Name __Solutions____ Test 1, Spring 2018

1) Using the graph below, find the following limits.
(2 points each)

$\lim_{x \to -2} f(x) = \infty$

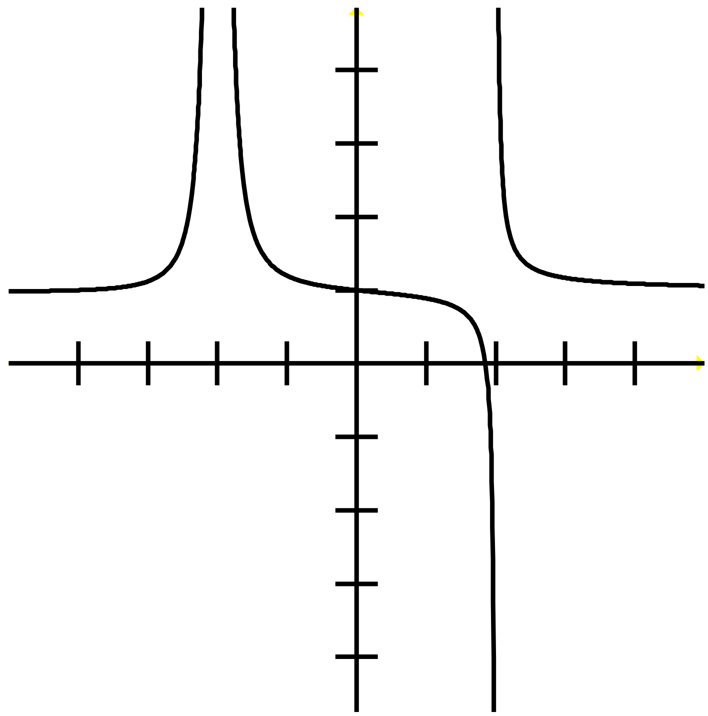
$\lim_{x \to 2^-} f(x) = -\infty$

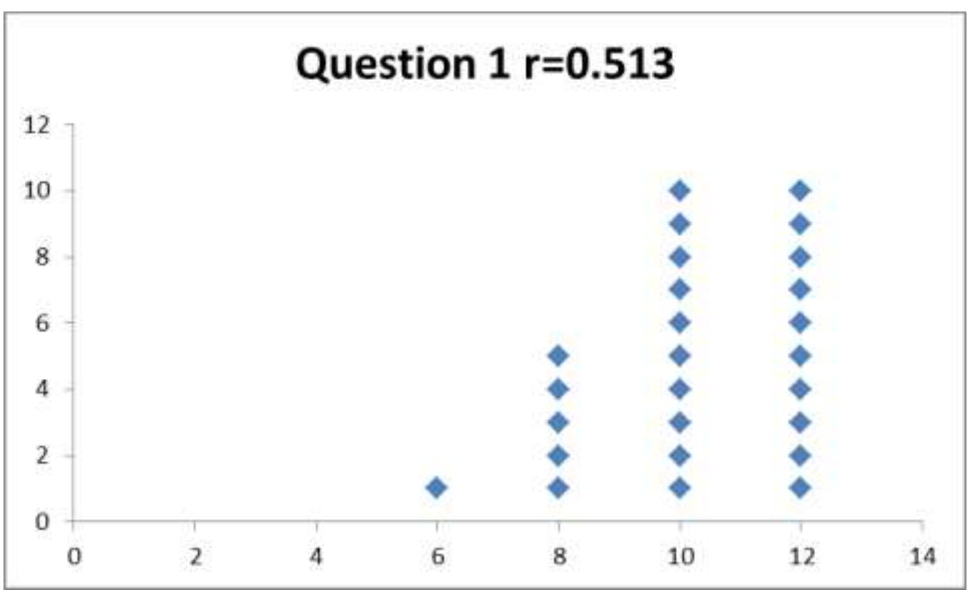
$\lim_{x \to 2^+} f(x) = \infty$

$\lim_{x \to 2} f(x) = \text{DNE}$

$\lim_{x \to 0} f(x) = 1$

$\lim_{x \to \infty} f(x) = 1$

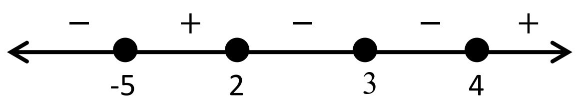




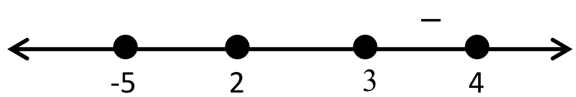
2) Find the limit below. (4 points)

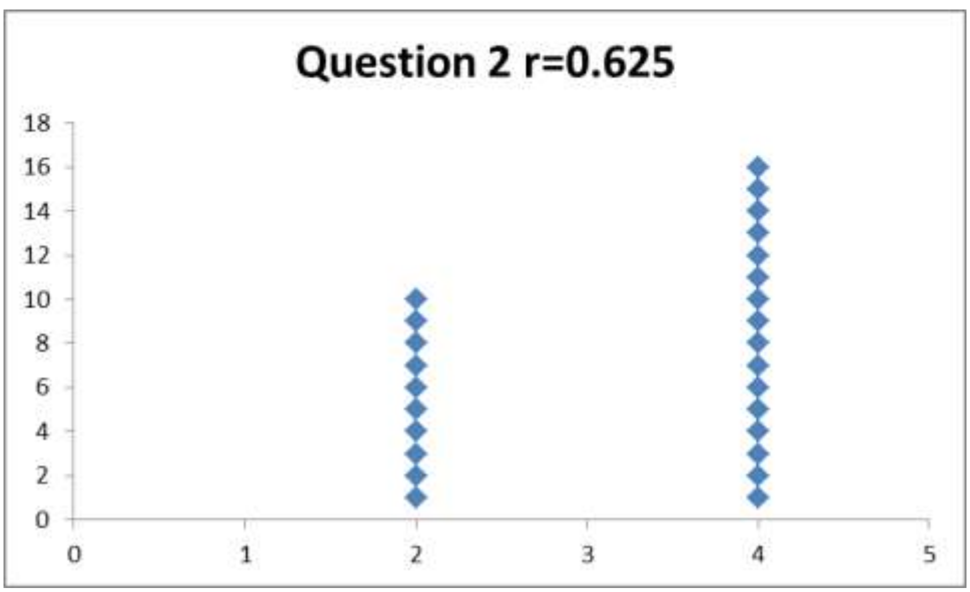
$$\lim_{x\to 3^+} \frac{(x-2)(x+5)}{(x-4)(x-3)^2} = -\infty$$

I found this by computing the whole sign chart:



But the reality is that we're only interested in one sign. So you can plug in any number between 3 and 4 to figure out what sign it is, if you like:



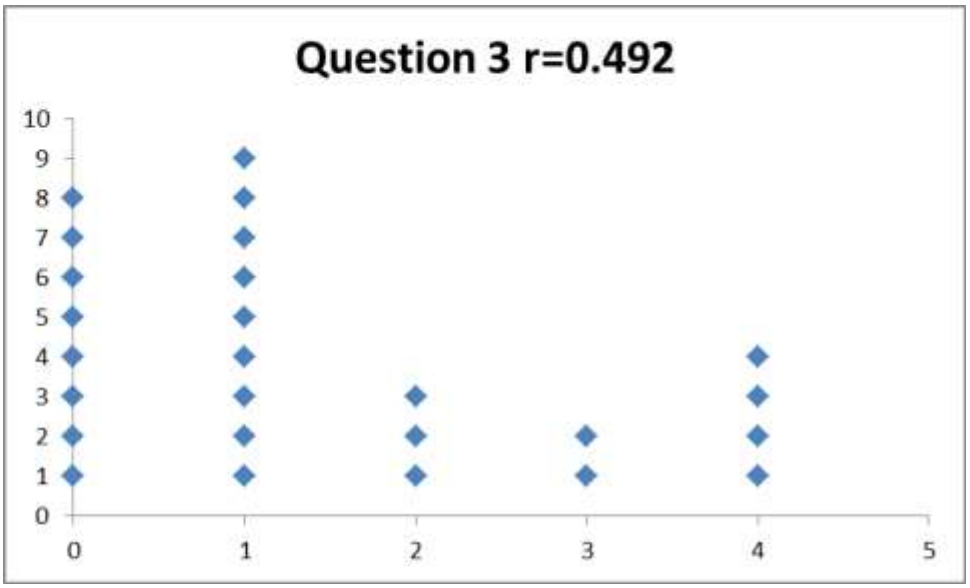


3) Let $r(x)$ be a rational function. That is, a polynomial divided by another polynomial. Use an English sentence to explain when the equation below is true. (4 points)

$$\lim_{x \to a} r(x) = r(a)$$

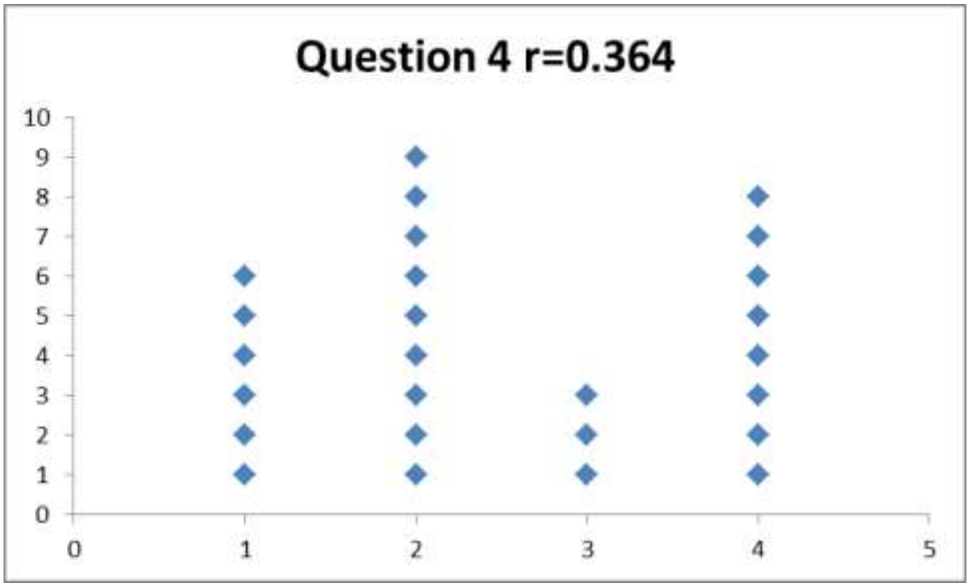
This is true whenever $r(x)$ is continuous. That occurs whenever the denominator is nonzero.

Note: Your response should be addressing the problem at hand: WHEN this is true.



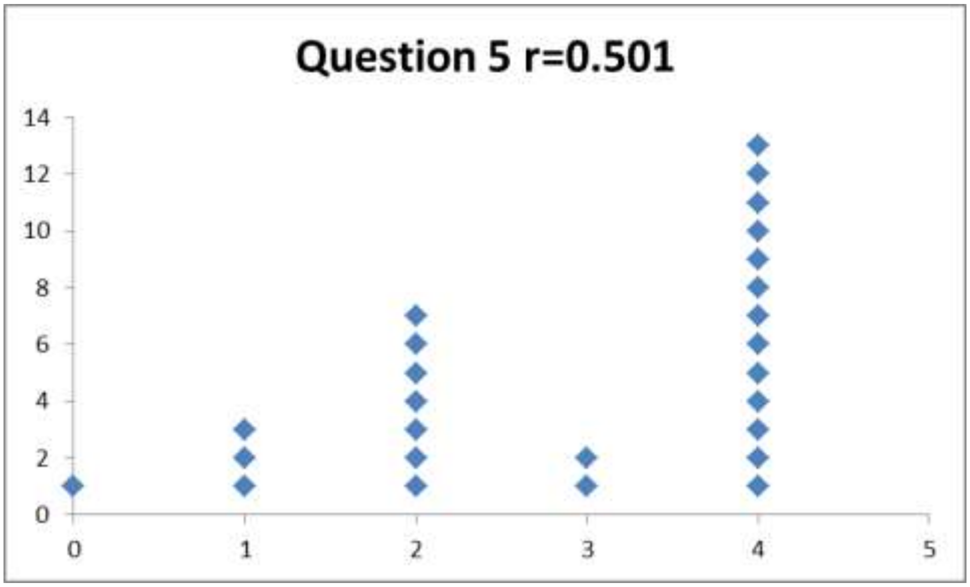
4) Find the limit below. (4 points)

$$\lim_{x\to 4^+} \frac{x^2-16}{4-x} = \lim_{x\to 4^+} \frac{(x+4)(x-4)}{4-x} = \lim_{x\to 4^+} \frac{(x+4)}{-1} = -8$$



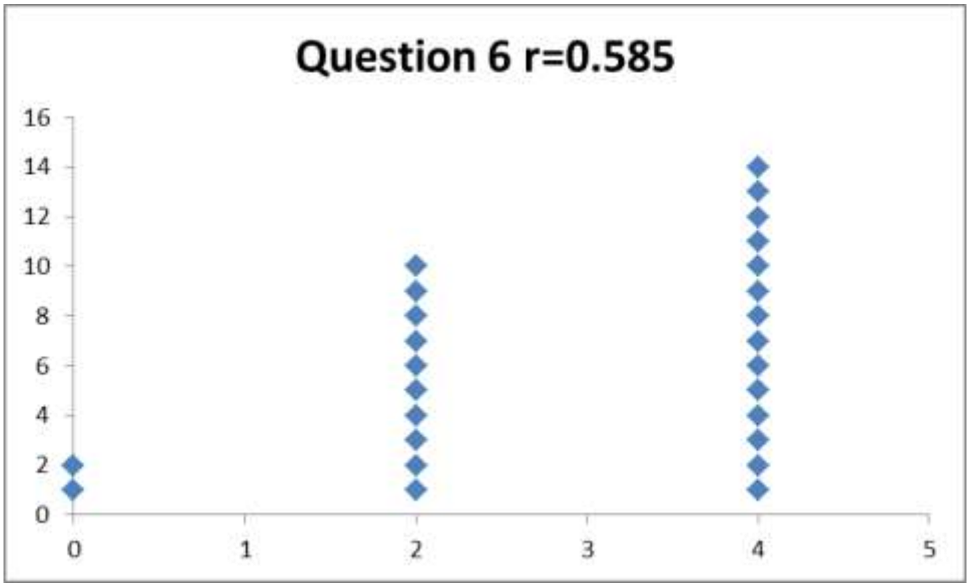
5) Find the limit below. (4 points)

$$\lim_{x\to 4^+}\frac{x^2-16}{(4-x)^2} = \lim_{x\to 4^+}\frac{(x+4)(x-4)}{(4-x)^2} = \lim_{x\to 4^+}\frac{(x+4)}{-(4-x)} = \lim_{x\to 4^+}\frac{(x+4)}{x-4} = \infty$$



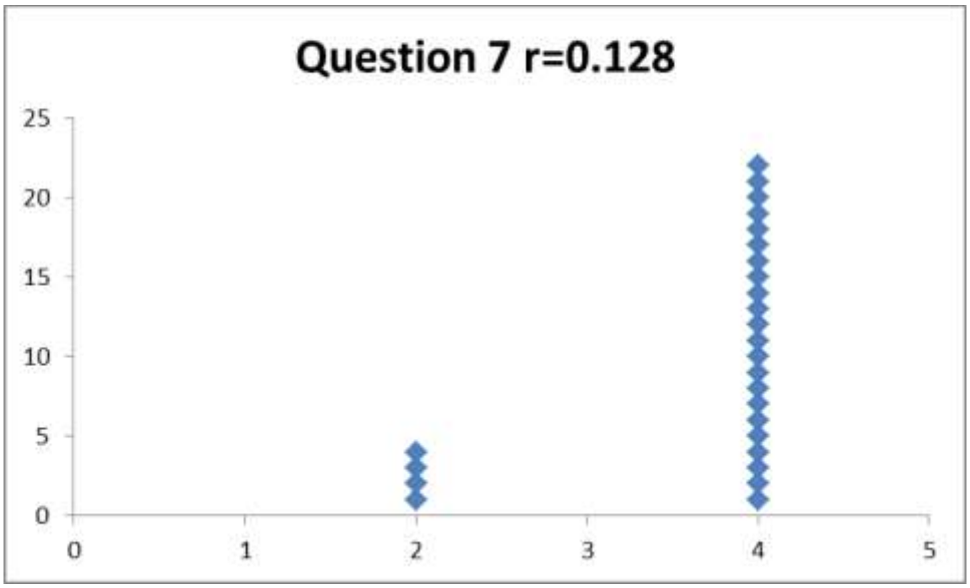
6) Find the limit below. (4 points)

$$\lim_{x\to\infty}\frac{c^3}{x^2} = 0$$



7) Find the limit below. (4 points)

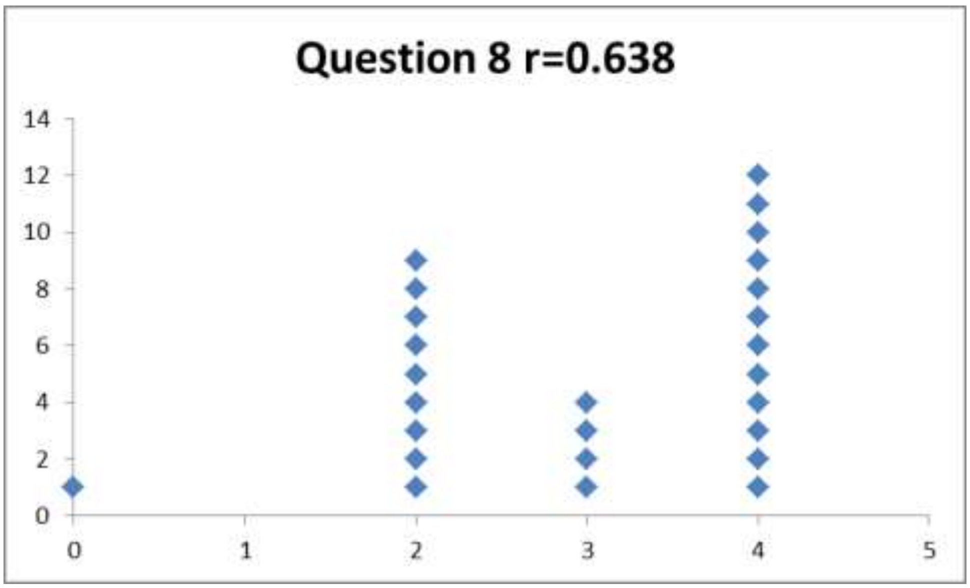
$$\lim_{x\to\infty} \frac{5x^2 - 2x + 3}{3x + 1} = \infty$$



8) Find the limit below. (4 points)

$$\lim_{x\to\infty} \frac{14x^2 + 2x + 1}{\sqrt{x^4 - 2x + 1} + x^2} = 7$$

(Note that the dominant term in the denominator has two copies of x^2. One as x^2, the other as $\sqrt{x^4}$.



) Is the function below continuous? Why or why not? (4 points)

$$f(x) = \begin{cases} \dfrac{x^2 + x}{x + 1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

It certainly is everywhere except $x = -1$ and $x = 1$.

At $x = -1$ it's obviously not continuous because the denominator is zero.

What about at $x = 1$? Let's check and see if $f(1) = \lim_{x \to 1} f(x)$.

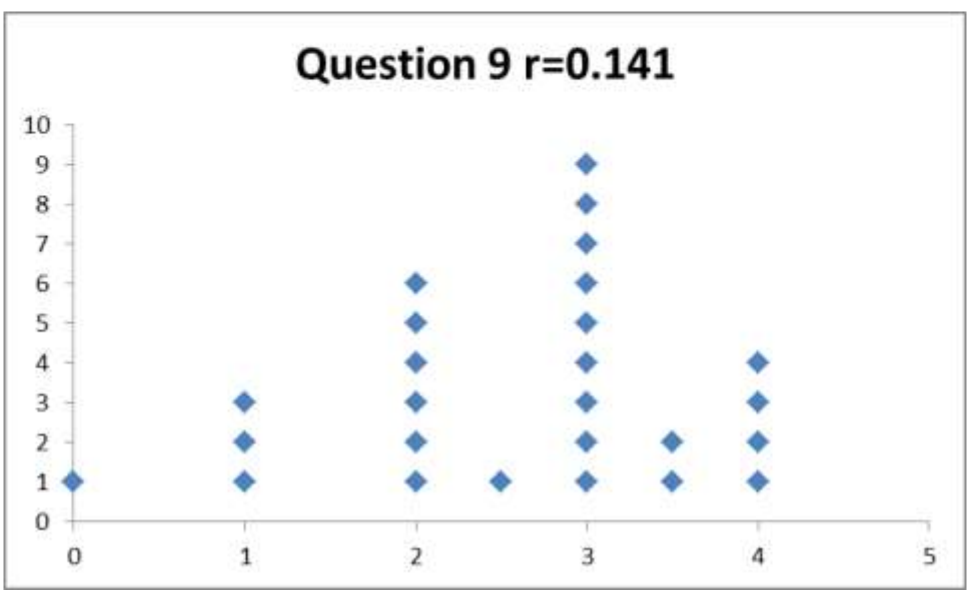
$$f(1) = 2$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^2 + x}{x + 1} = \lim_{x \to \infty} \frac{x(x + 1)}{x + 1} = \lim_{x \to \infty} x = 1$$

Those are not equal… so it's not continuous there either.

For full credit you should address one of those discontinuities.

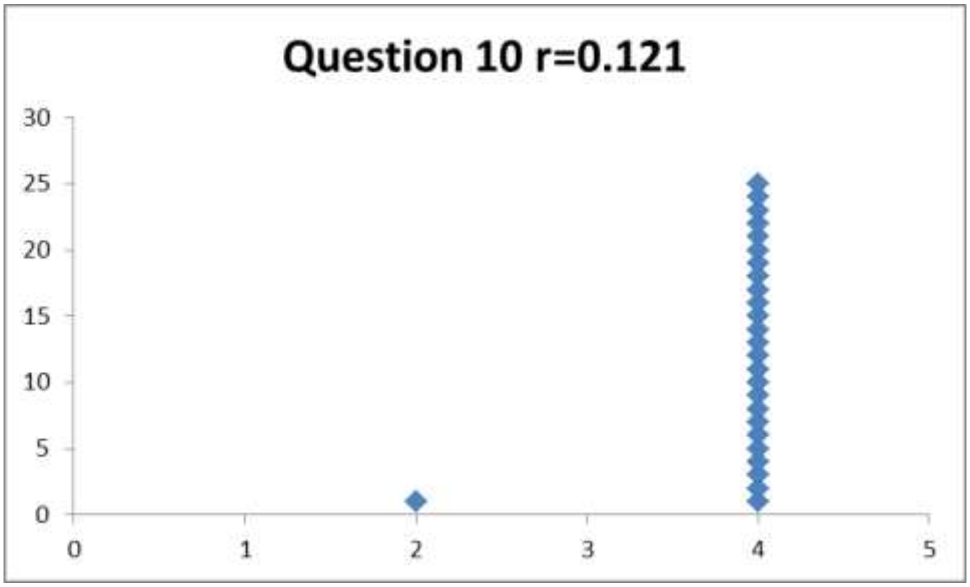
Note: The intention was to have the piecewise function be split at the obvious discontinuity at $x = -1$ to make it not obvious. But I copied the problem down incorrectly from the homework problem it came from and wrote $x = 1$ instead of $x = -1$ for the condition.



9) Find the derivative of the function below. (4 points)

$$f(x) = 2x^2 + 3x + 1$$

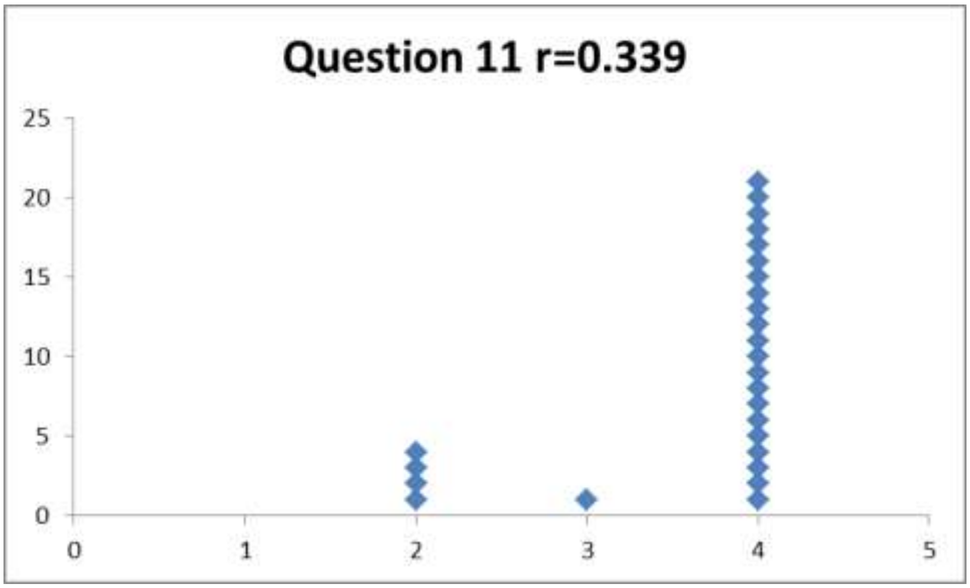
$$f'(x) = 4x + 3$$



10) Find the derivative of the function below. (4 points)

$$f(x) = (2x^7 + 4)(3x^4 - 2x^2)$$

$$f'(x) = (14x^6)(3x^4 - 2x^2) + (2x^7 + 4)(12x^3 - 4x)$$

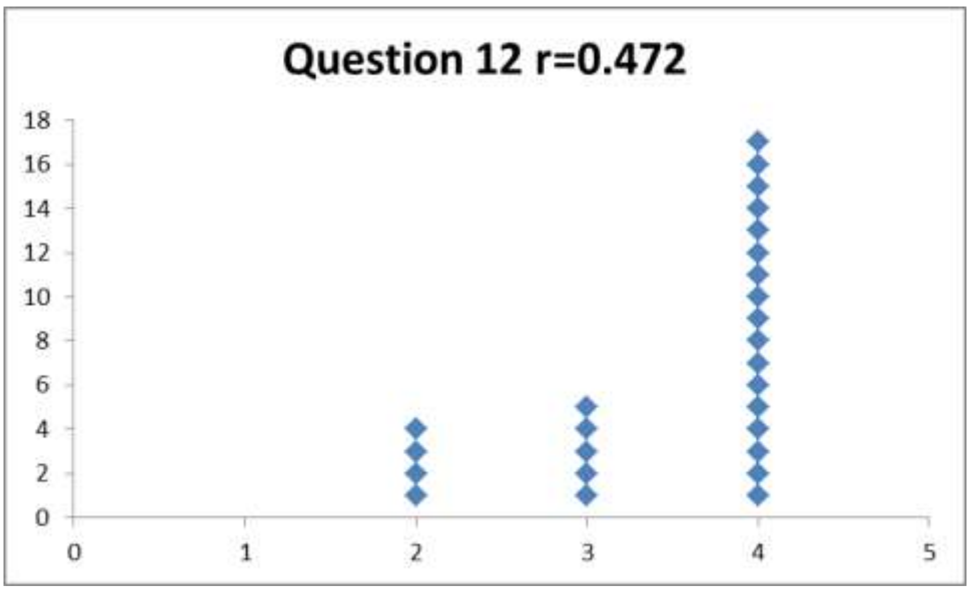


11) Find the derivative of the function below. (4 points)

$$f(x) = \frac{2x^2 + x}{x^3}$$

$$f'(x) = \frac{(4x+1)(x^3) - (2x^2+x)(3x^2)}{(x^3)^2}$$

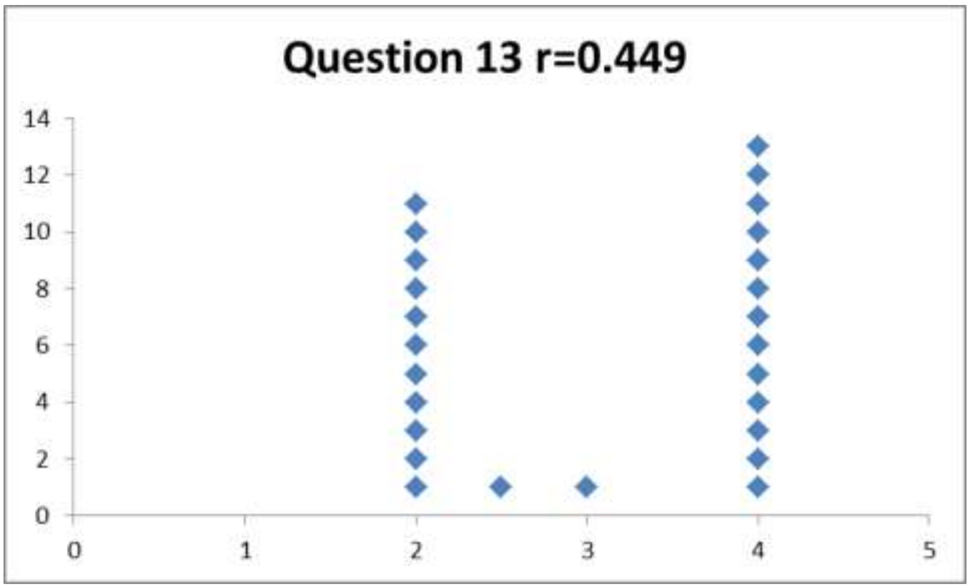
Note: You could also do this without the quotient rule by simplifying to $2x^{-1} + x^{-2}$ first and using the power rule.



12) Find the derivative of the function below. (4 points)

$$f(x) = \cos(2x)$$

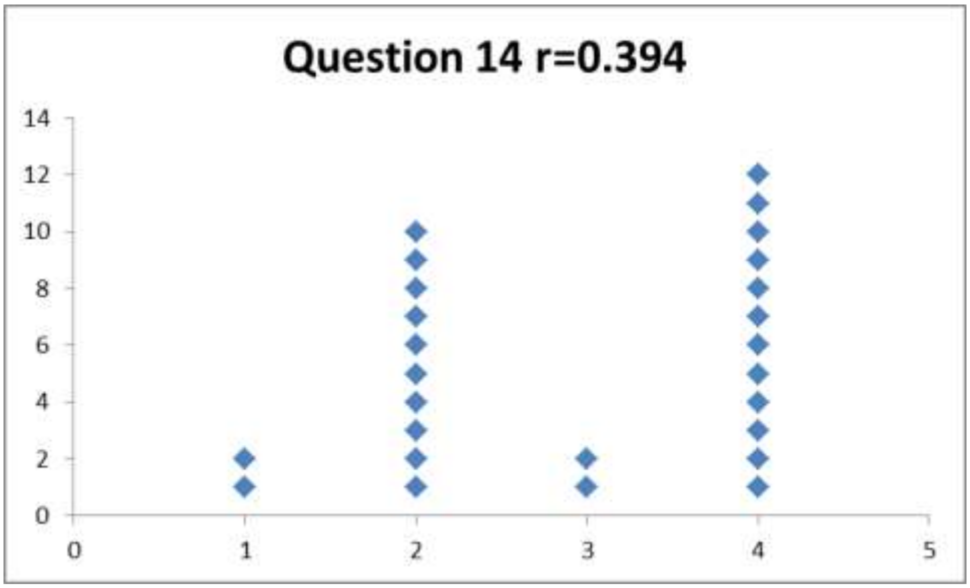
$$f'(x) = -\sin(2x) \cdot 2$$



13) Find the derivative of the function below. (4 points)

$$f(x) = 3^{2x}$$

$$f'(x) = 3^{2x}\ln(3) \cdot 2$$

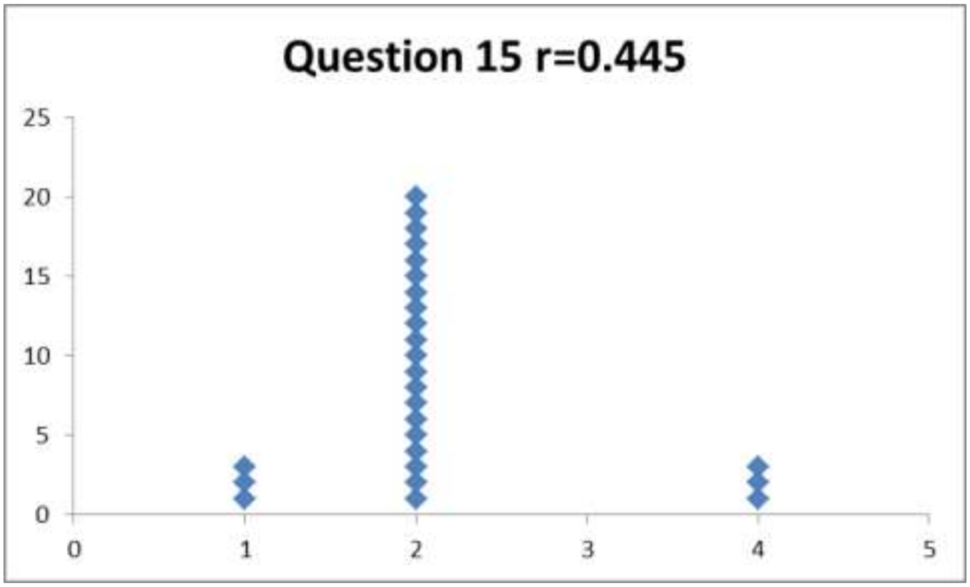


14) Find the derivative of the function below. (4 points)

$$f(x) = e^4$$

$$f'(x) = 0$$

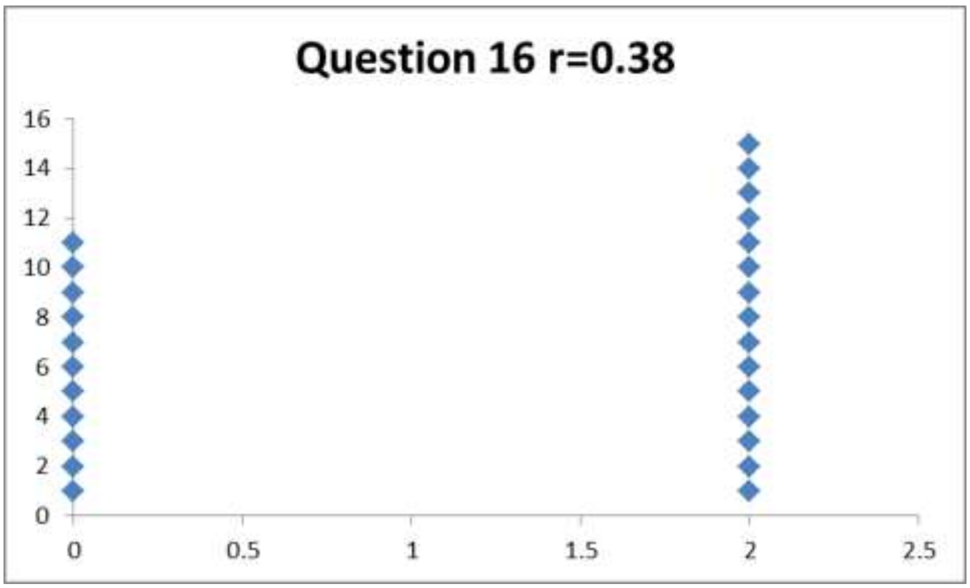
Note: $e^4 \approx 54$. It's just a number, meaning it is constant and never changes.



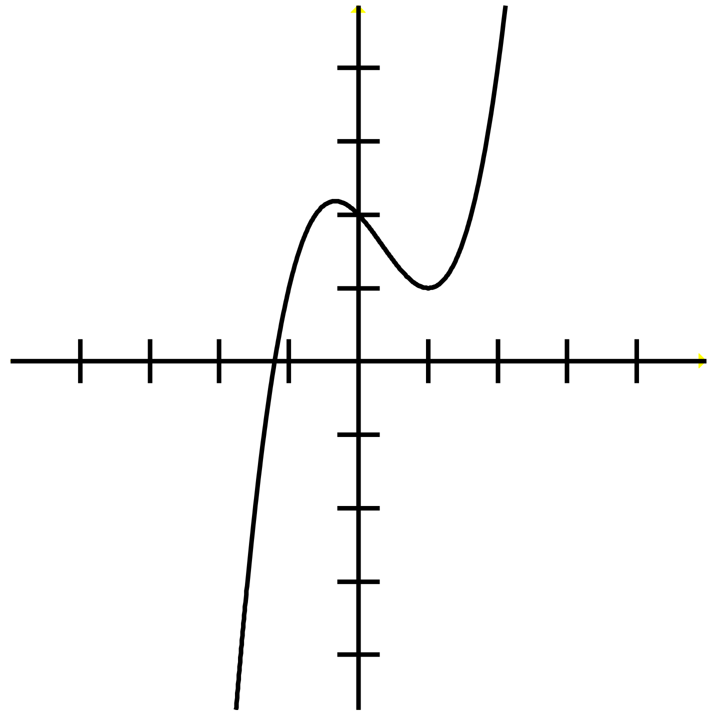
15) In the theory of calculus, one of the statements below is true. Which one is it? (2 points)
(No work required)

A) “Limits are used to define derivatives”

B) “Derivatives are used to define limits”



16) Using the graph below, estimate the following derivatives. (2 points each)



$f'(-1) = 5$

Anything in (1,10] is given full credit

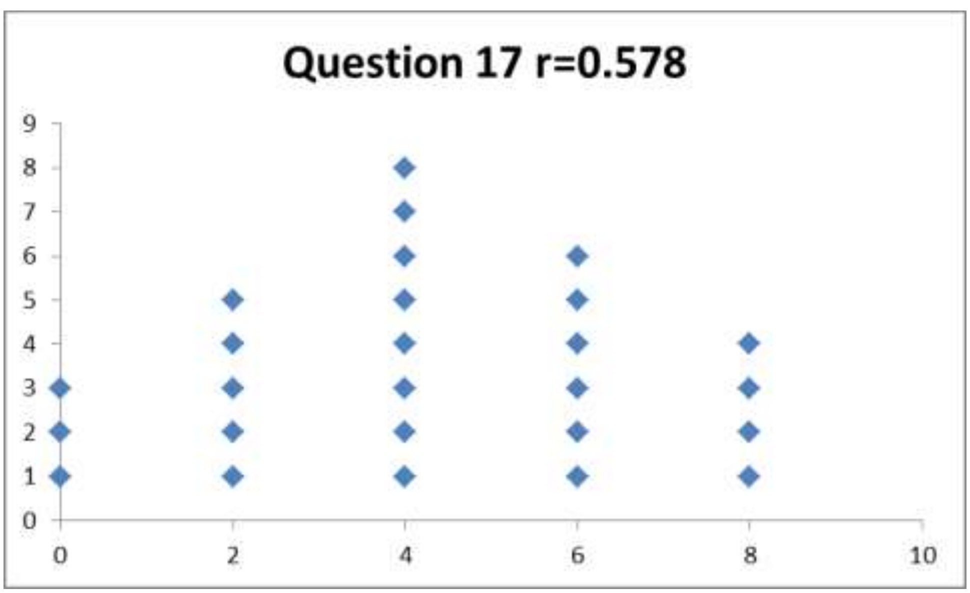
$f'(0) = -1.5$

Anything in $\left[-\frac{1}{2}, -5\right]$ is given full credit

$f'(1) = 0$

$f'(2) = 8$

Anything in [2,20] is given full credit



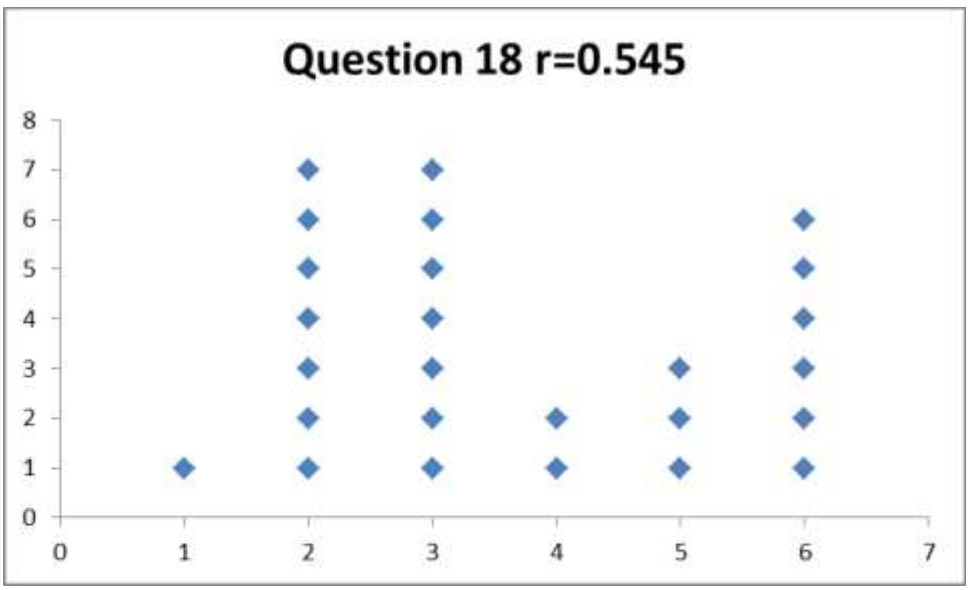
17) The position of a fly from a tree is given by $f(t) = 2t^2 - 9t + 12$ where t is measured in seconds and f is measured in feet. When is the fly motionless? (6 points)

To be motionless we need the derivative to be zero:

$$f'(t) = 4t - 9 = 0$$

$$t = \frac{9}{4}$$

It is motionless at $\frac{9}{4} = 2.25$ seconds.



18) A table of values is given below for a function $f(x)$. Using this table, estimate the derivative at $x = 3$.
(The better the approximation the more points you score. There are multiple answers that yield full credit)
(6 points)

x	$f(x)$
1	5
2	12
3	20
4	25
5	29

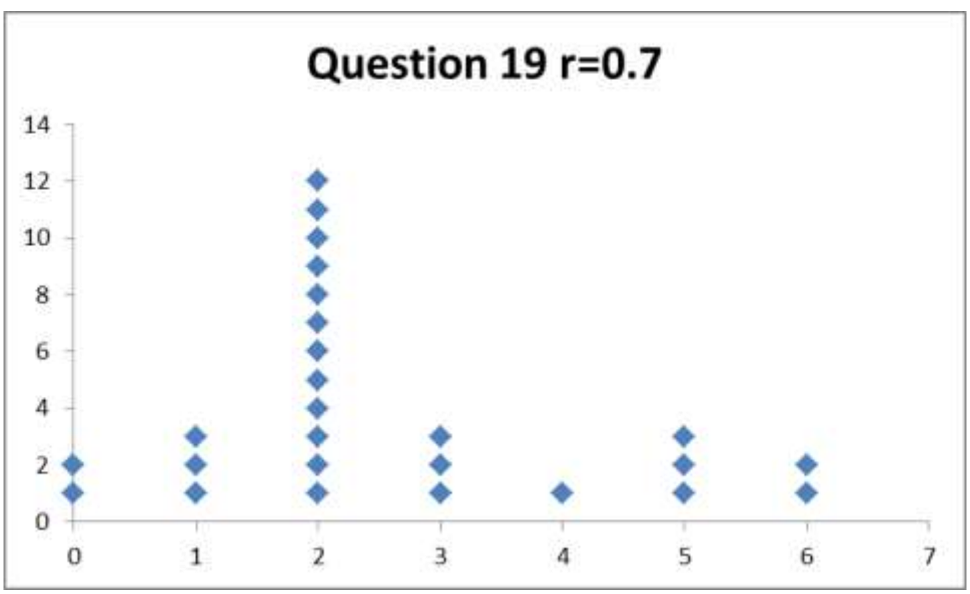
Recall that the derivative is rate of change. That is, $\frac{\Delta y}{\Delta x}$. There are three good ways to calculate this:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{25 - 12}{4 - 2} = \frac{13}{2} = 6.5$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{25 - 20}{4 - 3} = \frac{5}{1} = 5$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 12}{3 - 2} = \frac{8}{1} = 8$$

These are all good approximations of the derivative. There are other approximations using x-values further away from 3. But they're not as good and so have partial credit.



19) It is known that the derivative of $y = 3x^2$ is $y' = 6x$. Use the formal definition of the derivative to show this. OR for half credit correctly state the formal definition of the derivative using a limit.
(6 points)

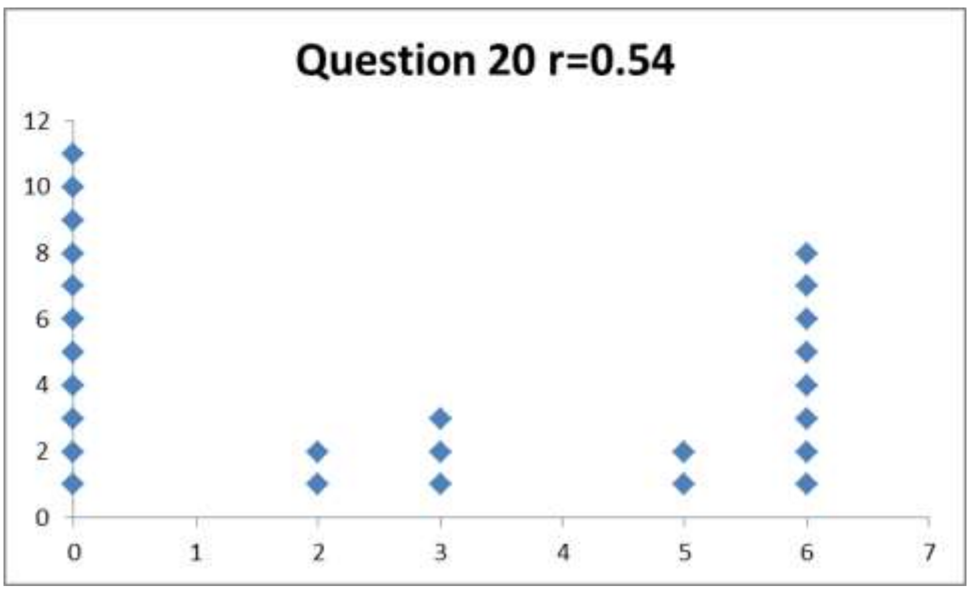
The formal definition of the derivative is:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Plugging in our function we get:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h} = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h} = \lim_{h \to 0} 6x + 3h = 6x$$



20) Find the derivative of the function below. (4 points)

$$f(x) = \tan^3((3x+1)^5)$$

$$f'(x) = 3\tan^2((3x+1)^5) \cdot \sec^2((3x+1)^5) \cdot 5(3x+1)^4 \cdot 3$$