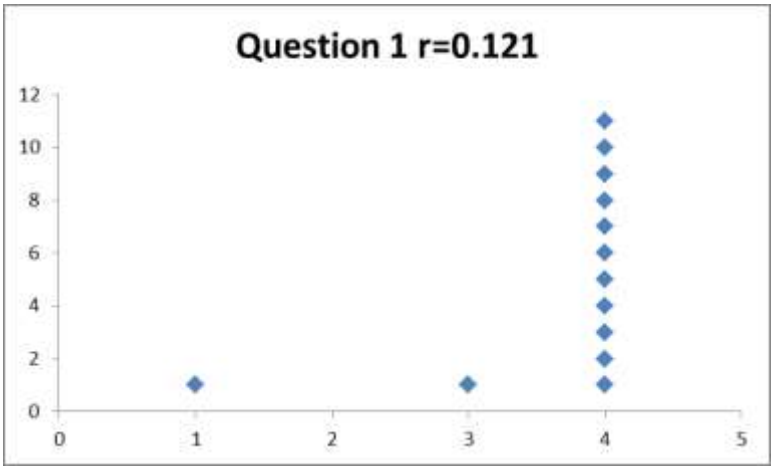
Name _____ Test 1, Fall 2018

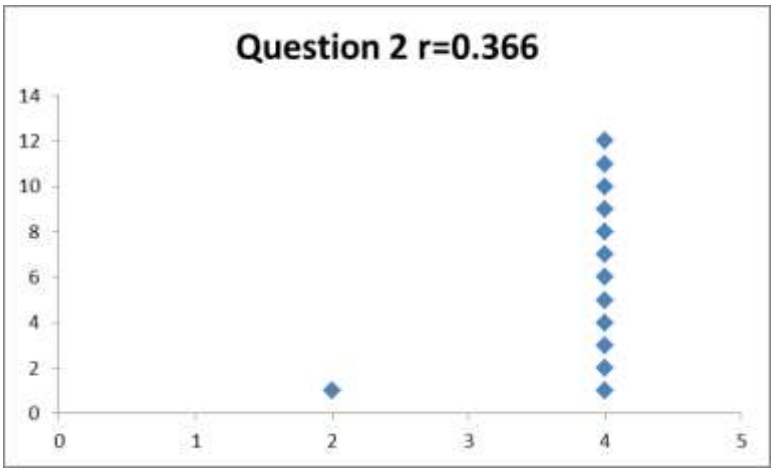
1) Find the limit below. (4 points)

$$\lim_{x\to 5}\frac{x^2-25}{x-5}=\lim_{x\to 5}\frac{(x-5)(x+5)}{x-5}=\lim_{x\to 5}(x+5)=10$$



2) Find the limit below. (4 points)

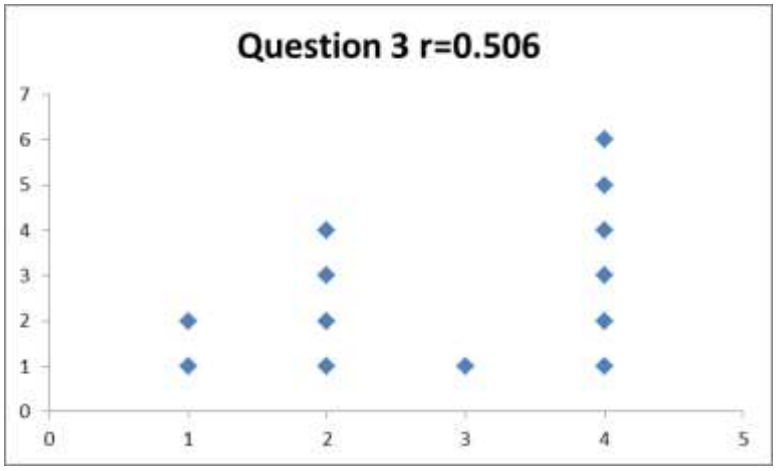
$\lim_{x \to 7} x^2 - x - 2 = 49 - 7 - 2 = 40$



3) Using the table to the right, guess the value of the limit below. (4 points)

$\lim_{x \to 2^-} f(x) = 26$

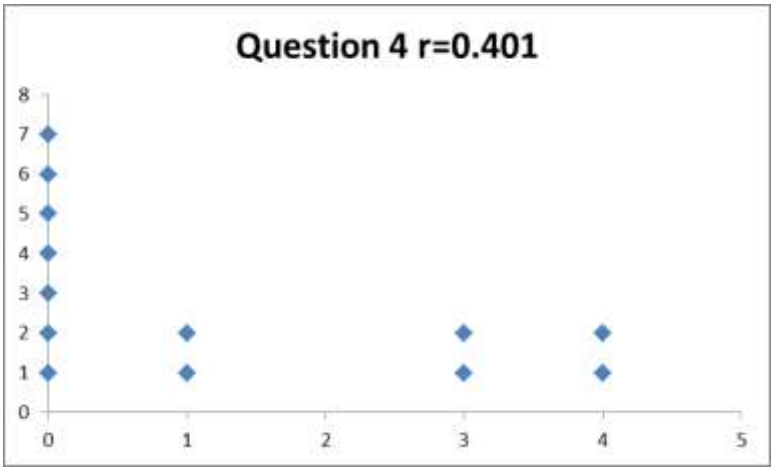
x	$f(x)$
-4	8
-3	10
-2.5	12.75
-2.1	13.8
-2.01	13.985
-2	14
-1.99	14.0023
-1.9	14.46
-1.5	15.2
-1	17
0	23
1	24
1.5	25.6
1.9	25.78
1.99	25.97
2	30
2.01	30.024
2.1	30.14
2.5	30.686
3	31.2
4	34



4) Using the table to the right, estimate the value of the derivative below. (4 points)

$$f'(-2) \approx \frac{y_2 - y_1}{x_2 - x_1} = \frac{14.0023 - 14}{-1.99 - (-2)} = \frac{0.0023}{0.01} = 0.023$$

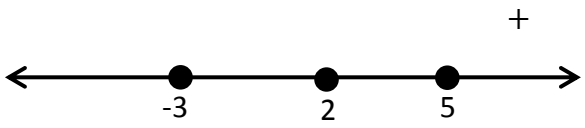
There are other answers: the slope of any secant line near -2 works as an estimate (Some estimates are better than others; for full credit choose to use values that result in a good estimate)

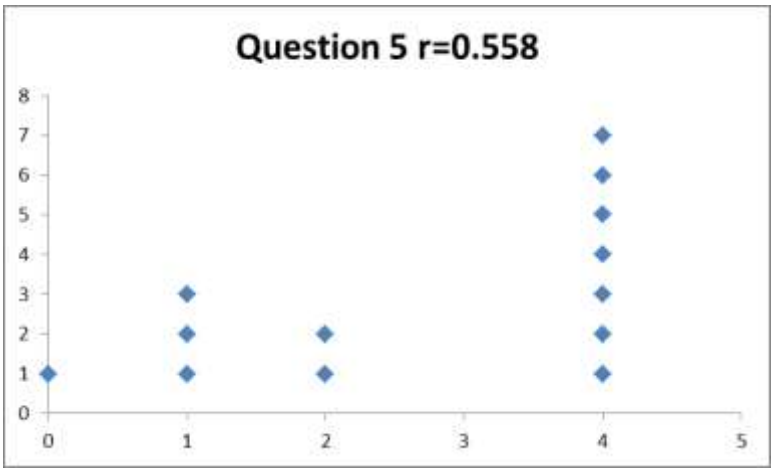


x	$f(x)$
-4	8
-3	10
-2.5	12.75
-2.1	13.8
-2.01	13.985
-2	14
-1.99	14.0023
-1.9	14.46
-1.5	15.2
-1	17
0	23
1	24
1.5	25.6
1.9	25.78
1.99	25.97
2	30
2.01	30.024
2.1	30.14
2.5	30.686
3	31.2
4	34

5) Find the limit below. (4 points)

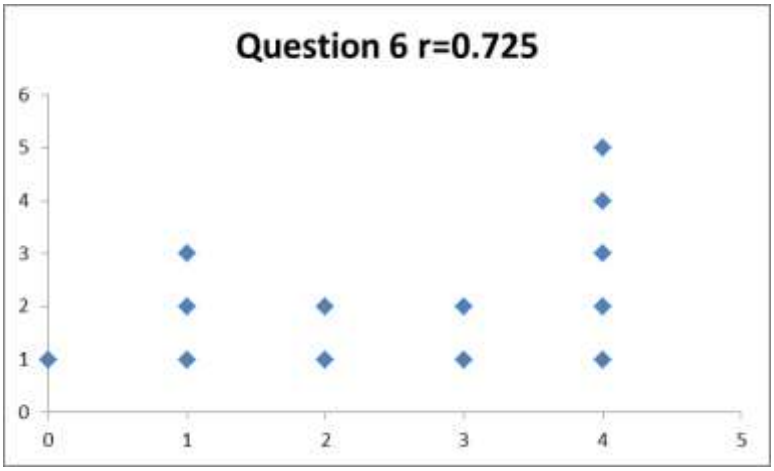
$$\lim_{x \to 5^+} \frac{(x-2)(x+3)}{x-5} = \infty$$





6) Find the limit below. (4 points)

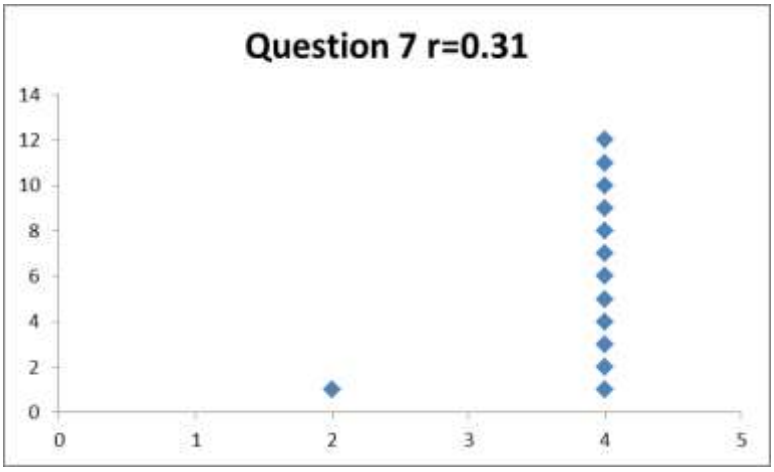
$$\lim_{x\to 5}\frac{\sqrt{x}-\sqrt{5}}{x-5}\cdot\frac{\sqrt{x}+\sqrt{5}}{\sqrt{x}+\sqrt{5}}=\lim_{x\to 5}\frac{x-\sqrt{5}\sqrt{x}+\sqrt{5}\sqrt{x}-5}{(x-5)\left(\sqrt{x}+\sqrt{5}\right)}=\lim_{x\to 5}\frac{x-5}{(x-5)\left(\sqrt{x}+\sqrt{5}\right)}\lim_{x\to 5}\frac{1}{\left(\sqrt{x}+\sqrt{5}\right)}=\frac{1}{2\sqrt{5}}$$



7) Find the limit below. (4 points)

$$\lim_{x\to\infty} \frac{(x-2)(x+3)}{x-5} = \infty$$

(The dominant term in the numerator is x^2; in the denominator x)



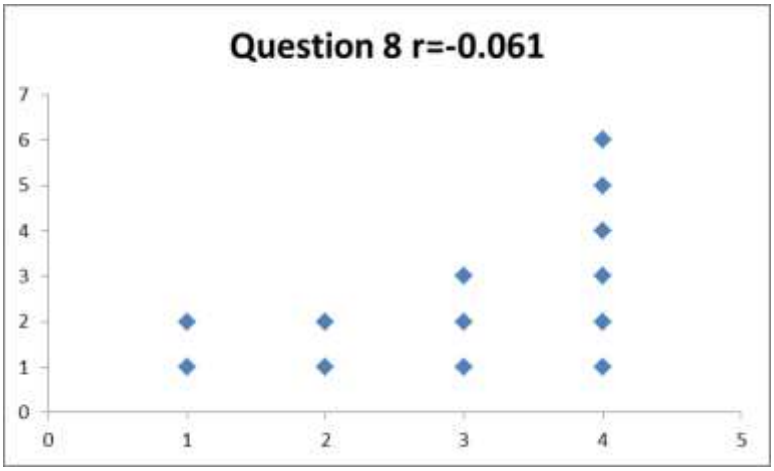
8) If $2x \leq f(x) \leq 2x^2$, what can be said about the limit below? (4 points)

$\lim_{x \to 3} f(x)$

$$\lim_{x \to 3} 2x \leq \lim_{x \to 3} f(x) \leq \lim_{x \to 3} 2x^2$$

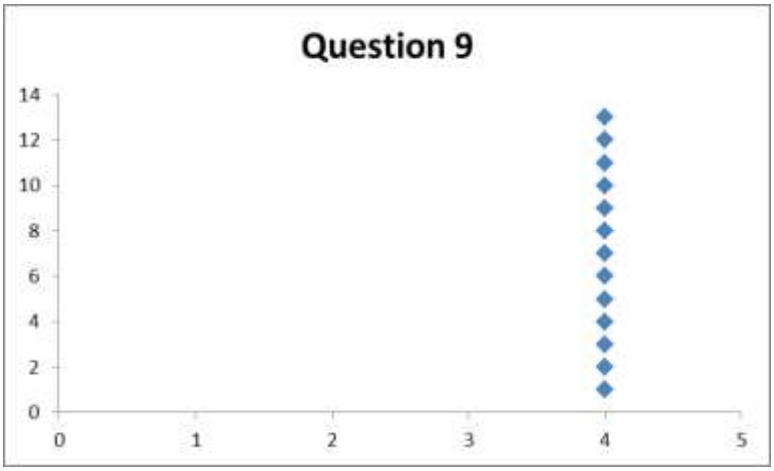
$$6 \leq \lim_{x \to 3} f(x) \leq 18$$

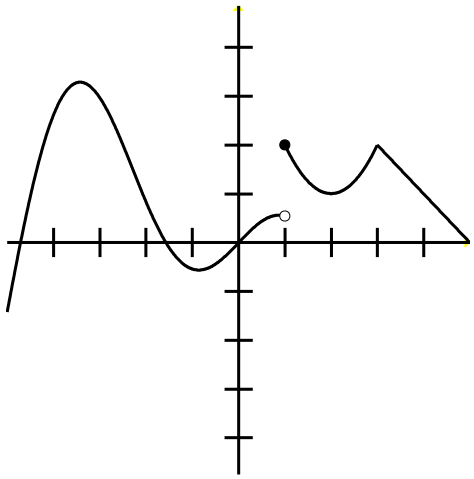
We don't know exactly what $\lim_{x \to 3} f(x)$ is, but we know if it exists, it's between 6 and 18.



9) Use the graph shown here to estimate the limit below. (4 points)

$\lim_{x \to 1^+} f(x) \approx 2$

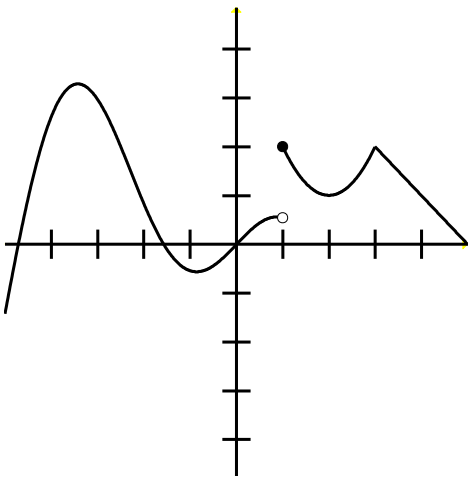


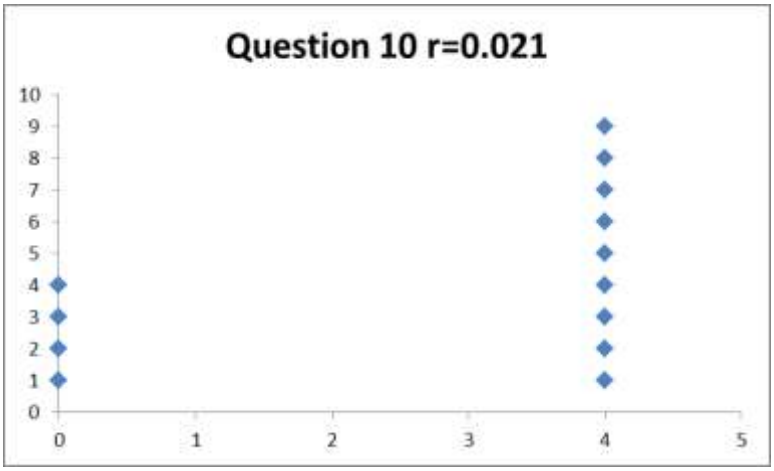


10) Use the graph shown here to estimate the derivative below. (4 points)

$f'(2.5) \approx 3$

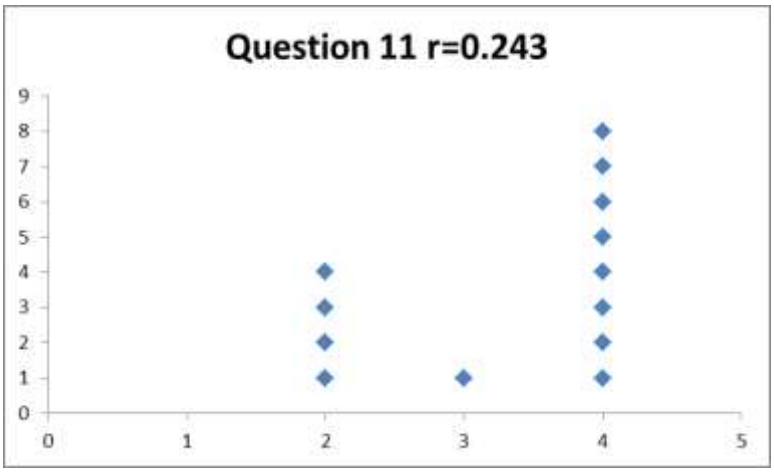
(Anything between 1 and 5 was given full credit)

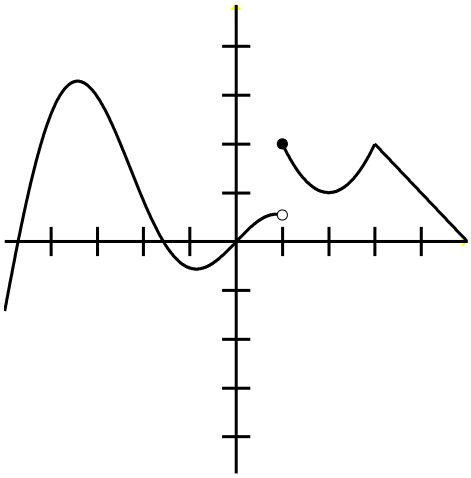




11) Use the graph shown here to identify all x-coordinates where the function is not continuous. (4 points)

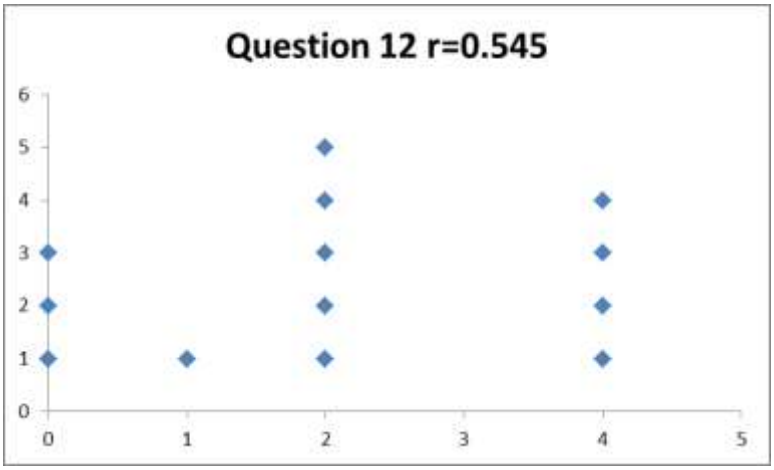
$x = 1$

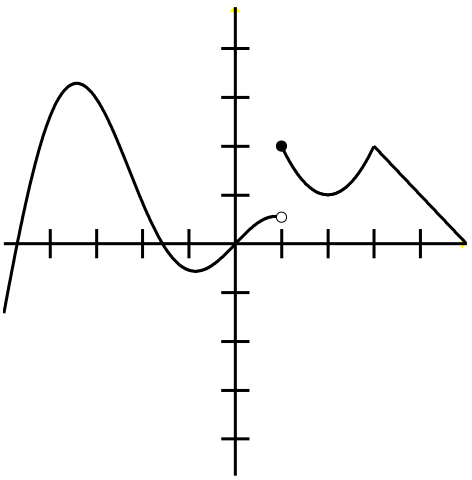




12) Use the graph shown here to identify all x-coordinates where the function is not differentiable. (4 points)

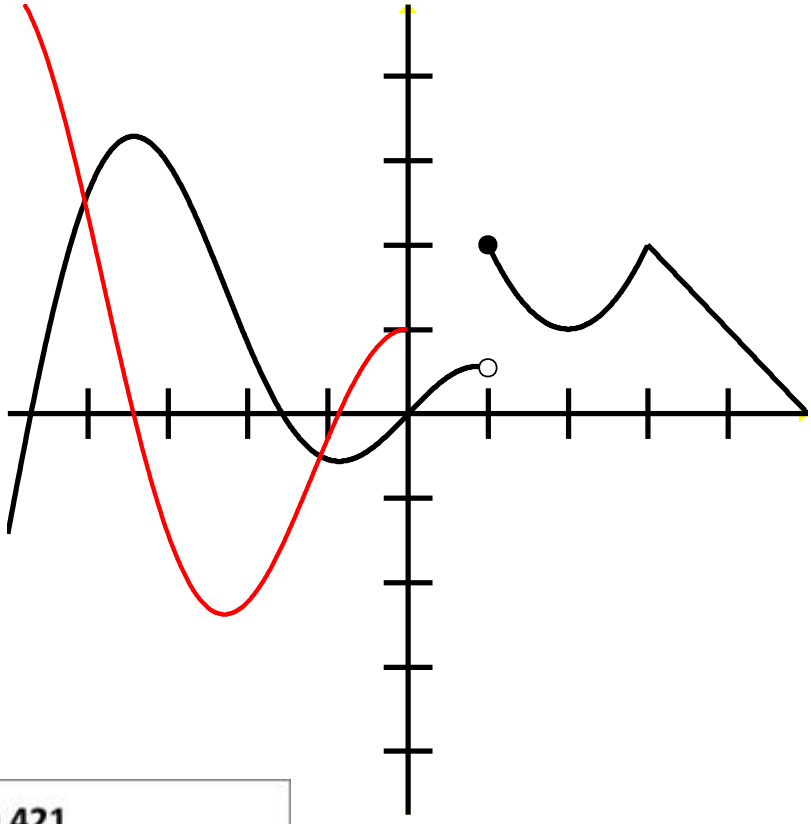
$x = 1, 3$

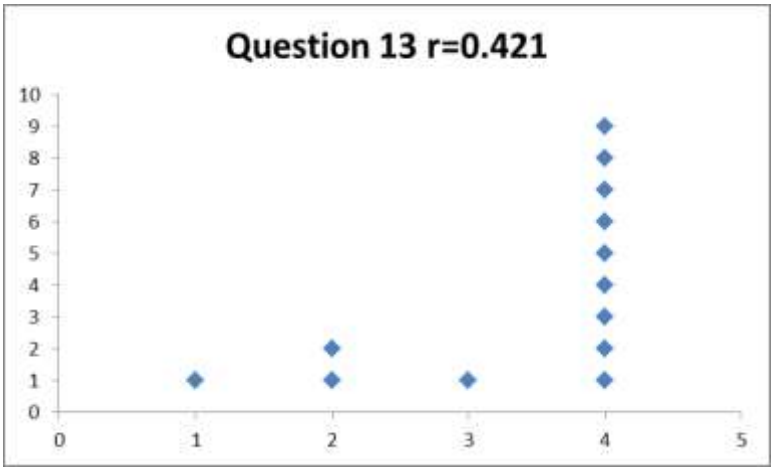




13) Sketch a graph of the derivative on the domain $(-\infty, 0)$. (4 points)

(Meaning you can ignore the right half of the graph)

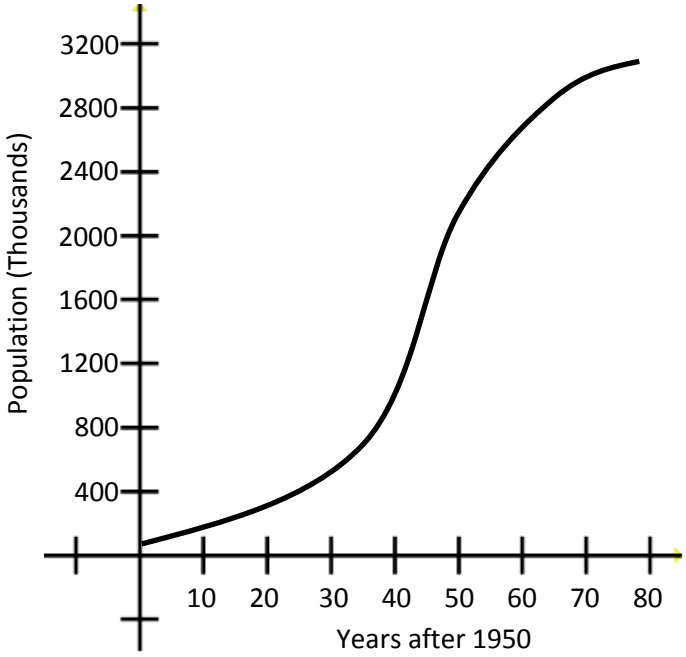


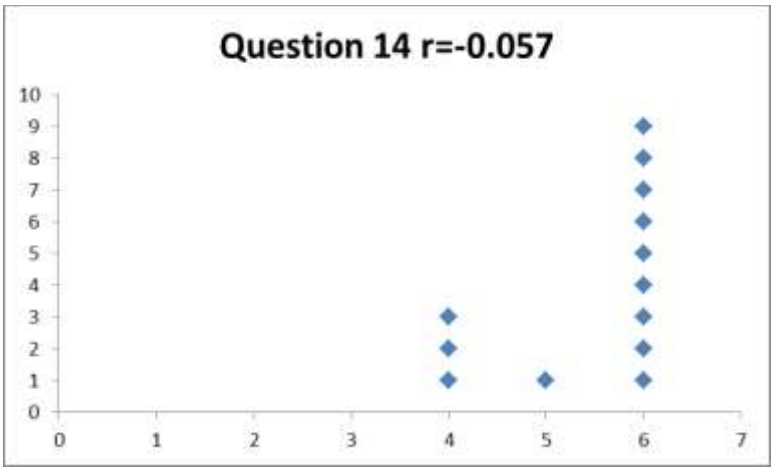


14) Shown here is a graph of the population of a certain city over a period of 60 years. Identify what year had the largest rate of growth. (6 points)

(Please remember that contextual problems have contextual answers. Don't lost easy points because you leave off the units!)

The instantaneous rate of change is steepest around $x = 45$. That corresponds to 1995.

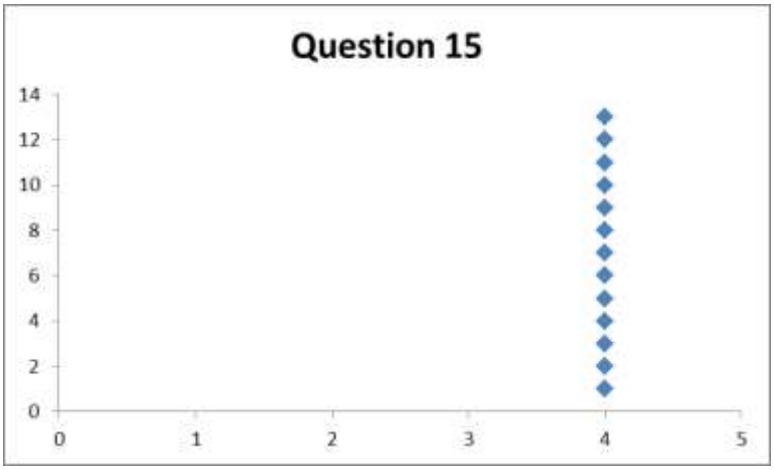




15) Find the derivative of the function below. (4 points)

$f(x) = x^5$

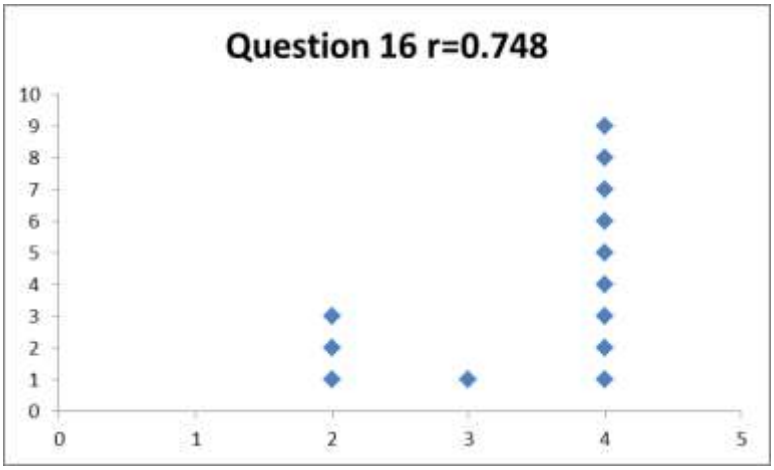
$f'(x) = 5x^4$



16) Find the derivative of the function below. (4 points)

$f(x) = 2g(x) + h(x) + c$

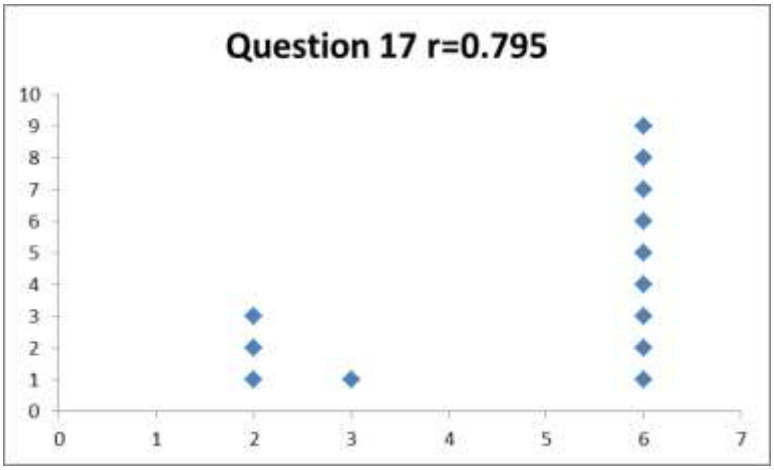
$f'(x) = 2g'(x) + h'(x)$



17) Find the derivative of the function below. (6 points)

$f(x) = x^2 g(x)$

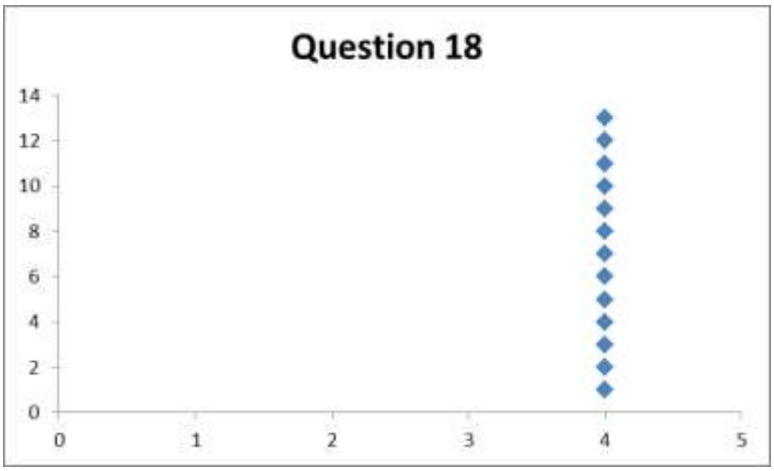
$f'(x) = 2xg(x) + x^2 g'(x)$



18) Find the derivative of the function below. (4 points)

$f(x) = 6x^2$

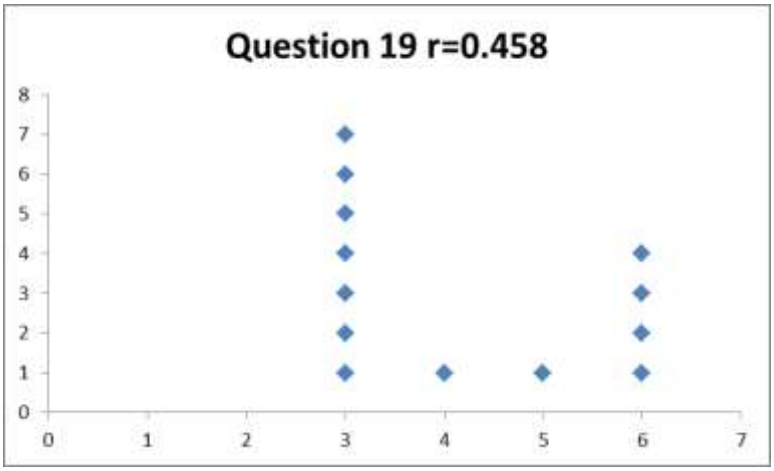
$f'(x) = 12x$



19) Find the derivative of the function below. (6 points)

$f(t) = 3t^2 + \frac{6}{t^7} = 3t^2 + 6t^{-7}$

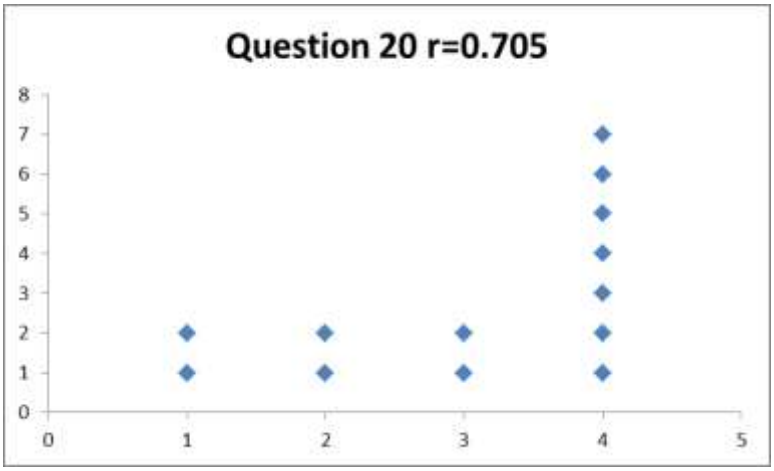
$$f'(t) = 6t - 42t^{-8}$$



20) Find the derivative of the function below. (4 points)

$f(x) = \sqrt{x}(\sqrt{x} - 1) = x - \sqrt{x}$

$f'(x) = 1 - \frac{1}{2}x^{-\frac{1}{2}}$

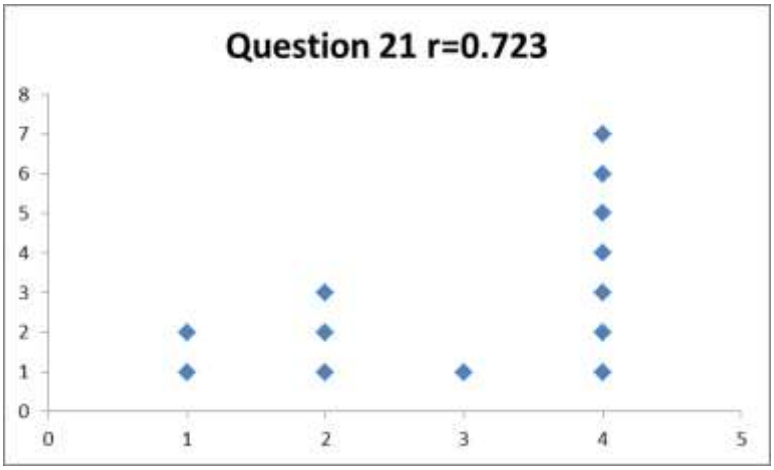


21) Find the second derivative of the function below. (4 points)

$f(x) = \sqrt{x}(\sqrt{x} - 1)$

$f'(x) = 1 - \frac{1}{2}x^{-\frac{1}{2}}$

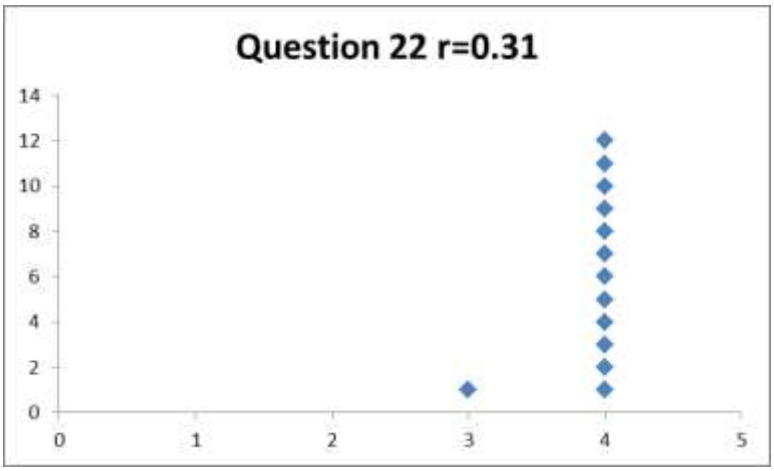
$f''(x) = \frac{1}{4}x^{-\frac{3}{2}}$



22) Find the derivative of the function below. (4 points)

$$f(x) = \frac{x^4+5x^2+x}{x^2}$$

$$f'(x) = \frac{(4x^3 + 10x + 1)x^2 - 2x(x^4 + 5x^2 + x)}{x^4}$$



23) On the axis below, sketch a graph of a function with the following properties. (6 points)

$\lim_{x\to 2^-} f(x) = 1$

$\lim_{x\to 2^+} f(x) = 2$

$\lim_{x\to 2^+} f'(x) = -1$