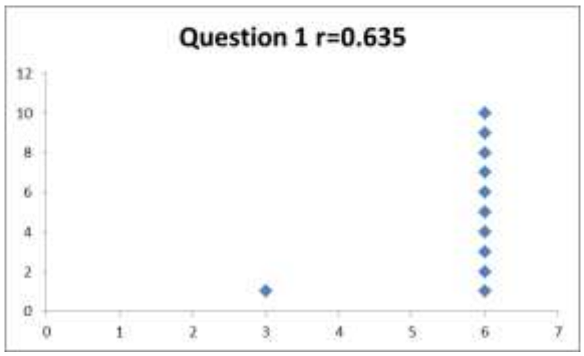
Name ____ Test 3, Fall 2018

1) Calculate the definite integral below.
(6 points)

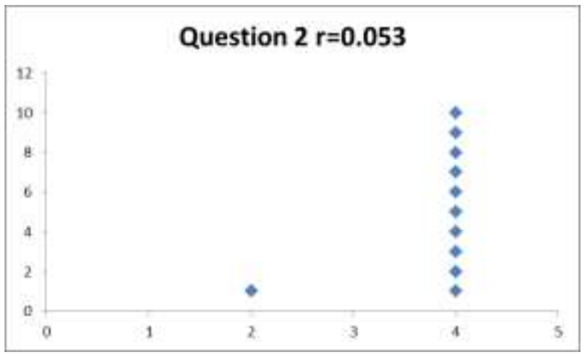
$$\int_0^4 8 - 2x\,dx = 8x - x^2\Big|_0^4 = 32 - 16 - (0 - 0) = 16$$



2) Suppose $\int_1^4 f(x)dx = 8$. Find the integral below.

(4 points)

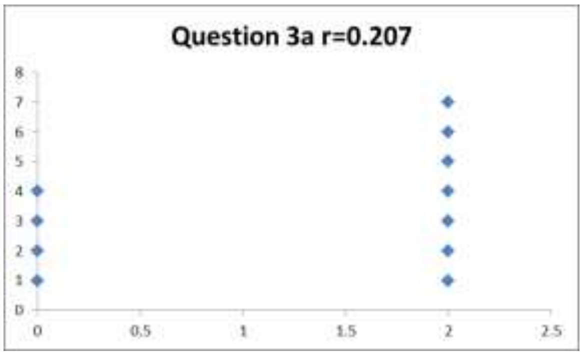
$$\int_1^4 3f(x)dx = 3\int_1^4 f(x)dx = 3 \cdot 8 = 24$$

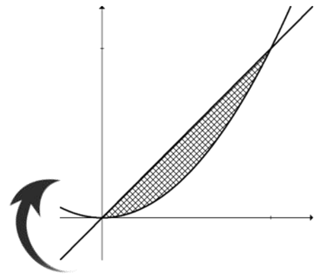


3) Consider the region bounded by $y = x$ and $y = x^2$ in the first quadrant, rotated around the x-axis.

a. Which figure best describes the solid created? (2 points)
(See pictures on projector)

#8, which is shown here.





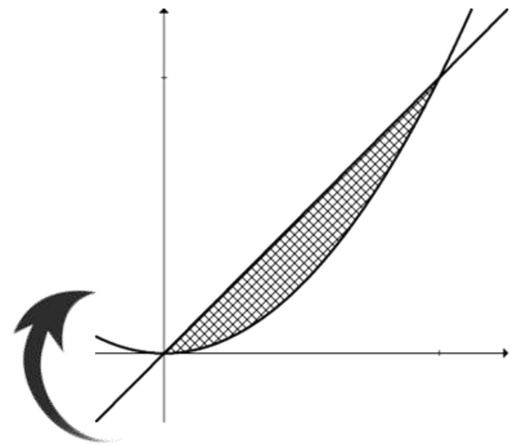
b. Set up an integral that gives the **volume** of the solid created. You do NOT need to calculate the integral. Specify which method you're using. (4 points)
(Disk/washer) (Cylindrical Shell)

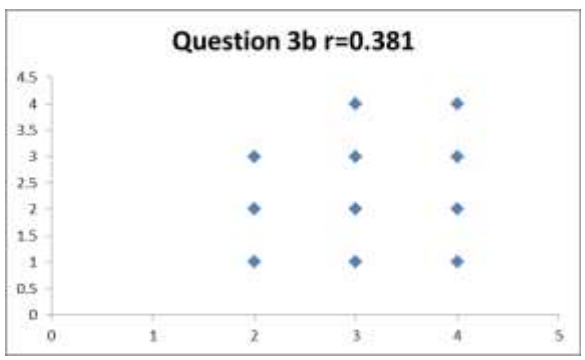
Disk/washer:

$$\int_0^1 \pi x^2 - \pi(x^2)^2 dx$$

Shell:

$$\int_0^1 2\pi y(\sqrt{y} - y)dy$$

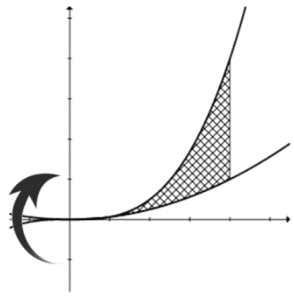


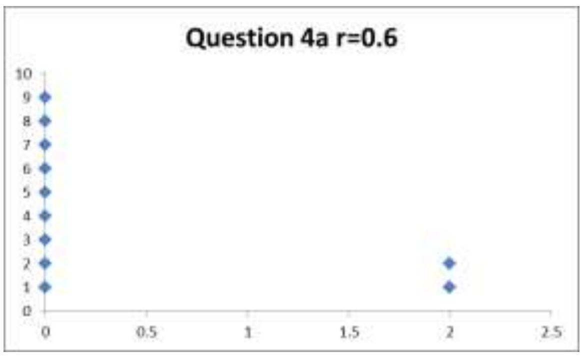


4) Consider the region bounded by $y = x^3, y = x^2, x = 1$, and $x = 3$, rotated around the x-axis.

a. Which figure best describes the solid created? (2 points)
   (See pictures on projector)

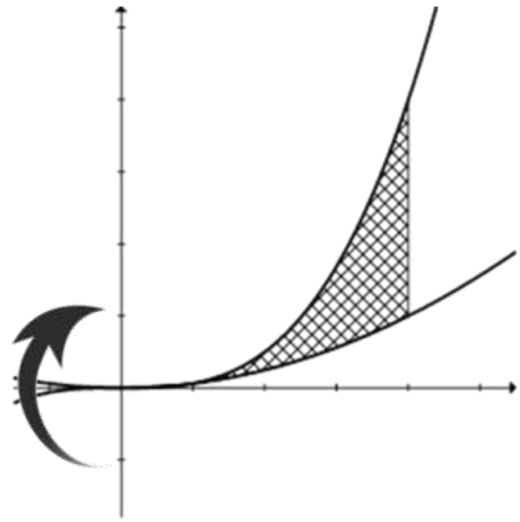
#3, which is shown below

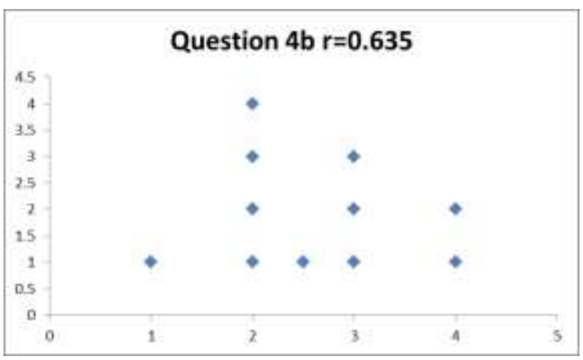




b. Set up an integral that gives the **surface area** of the solid created. You do NOT need to calculate the integral. You do NOT need to worry about any flat part(s). Only the curved part(s). (4 points)

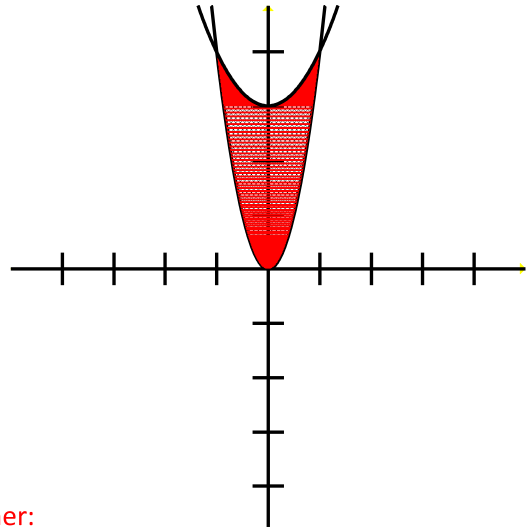
$\int_0^1 2\pi x^3\sqrt{1+(3x^2)^2}dx = \int_0^1 2\pi x^2\sqrt{1+(2x)^2}dx$





5) Graph these two curves, then find the area enclosed by the them. (10 points)

$$y = 4x^2$$
$$y = x^2 + 3$$

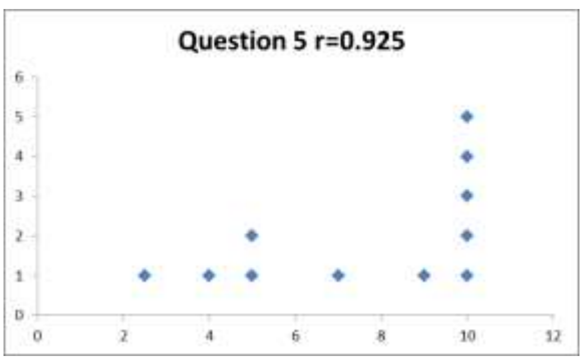


First find the points of intersection by setting them equal to each other:

$$4x^2 = x^2 + 3$$
$$3x^2 = 3$$
$$x^2 = 1$$
$$x = \pm 1$$

Now, note that between -1 and 1, $y = x^2 + 3$ is larger, so the area is given by:

$$\int_{-1}^{1} \left((x^2+3) - 4x^2\right)dx = \int_{-1}^{1} (3 - 3x^2)dx = 3x - x^3\Big|_{-1}^{1} = (3-1) - \left(-3-(-1)\right) = 2 - (-2) = 4$$



6) Find the integral below.
(6 points)

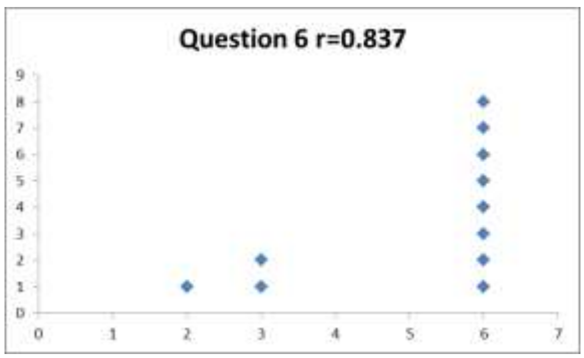
$$\int \cos(ax)\,dx = \frac{1}{a}\int \cos(u)\,du = \frac{1}{a}\sin(u) + C = \frac{1}{a}\sin(ax) + C$$

(a is a nonzero constant)

$u = ax$

$du = adx$

$\frac{1}{a}du = dx$



7) Find the integral below.

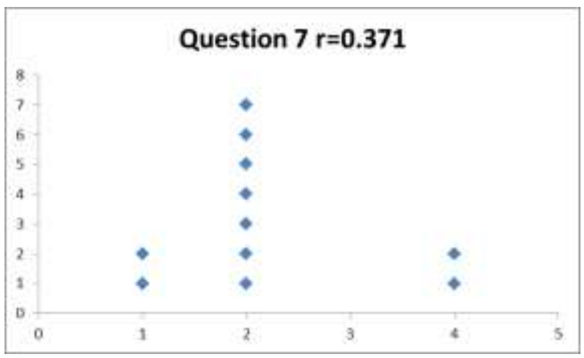
(4 points)

$$\int \frac{1}{1+4x^2}dx = \int \frac{1}{1+(2x)^2}dx = \frac{1}{2}\int \frac{1}{1+u^2}du = \frac{1}{2}\tan^{-1}(u) + C = \frac{1}{2}\tan^{-1}(2x) + C$$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$



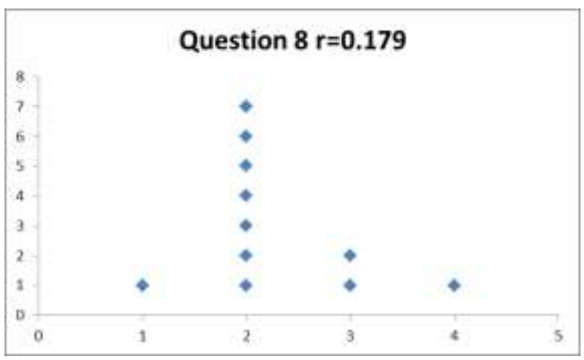
8) Find the integral below.
(4 points)

$$\int \frac{x}{\sqrt{x-5}}dx = \int \frac{u+5}{\sqrt{u}}du = \int \frac{u}{\sqrt{u}} + \frac{5}{\sqrt{u}}du = \int u^{\frac{1}{2}} + 5u^{-\frac{1}{2}}du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{5u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{2}{3}(x-5)^{\frac{3}{2}} + 10(x-5)^{\frac{1}{2}} + C$$

$u = x - 5$
$du = dx$

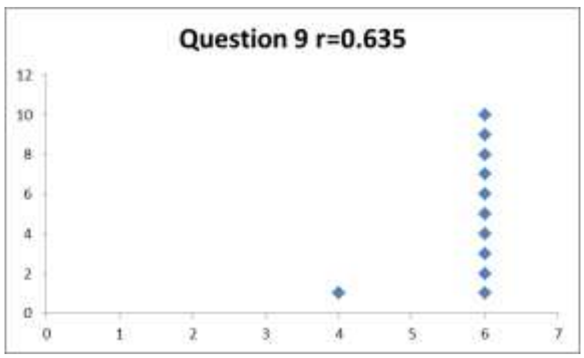
Also,
$u + 5 = x$



9) Find the integral below.
(6 points)

$$\int 2x(x^2-1)^{99}dx = \int u^{99}du = \frac{u^{100}}{100} + C$$

$u = x^2 - 1$
$du = 2xdx$



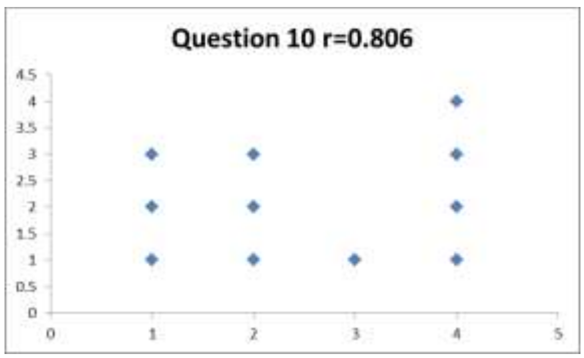
10) Find the integral below.

(4 points)

$$\int \frac{\cos(x)}{\sin^2(x)}dx = \int \frac{du}{u^2} = \int u^{-2}du = \frac{u^{-1}}{-1} + C = -(\sin(x))^{-1} + C = -\frac{1}{\sin(x)} + C$$

$u = \sin(x)$

$du = \cos(x)\,dx$

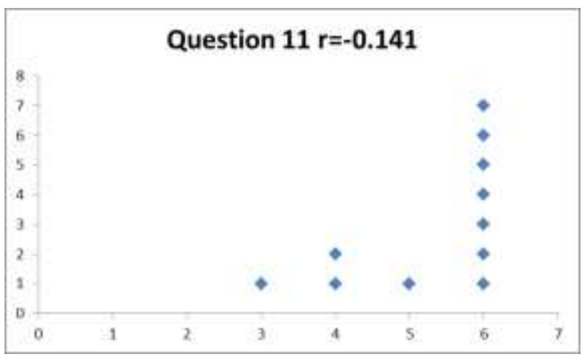


11) Calculate the definite integral below.
(6 points)

$$\int_{-1}^{2} x^2 e^{x^3+1} dx = \frac{1}{3}\int_{0}^{9} e^u du = \frac{1}{3}e^u \Big|_0^9 = \frac{1}{3}e^9 - \frac{1}{3}e^0 = \frac{1}{3}e^9 - \frac{1}{3}$$

$u = x^3 + 1$
$du = 3x^2 dx$
$\frac{1}{3} du = x^2 dx$

When $x = -1, u = 0$
When $x = 2, u = 9$



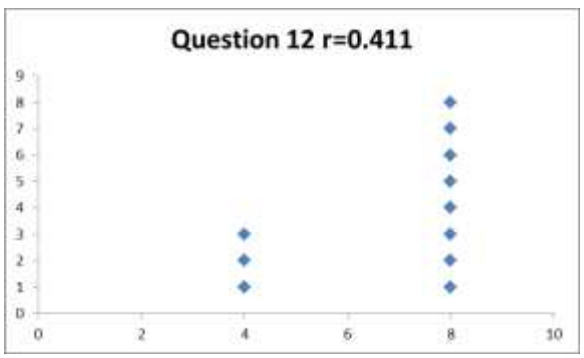
12) Consider the function f with derivative $f'(x) = 3x^2 + 3$ and initial value $f(1) = 8$. Find $f(x)$.
(8 points)

$$\int 3x^2 + 3dx = x^3 + 3x + C$$

$$1^3 + 3 \cdot 1 + C = 8$$
$$C = 4$$

Answer:

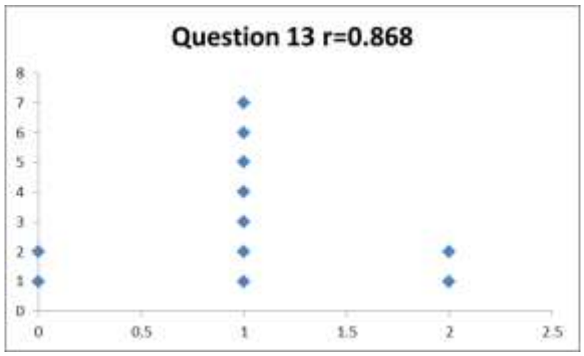
$$f(x) = x^3 + 3x + 4$$



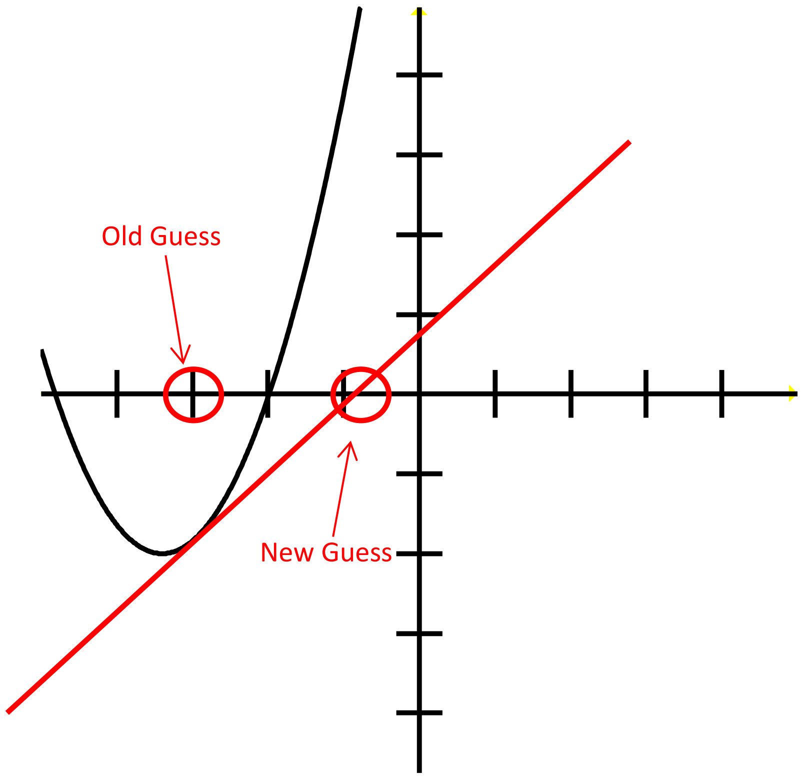
13) Evaluate the expression below.

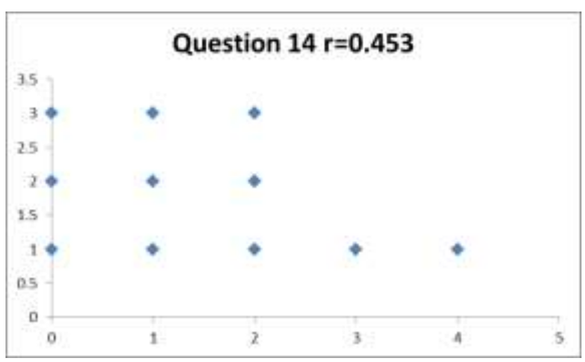
(2 points)

$$\frac{d}{dx}\int_{3}^{x^2}(t^2+t+1)dt=((x^2)^2+x^2+1)\cdot 2x$$



14) Below is a graph of a function. If Dr. Beyerl guesses that a root of the function is $x = -3$, use Newton's Method to improve his guess. Illustrate what you do on the graph, and circle the new guess. (4 points)

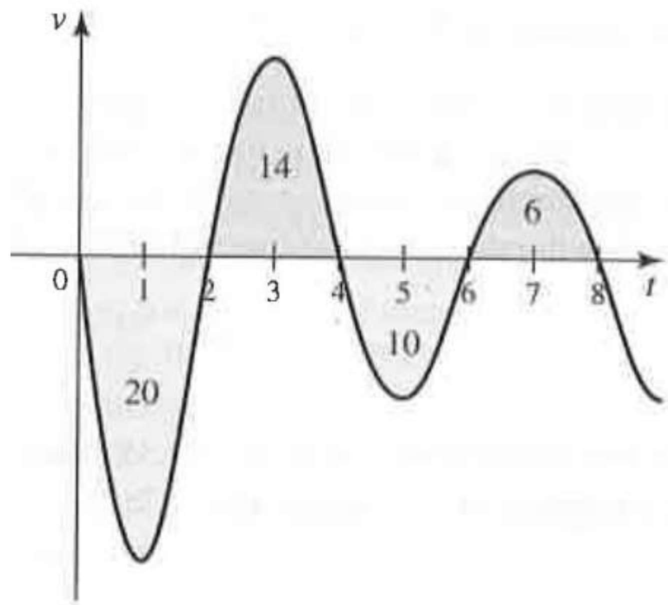


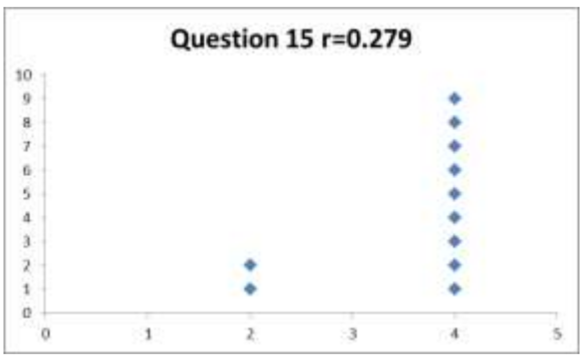


Consider the velocity function shown below of an object moving along a line. Assume time is measured in seconds and distance is measured in meters. The area of four regions bounded by the velocity curve and the t-axis are also given.

15) On what intervals is the objet moving in the negative direction? (4 points)

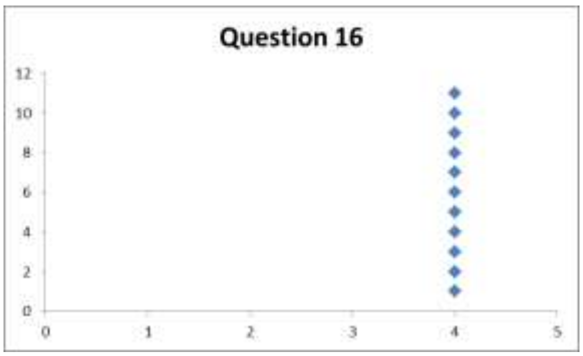
(0,2) and (4,6)

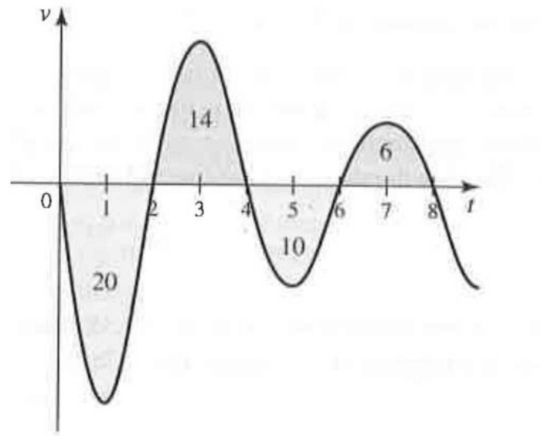




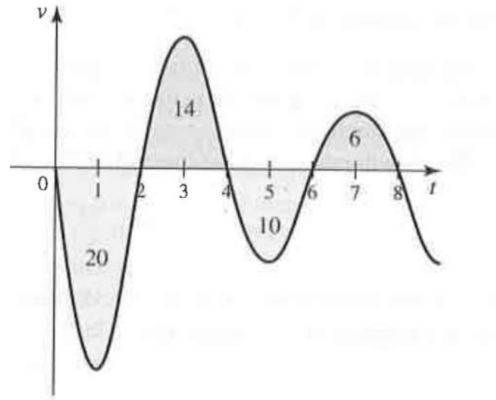
16) What is the displacement of the object over the interval $[2,6]$? (4 points)

4 meters

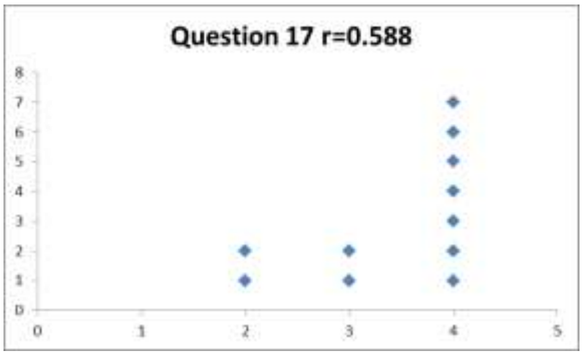




17) How far does the object travel over the interval $[0,6]$? (4 points)

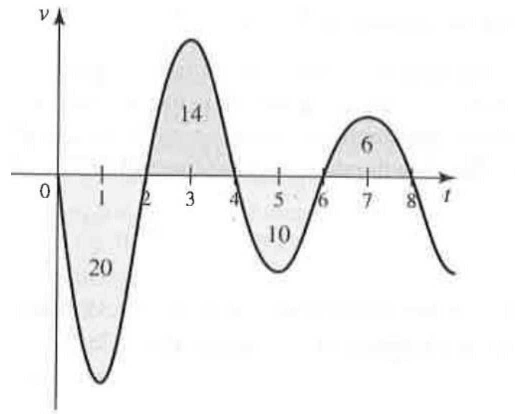


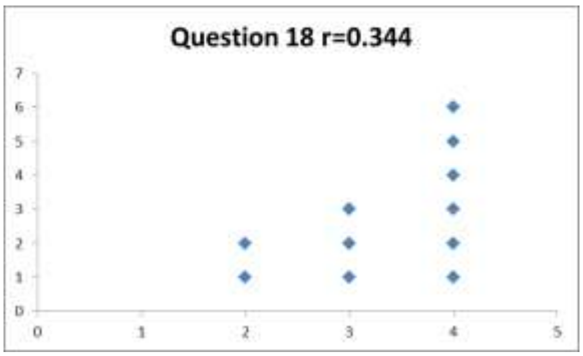
44 meters



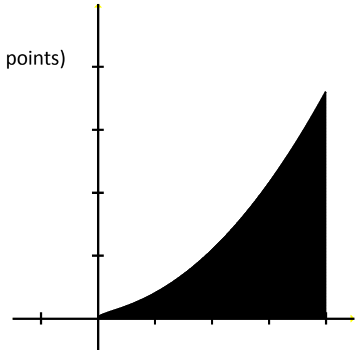
18) Describe the position of the object relative to its initial position after 8 seconds. (4 points)

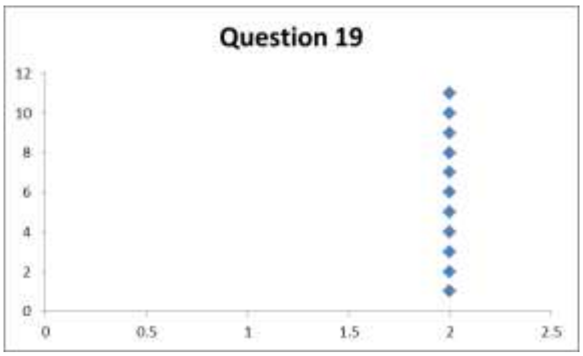
10 meters in the negative direction.





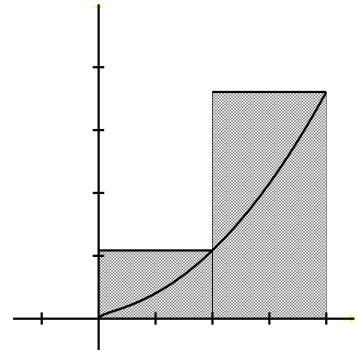
19) Illustrate (do not calculate) the area under the curve given below. (2 points)

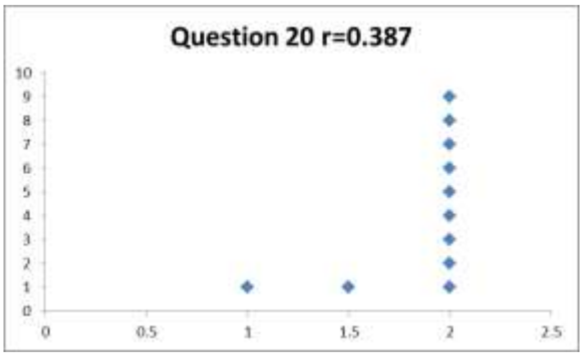




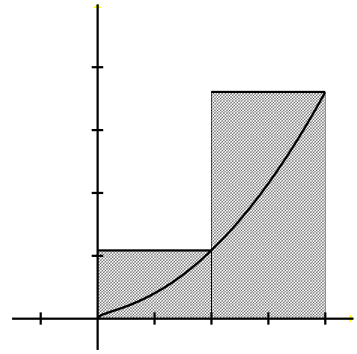
20) Illustrate (do not calculate) an approximation to the area under the curve given below. (2 points)

(There are multiple answers)



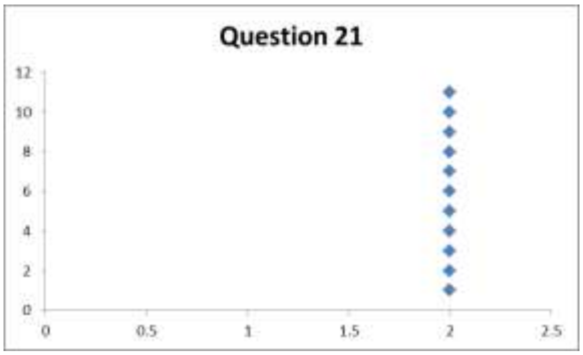


21) Calculate the approximation you illustrated in #20. (2 points)



(There are multiple answers, but each is unique and based on your answer to #19)

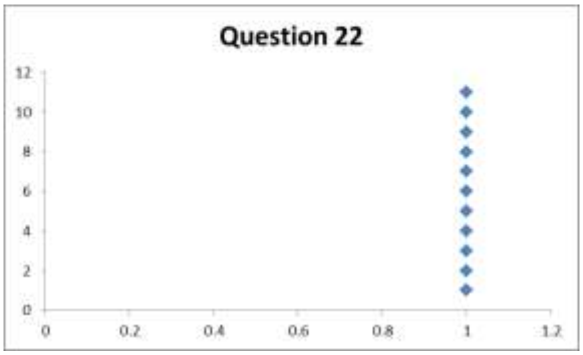
$1.1 \cdot 2 + 3.5 \cdot 2$

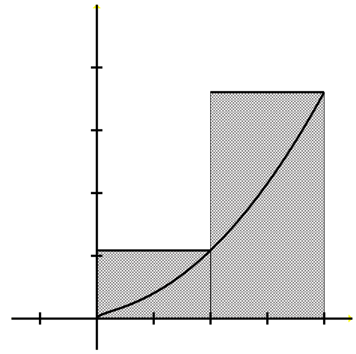


22) Is your approximation in #20 an overestimate or an underestimate? (1 point)

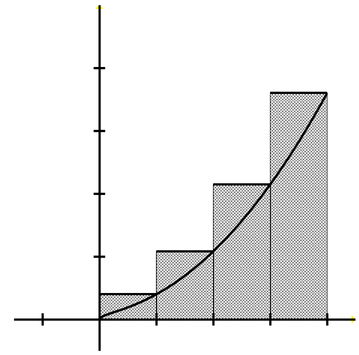
(Again based on #3)

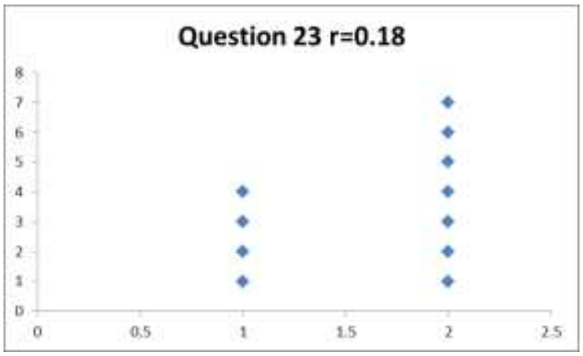
Overestimate





23) Illustrate a better approximation than you came up with in #20. (2 points)





24) Illustrate an even better approximation than you came up with in the previous question. (1 points)