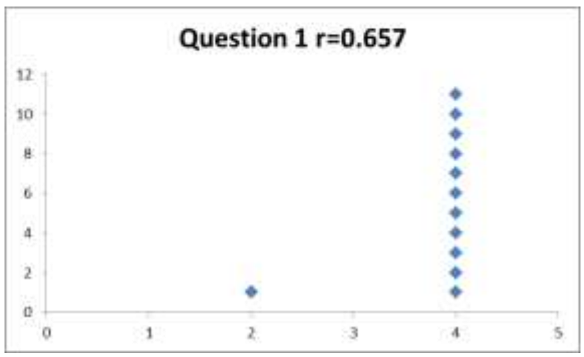
Name ____ Test 2, Fall 2018

1) Find the derivative of $f(x)$ below.
(4 points)

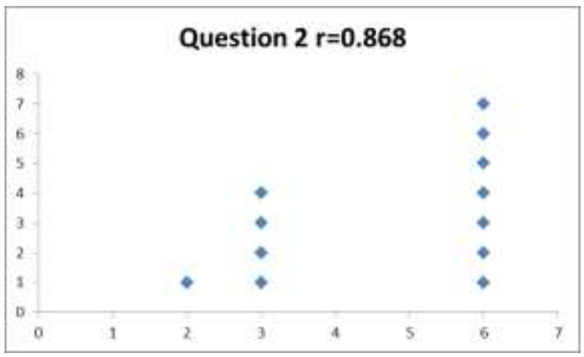
$f(x) = \sin(2x)$

$$f'(x) = \cos(2x) \cdot 2 = 2\cos(2x)$$

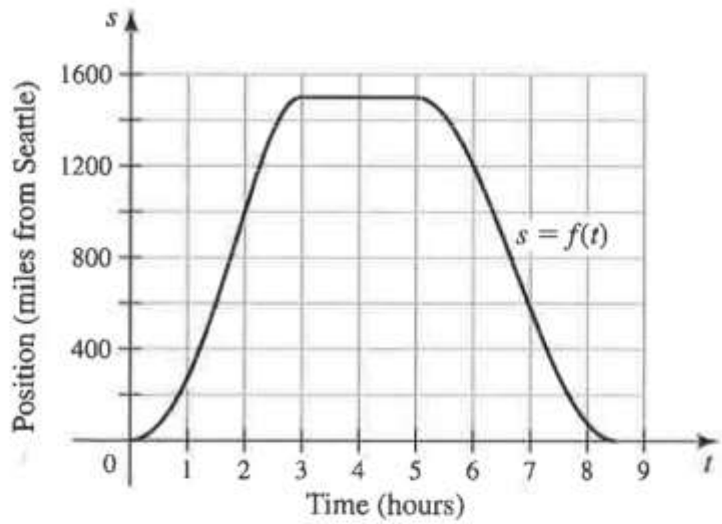


2) Given the equation $x + y^2 = y$, find y'.
(6 points)

$$1 + 2yy' = y'$$
$$1 = y' - 2yy'$$
$$1 = y'(1 - 2y)$$
$$\frac{1}{1 - 2y} = y'$$

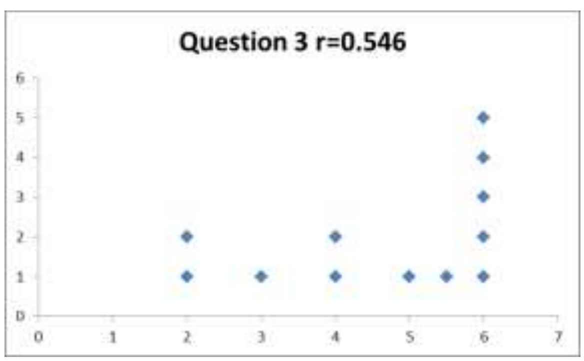


3) The graph below shows the position function of an airliner on an out-and-back trip from Seattle to Minneapolis where $s = f(t)$ is the number of round miles from Seattle t hours after take-off at 6am. When is the plane traveling the fastest toward Seattle?
(6 points)



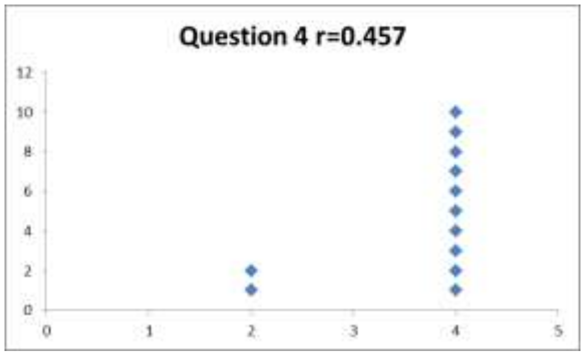
$t = 7$ which is 1pm.

(Anything between 6 (noon) and 7.5 (1:30pm) was given full credit)



4) Find the derivative of $f(x) = \tan(sin(cos(x)))$.
(4 points)

$$\sec^2(\sin(cos(x))) \cdot \cos(cos(x)) \cdot (-\sin(x))$$

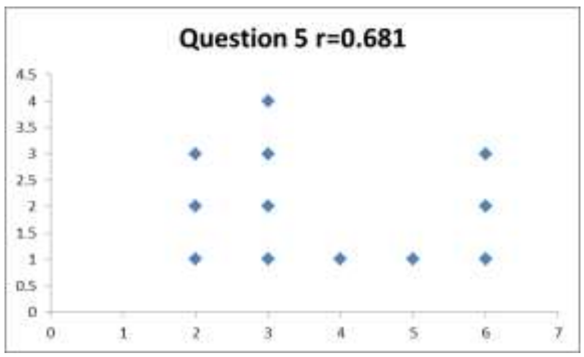


5) Given the equation $\sin(y) + \cos(x) = e^x$ where both x and y are functions t, find $\frac{dy}{dt}$.

(6 points)

$$\sin(y)\,y' - \sin(x)\,x' = e^x x'$$
$$\sin(y)\,y' = e^x x' + \sin(x)\,x'$$
$$y' = \frac{e^x x' + \sin(x)\,x'}{\sin(y)}$$

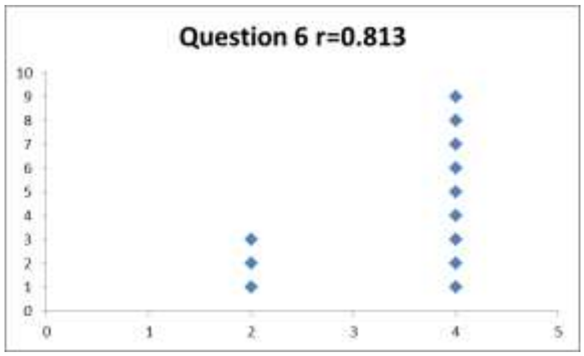
Note that in this problem $y' = \frac{dy}{dt}$ and $x' = \frac{dx}{dt}$



6) Find the derivative of $\ln(x^7 + 2x)$.

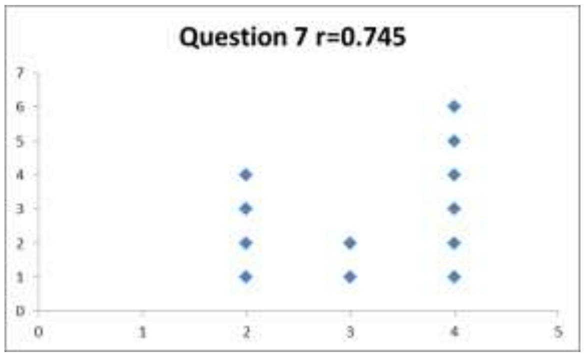
(4 points)

$$\frac{d}{dx}\ln(x^7 + 2x) = \frac{1}{x^7 + 2x} \cdot (7x^6 + 2)$$



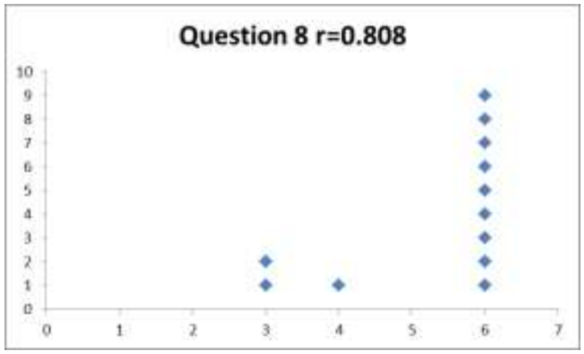
7) Find the derivative of $\tan^{-1}(x^2+1)$.
(4 points)

$$\frac{d}{dx}\tan^{-1}(x^2+1) = \frac{1}{1+(x^2+1)^2}\cdot 2x = \frac{2x}{1+(x^2+1)^2}$$



8) Find the derivative of $\left((x+2)(x^2+1)\right)^4$.
(6 points)

$$y' = 4\left((x+2)(x^2+1)\right)^3 \cdot [1 \cdot (x^2+1) + (x+2) \cdot 2x]$$



9) A certain cone starts out with a radius of 1mm and height of 3mm. It maintains the same aspect ratio while its radius expands at a rate of 4mm/s. When the volume is 27π mm^3, how quickly is the volume increasing?

(Note that the volume of a right circular cone is given by $V = \frac{1}{3}\pi r^2 h$)

(10 points)

Variables:

$V = 27\pi$

$V = ?$

$r = ?$

$r' = 4$

$h = 3r$

$h' = 3r'$

Equation:

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 3r = \pi r^3$

Derivative:

$V' = 3\pi r^2 r'$

Solve for r

$V = \pi r^3$

$27\pi = \pi r^3$

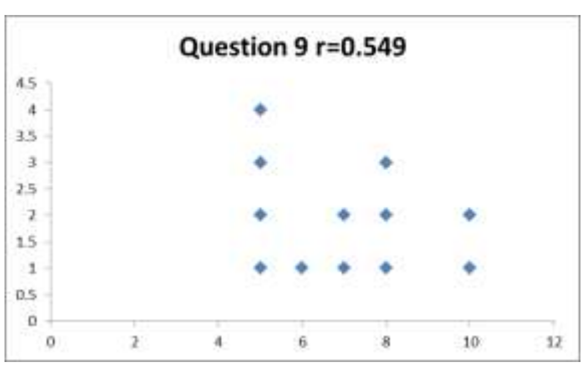
$27 = r^3$

$3 = r$

Solution:

$V' = 3\pi r^2 r'$

$V' = 3\pi(3)^2 4 = 108\pi$ mm^3/s



10) Use the first derivative test to graph the function below.

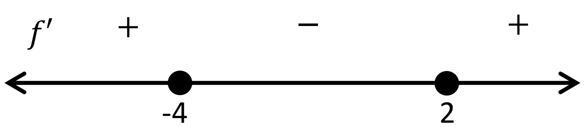
$$f(x) = x^3 + 3x^2 - 24x$$

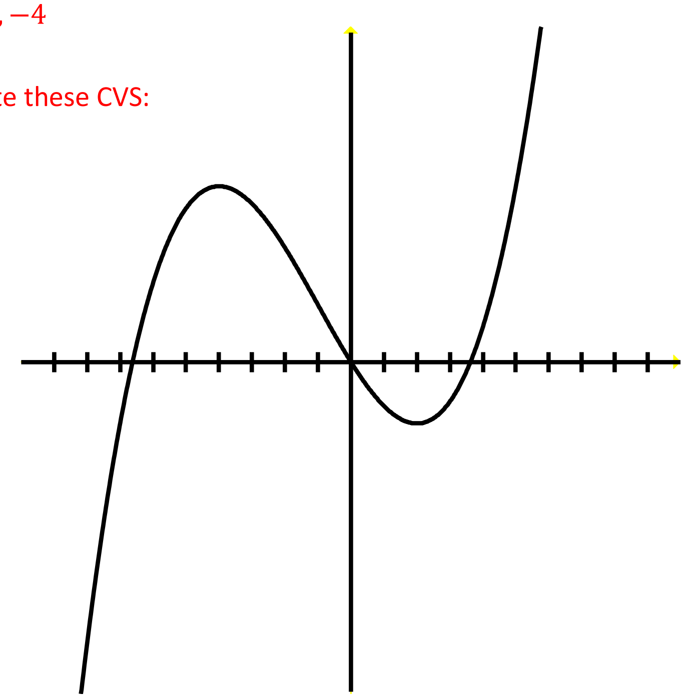
(10 points)

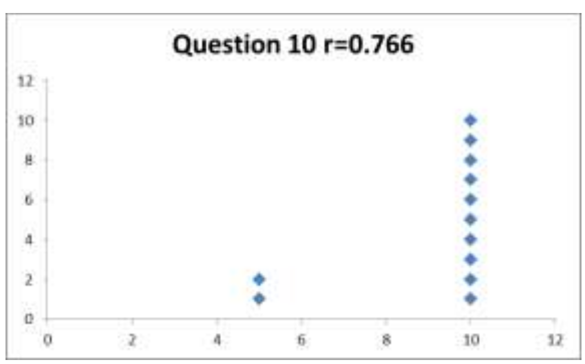
$f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = 3(x - 2)(x + 4)$

The critical values are when $f'(x) = 0$, which are $x = 2, -4$

The first derivative test calls for a sign chart to investigate these CVS:







11) Use the second derivative test to graph the function below.

$$f(x) = 2x^3 - 6x^2 - 90x$$

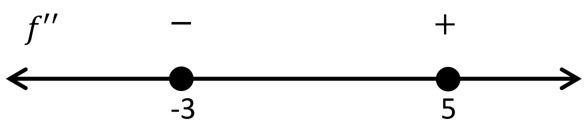
(10 points)

$f'(x) = 6x^2 - 12x - 90 = 6(x^2 - 2x - 15) = 6(x-5)(x+3)$

The critical values are when $f'(x) = 0$, which are $x = 5, -3$

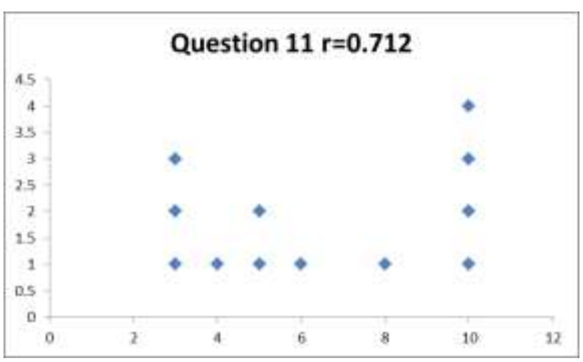
The second derivative test calls to evaluate the second derivative at these CVs:

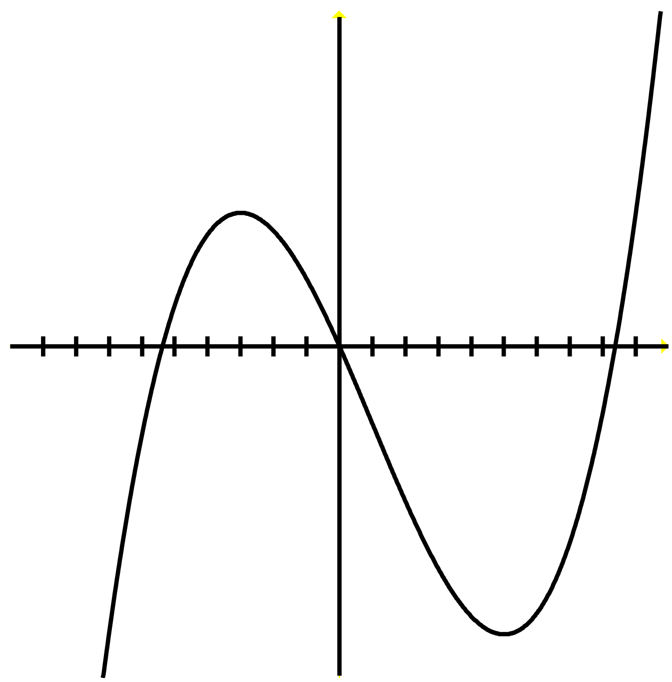
$f''(x) = 12x - 12$



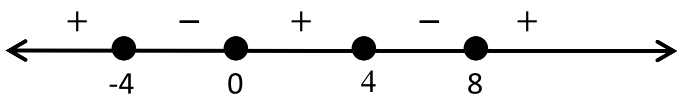
$f''(5) > 0$ --- concave up, so it's a minimum.

$f''(-3) < 0$ --- concave down, so it's a maximum.

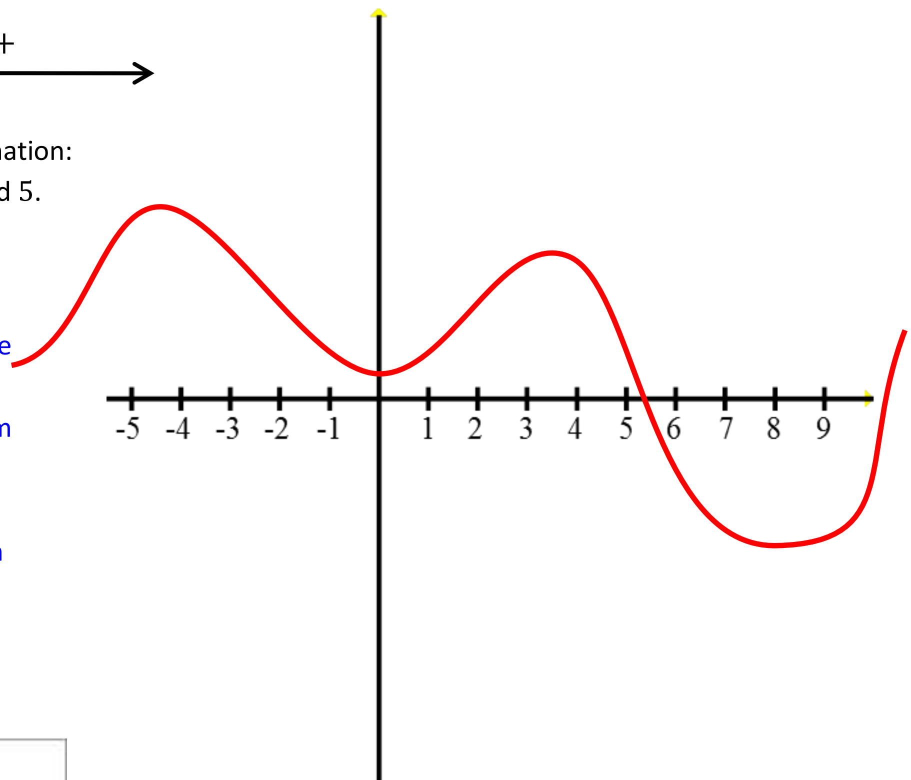




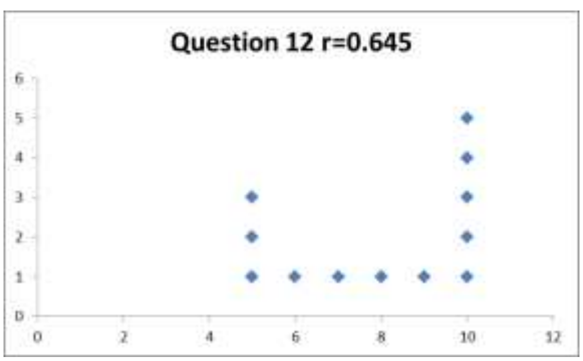
12) Graph a function f that is defined everywhere; the sign chart for $\frac{d}{dx}f$ is given below. Each value labelled on the sign chart is a critical value.



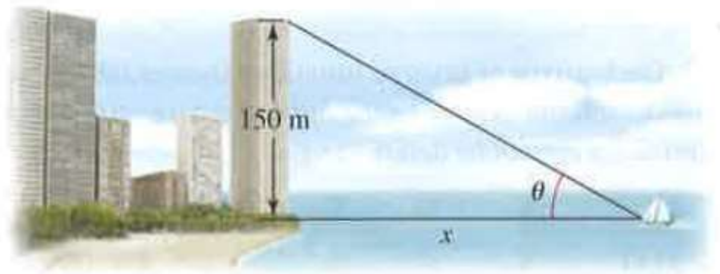
Assume we also know the following information:
$f(x)$ has inflection points at $x = -2, 2,$ and 5.
(10 points)

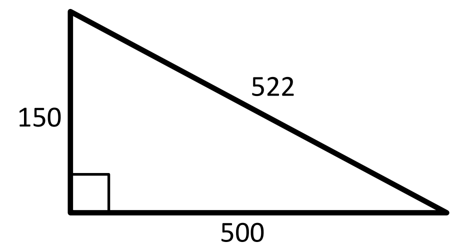


Note that the first few critical values and inflection points turn out fairly smooth. The inflection point at $x = 5$ is the tough one because that means that it's switching from curving downward out of the maximum to curving upward into the minimum at 5, which is much closer to the maximum than the minimum.



13) A boat sails directly toward a 150-meter skyscraper that stands on the edge of a harbor. The angular size θ of the building is the angle formed by lines from the top and bottom of the building to the observer (see figure). A particular triangle that might be helpful is also given.





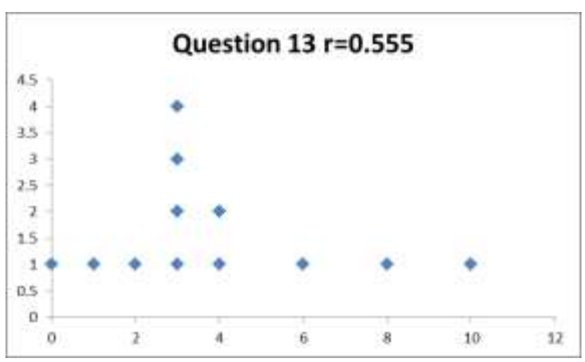
What is the rate of change of the angular size $\frac{d\theta}{dx}$ when the boat is $x = 500m$ from the building?

(10 points)

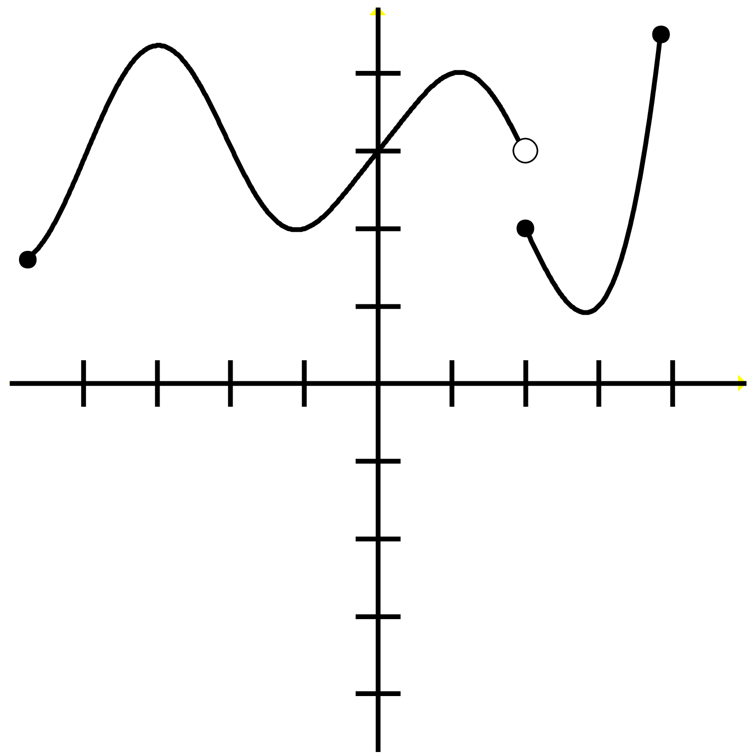
$$\tan(\theta) = \frac{150}{x}$$

$$\sec^2(\theta)\,\theta' = -\frac{150}{x^2}$$

$$\theta' = \frac{-150}{x^2\sec^2(\theta)} = \frac{-150\cos^2(\theta)}{x^2} = \frac{-150\cdot 500^2}{500^2\cdot 522^2} = \frac{-150}{522^2}\ rad/m$$



Use the graph below to answer the questions on this page.



14) Find all local maximizers of the function.
(4 points)

$x = -3, 1, 4$

15) Find all local minimum values of the function.
(4 points)

$y = 1.5, 2, 1$

16) Find all x-values where the function changes concavity.
(2 points)

$x = -4, -2, 0, 2$