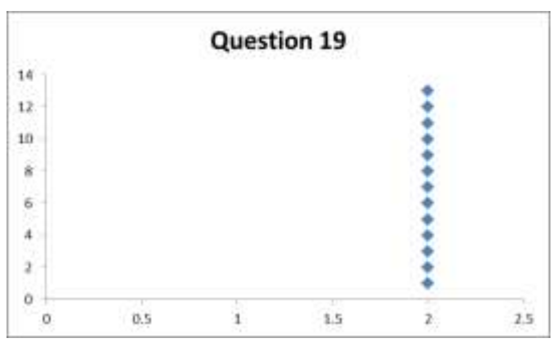
$1.1 \cdot 2 + 3.5 \cdot 2$

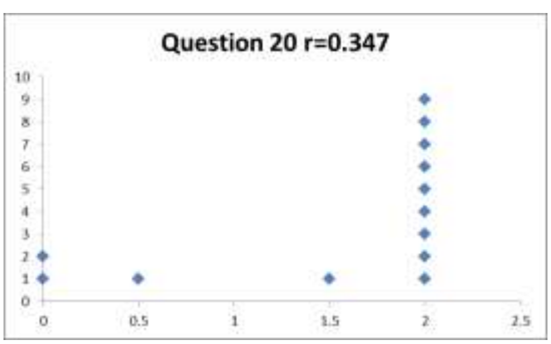
21) Is your approximation in #19 an overestimate or an underestimate? (1 point)

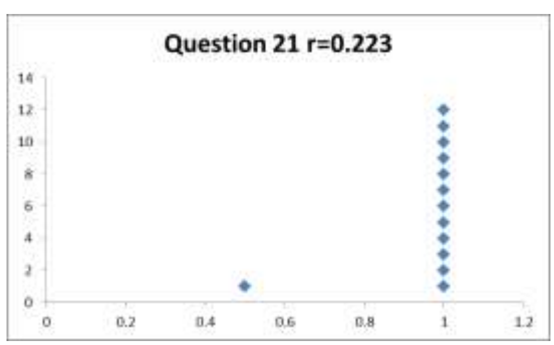
(Again based on #3)

Overestimate

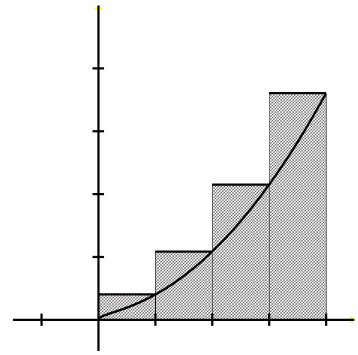








22) Illustrate a better approximation than you came up with in #19. (2 points)



23) Illustrate an even better approximation than you came up with in the previous question. (1 points)