Name ____Solutions____ Quiz 4

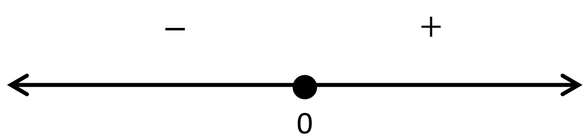
1) Graph the function $y = (x^4 + 2x^2 + 1)^5$ on the axis below. Your graph should accurately reflect all maxima and minima, however you need not worry about concavity. Show your work.

$$y' = 5(x^4 + 2x^2 + 1)^4(4x^3 + 4x)$$

$$5(x^4 + 2x^2 + 1)^4(4x^3 + 4x) = 0$$
$$\therefore (4x^3 + 4x) = 0$$
$$\therefore 4x(x^2 + 1) = 0$$
$$\therefore x = 0$$

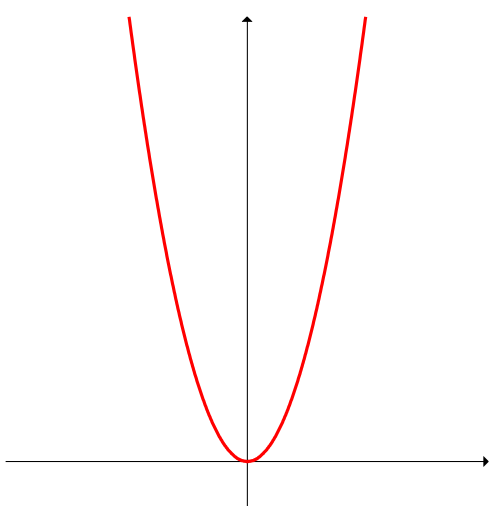
There is one critical value at $x = 0$.

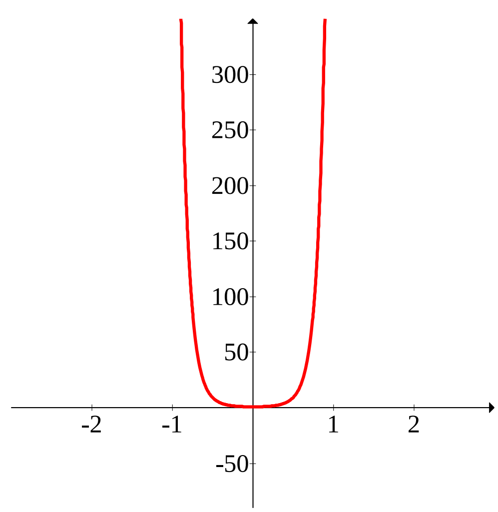
Doing the first derivative test, we see that this is a minimum. Note that we plug values into the derivative, $y' = 5(x^4 + 2x^2 + 1)^4(4x^3 + 4x)$.



Based on this information, we can make the rough sketch below on the left. I didn't give a scale to the axes because it's only meant to illustrate the shape of the graph.

The reality is that this function grows very quickly, however. Notice that the dominant term is x^{20} ! That's huge. So if you give it a scale, it actually looks more like the function on the right.





2) A spherical balloon is increasing in size as air fills the inside. Air is being pumped in such that the radius is increasing at 2in/min. When the radius is exactly 5 inches, how fast is the surface area increasing? Note that the formula for surface area of a sphere is $SA = \frac{4}{3}\pi r^2$.

$SA' = \frac{8}{3}\pi r r'$

$r = 5$
$r' = 2$

$SA' = \frac{8}{3}\pi \cdot 5 \cdot 2 = \frac{80}{3}\pi$ inches per minute.