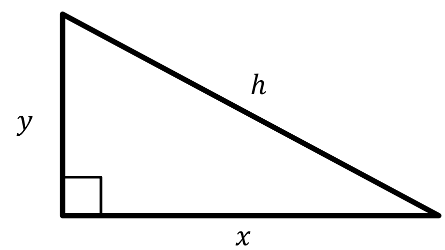
Name __Solutions____ Test 2, Spring 2018

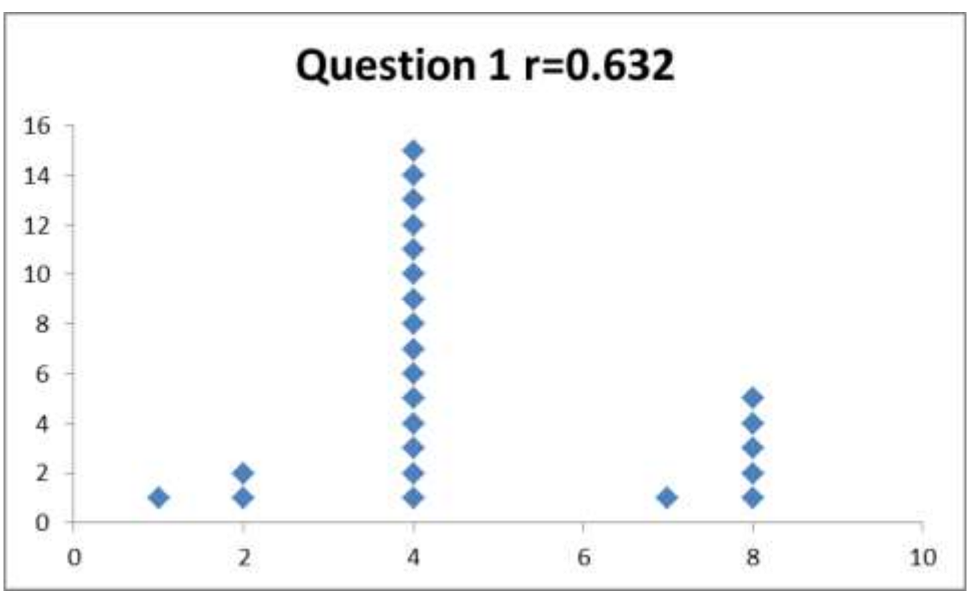
1) A 13-foot ladder is leaning against a vertical wall (see figure) when Jack begins pulling the foot of the ladder away from the wall at a rate of 0.5ft/s. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5ft from the wall?

$$x = 5$$
$$x' = \frac{1}{2}$$
$$h = 13$$
$$h' = 0$$
$$y = \sqrt{13^2 - 5^2} = 12$$
$$y' = ??$$



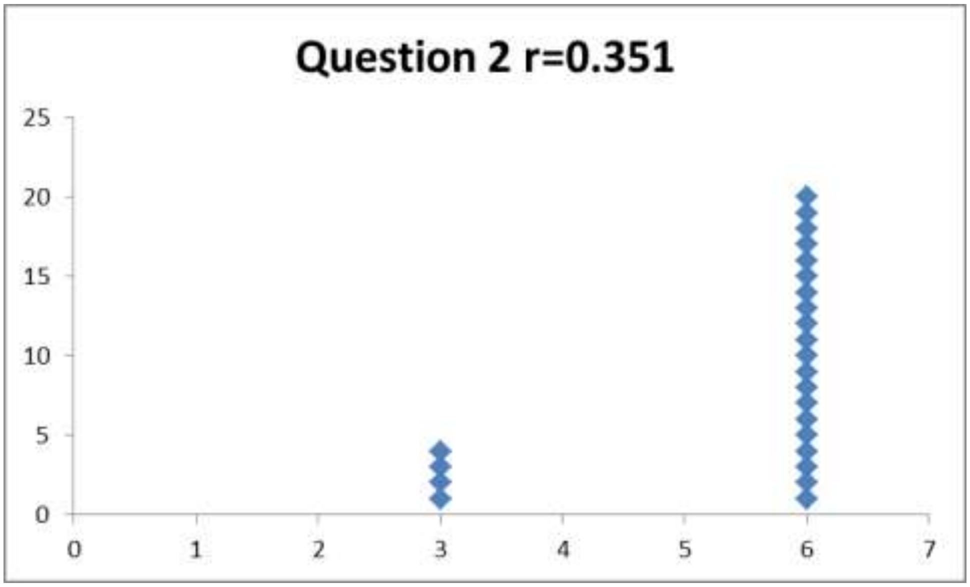
$$x^2 + y^2 = h^2$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}h^2$$
$$\therefore 2xx' + 2yy' = 2hh'$$
$$\therefore 2 \cdot 5 \cdot \frac{1}{2} + 2 \cdot \sqrt{13^2 - 5^2} \cdot y' = 0$$
$$\therefore y' = \frac{-5}{2\sqrt{13^2 - 5^2}} ft/s \qquad \left(= -\frac{5}{24} ft/s\right)$$



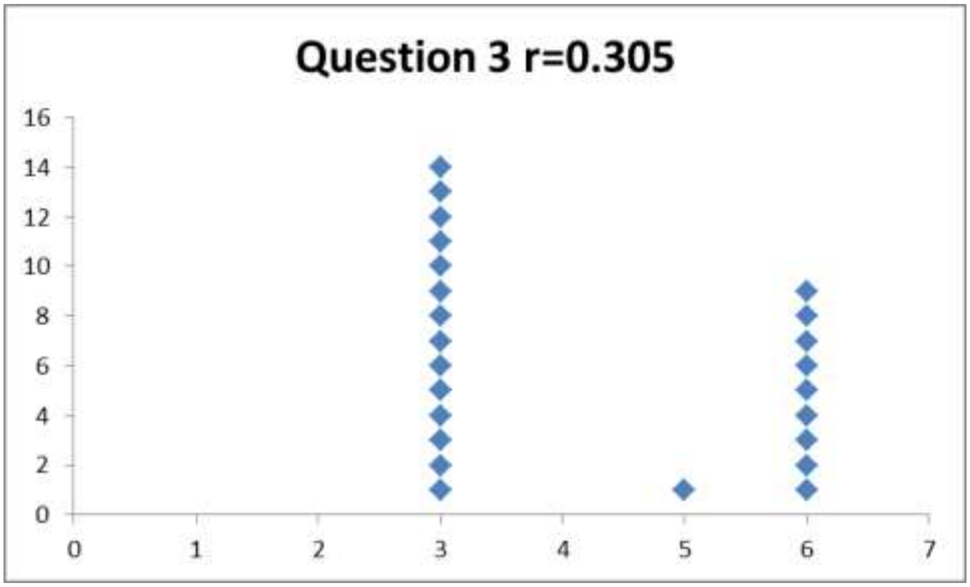
2) Find the derivative of $y = \tan(e^x)$.

$$y' = \sec^2(e^x)\, e^x$$



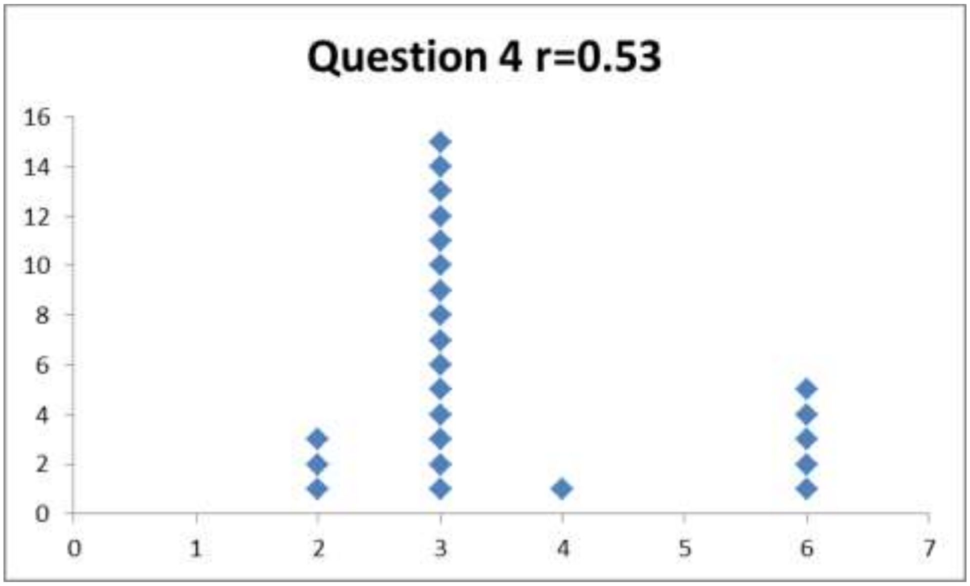
3) Find the derivative of $\left((x+2)(x^2+1)\right)^4$

$$y' = 4\left((x+2)(x^2+1)\right)^3 \cdot [1 \cdot (x^2+1) + (x+2) \cdot 2x]$$



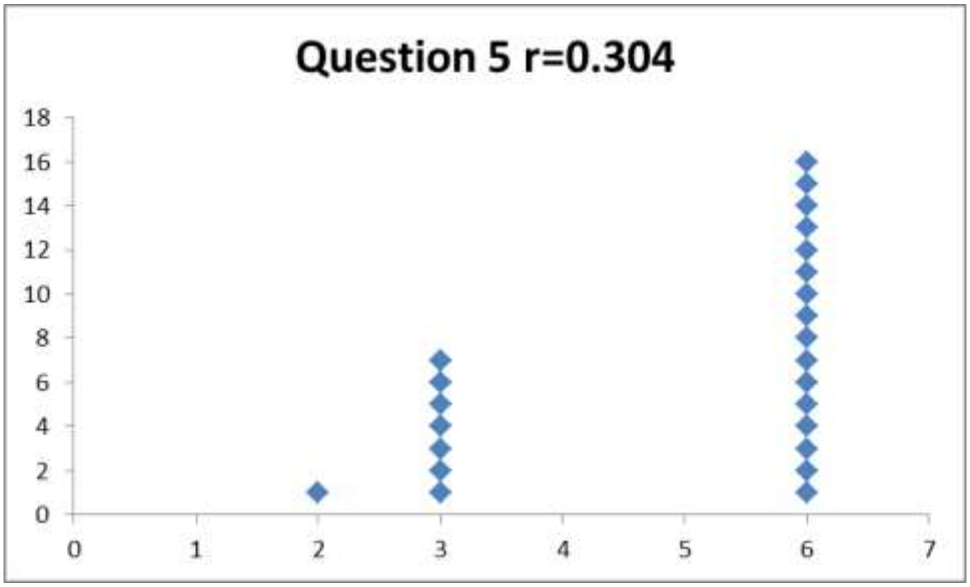
4) Given that $\cos(y^2) + x = e^y$, find $\frac{dy}{dx}$.

$$-\sin(y^2) \cdot 2yy' + 1 = e^y y'$$
$$1 = e^y y' + 2\sin(y^2)\, yy'$$
$$1 = y'(e^y + 2\sin(y^2)\, y)$$
$$y' = \frac{1}{e^y + 2\sin(y^2)\, y}$$



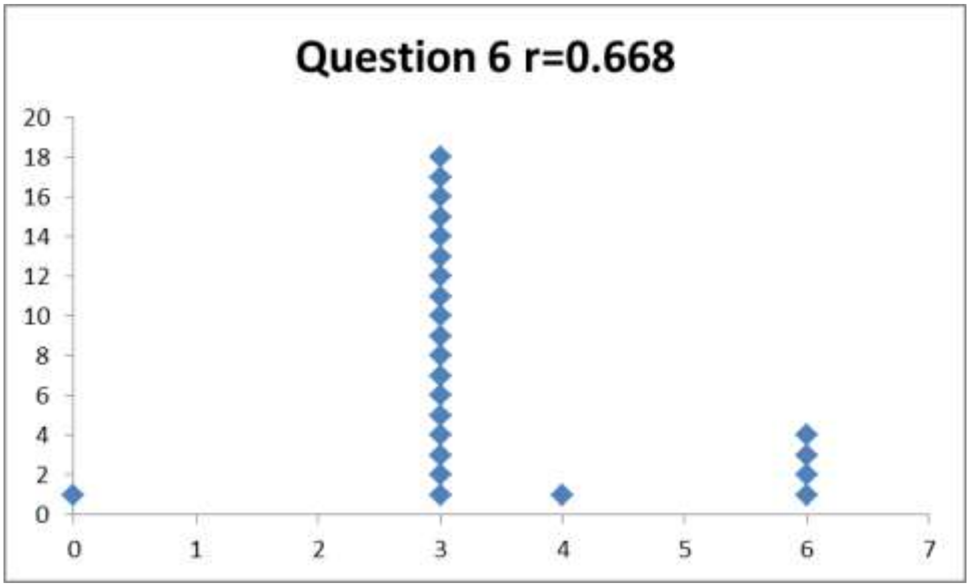
5) Given that $y = x^{\frac{5}{4}}$, find $\frac{dy}{dx}$.

$$y' = \frac{5}{4}x^{\frac{1}{4}}$$



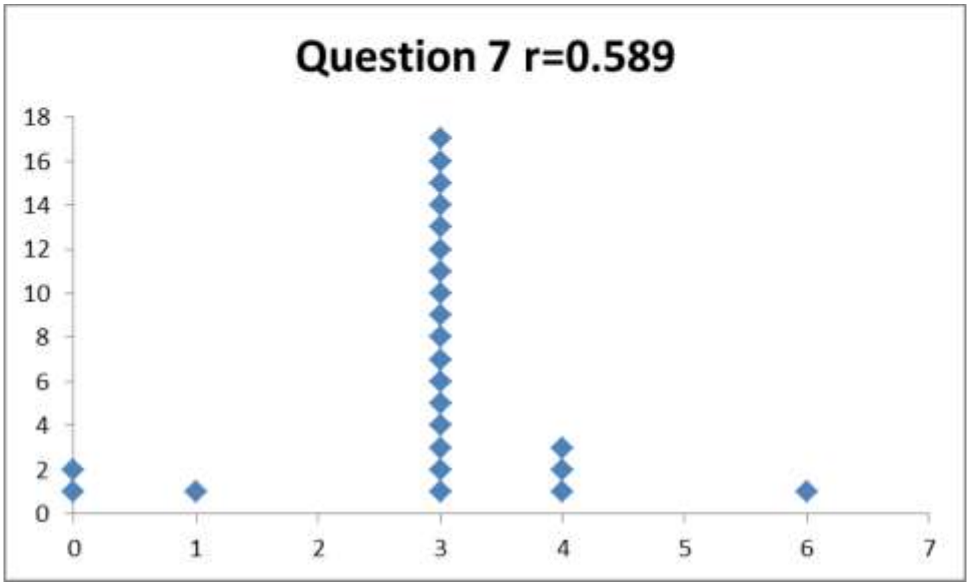
6) Given that $g(y) = e^y y^e$, find the derivative of $g(y)$.

$$g'(y) = e^y y^e + e^y e y^{e-1}$$

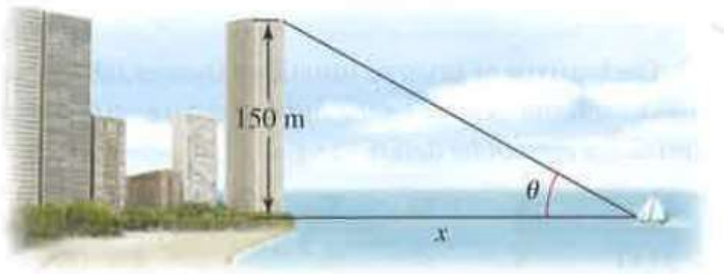


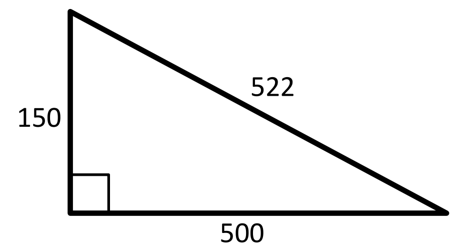
7) Calculate the derivative of $y = \frac{1}{\log_4(x)} = (\log_4(x))^{-1} = \left(\frac{\ln(x)}{\ln(4)}\right)^{-1} = \ln(4)\,(\ln(x))^{-1}$

$$y' = \ln(4) \cdot (-(\ln(x))^{-2} \cdot \frac{1}{x}$$



8) A boat sails directly toward a 150-meter skyscraper that stands on the edge of a harbor. The angular size θ of the building is the angle formed by lines from the top and bottom of the building to the observer (see figure). A particular triangle that might be helpful is also given.



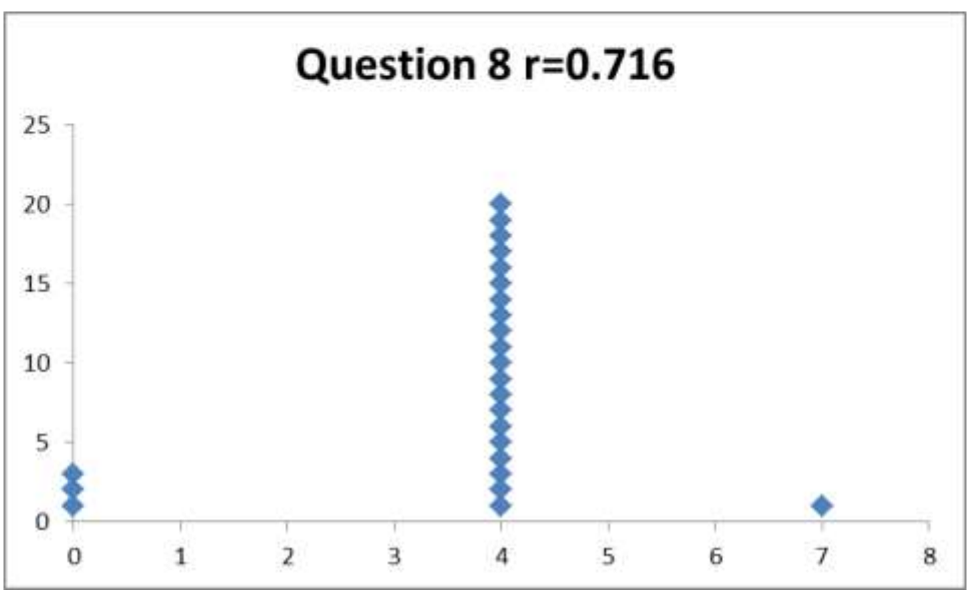


What is the rate of change of the angular size $\frac{d\theta}{dx}$ when the boat is $x = 500m$ from the building?

$$\tan(\theta) = \frac{150}{x}$$

$$\sec^2(\theta)\,\theta' = -\frac{150}{x^2}$$

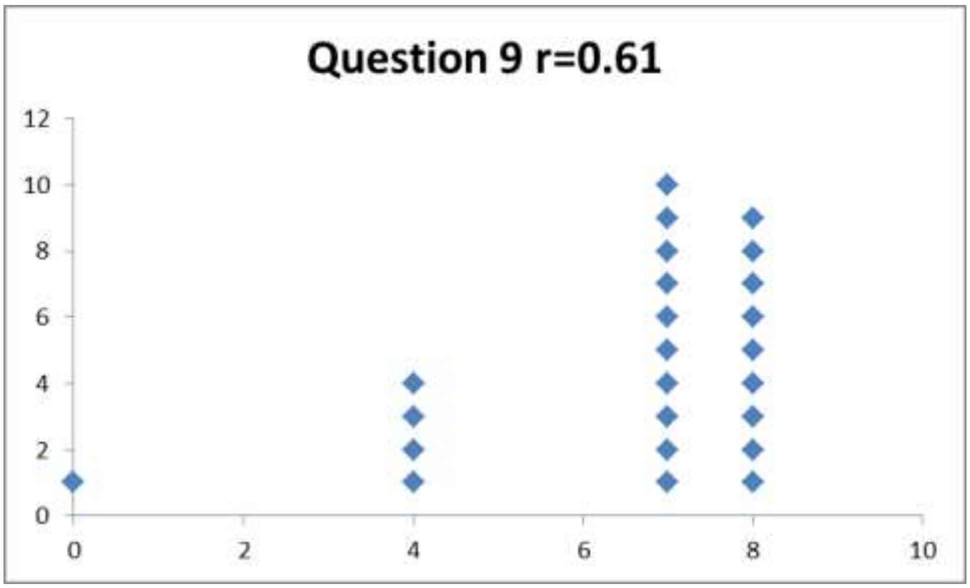
$$\theta' = \frac{-150}{x^2\sec^2(\theta)} = \frac{-150\cos^2(\theta)}{x^2} = \frac{-150\cdot 500^2}{500^2\cdot 522^2} = \frac{-150}{522^2}\ rad/m$$



9) The area of a circle increases at a rate of $1cm^2/s$. How fast is the radius changing when the radius is $2cm$?

$$r = 2$$
$$r' = ??$$
$$A' = 1$$

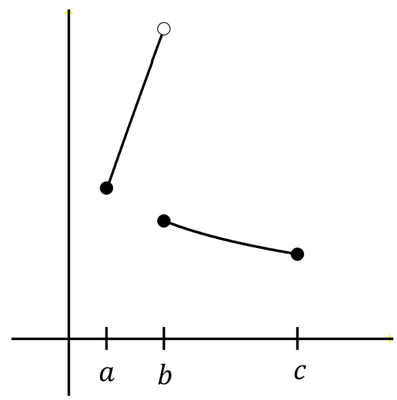
$$A = \pi r^2$$
$$A' = 2\pi r r'$$
$$1 = 2\pi \cdot 2 \cdot r'$$
$$r' = \frac{1}{4\pi} cm/s$$

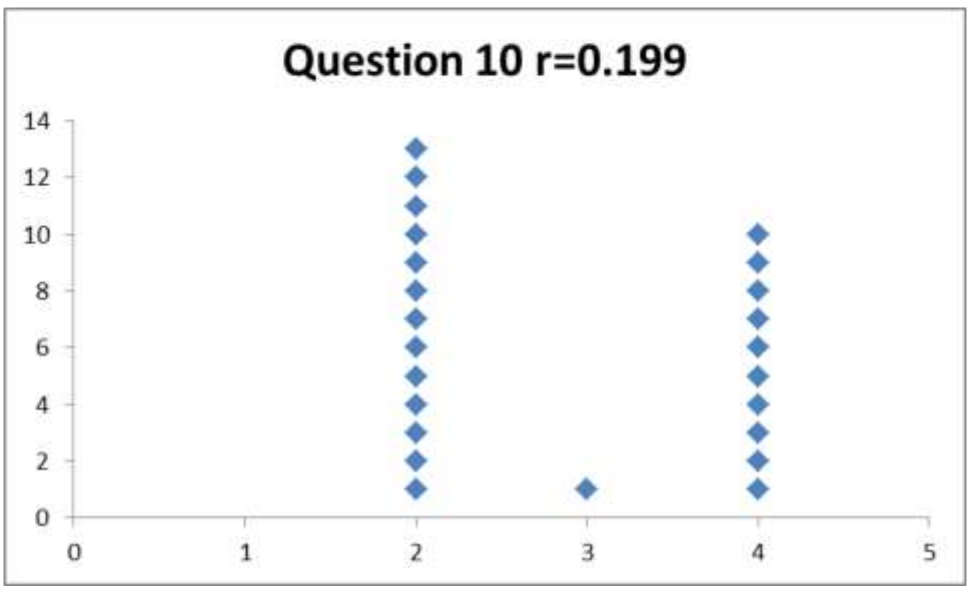


10) Use the graph below to identify the points (if any) on the interval $[a, c]$ at which the function has an absolute maximum value or an absolute minimum value.

Absolute Maximum: __None_____________

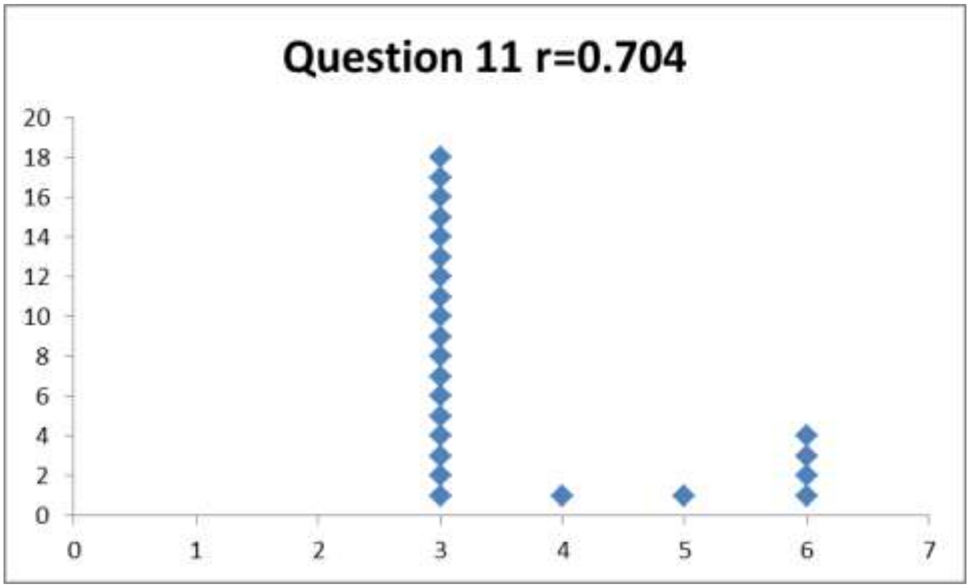
Absolute Minimum: __At $x = c$____________





11) Find the derivative of $f(x) = \sin^{-1}(2x)$

$$f'(x) = \frac{1}{\sqrt{1+(2x)^2}} \cdot 2$$



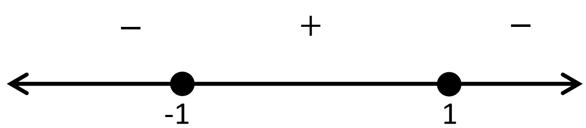
12) Graph the function below, paying particular attention to the critical values and the end behavior.
(On this problem you need not calculate the location of the inflection point(s) precisely)

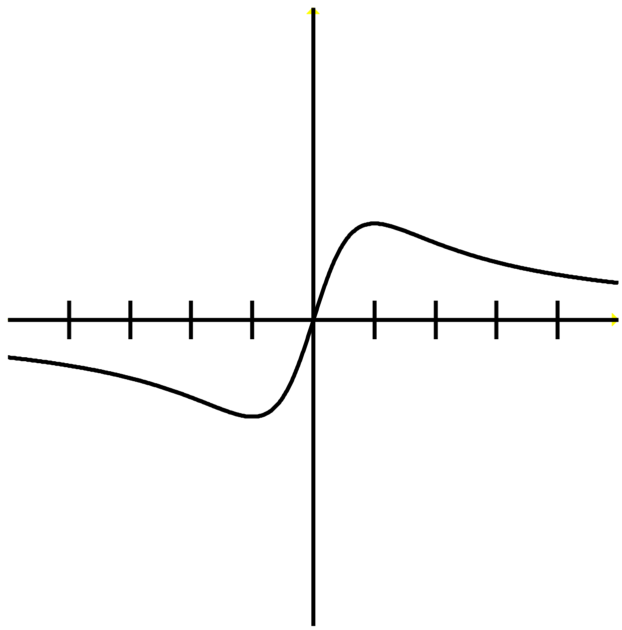
$$y = \frac{x}{x^2+1}$$

$$y' = \frac{x^2+1-x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

CVs: $x = \pm 1$

First derivative test:

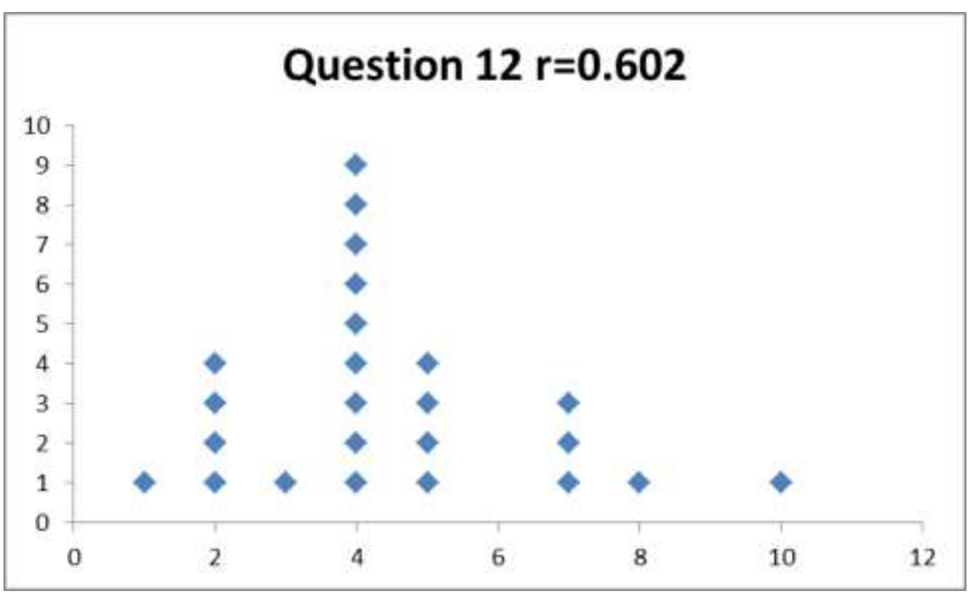




End behavior:

$$\lim_{x\to\infty} \frac{x}{x^2+1} = 0$$

$$\lim_{x\to-\infty} \frac{x}{x^2+1} = 0$$



13) Graph the function $y = -x^3 + 9x$.

$y' = -3x^2 + 9 = 3(3 - x^2)$

CVs: $x = \pm\sqrt{3}$

$y'' = -6x$

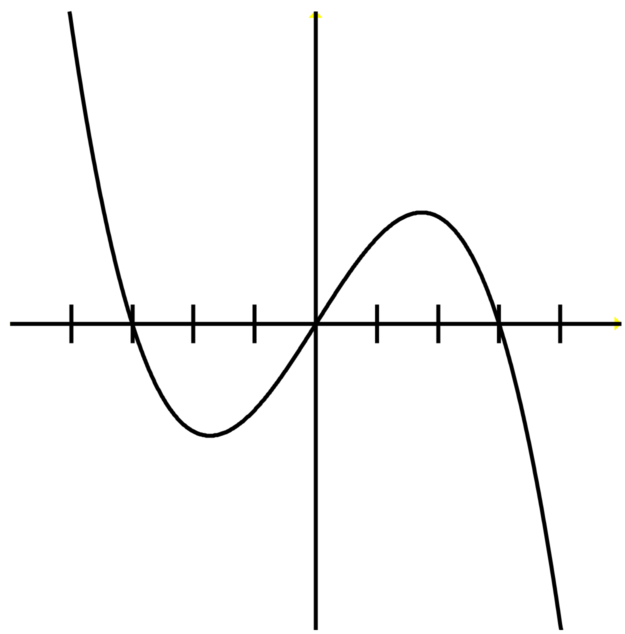
Second derivative test: $y'' > 0$ at $x = -\sqrt{3}$ making that a max. $y'' < 0$ at $x = \sqrt{3}$ making that a min.

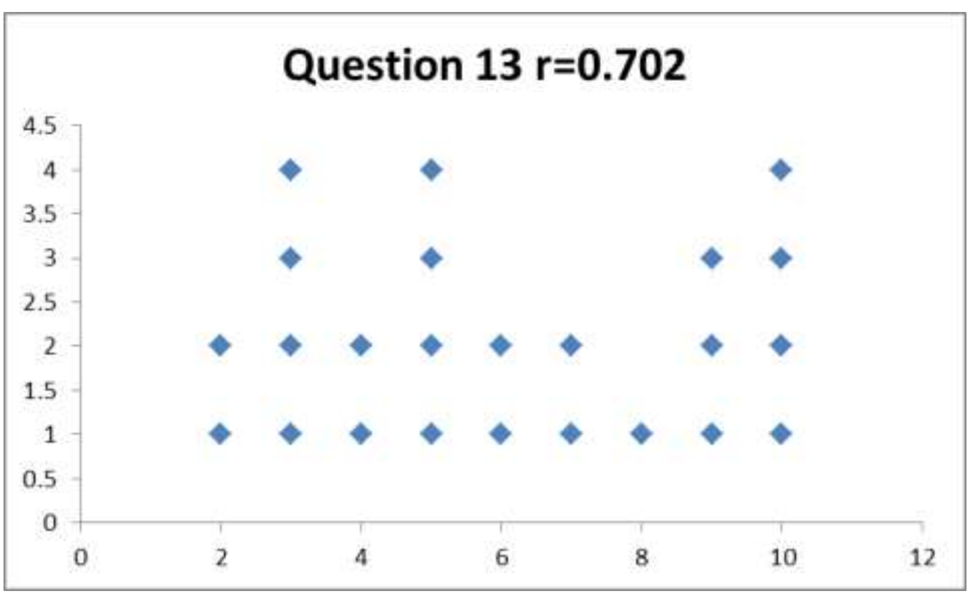
Inflection point at $x = 0$

End behavior:

$\lim_{x\to\infty} -x^3 + 9x = -\infty$

$\lim_{x\to-\infty} -x^3 + 9x = \infty$





14) Graph the function $y = x + 2\cos(x)$ on $[-2\pi, 2\pi]$.

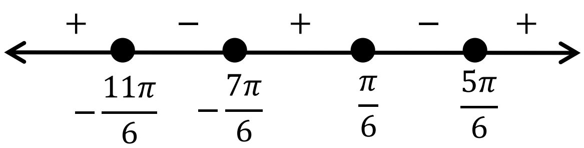
(On this problem you need not calculate the location of the inflection point(s) precisely)

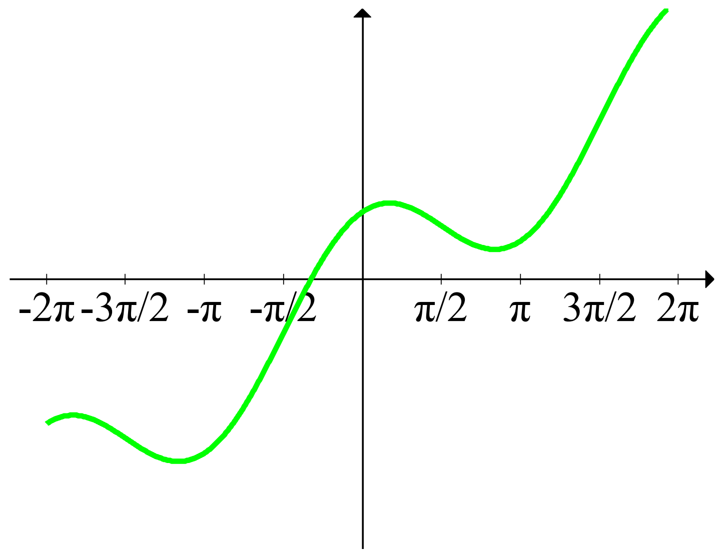
$y' = 1 - 2\sin(x)$

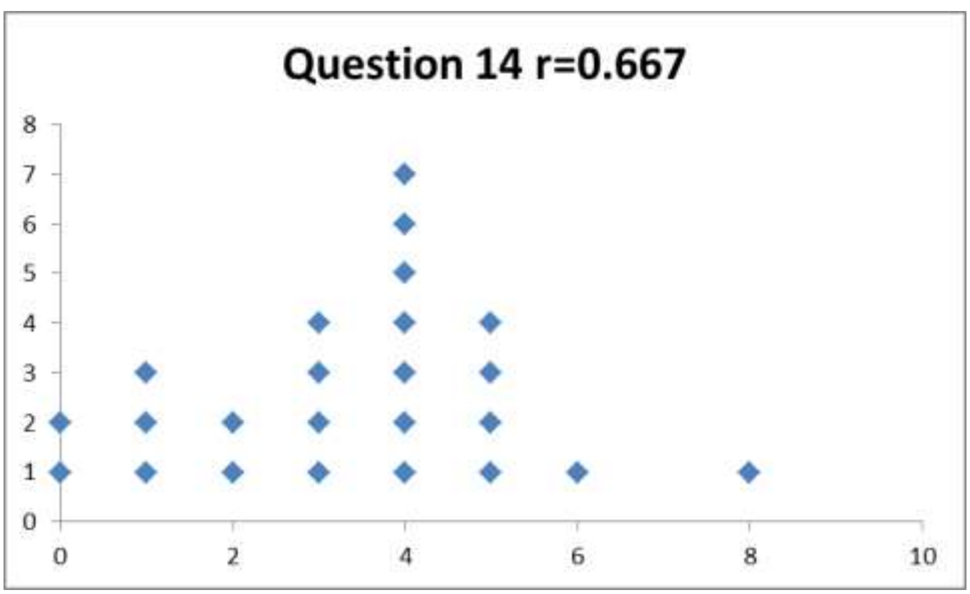
CVs: whenever $\sin(x) = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$

Sign chart







15) Find positive numbers x and y satisfying the equation $xy = 12$ such that the sum $2x + y$ is as small as possible.

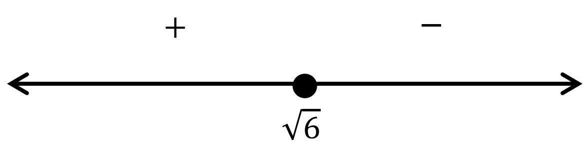
$$y = \frac{12}{x}$$

$$f(x) = 2x + \frac{12}{x}$$

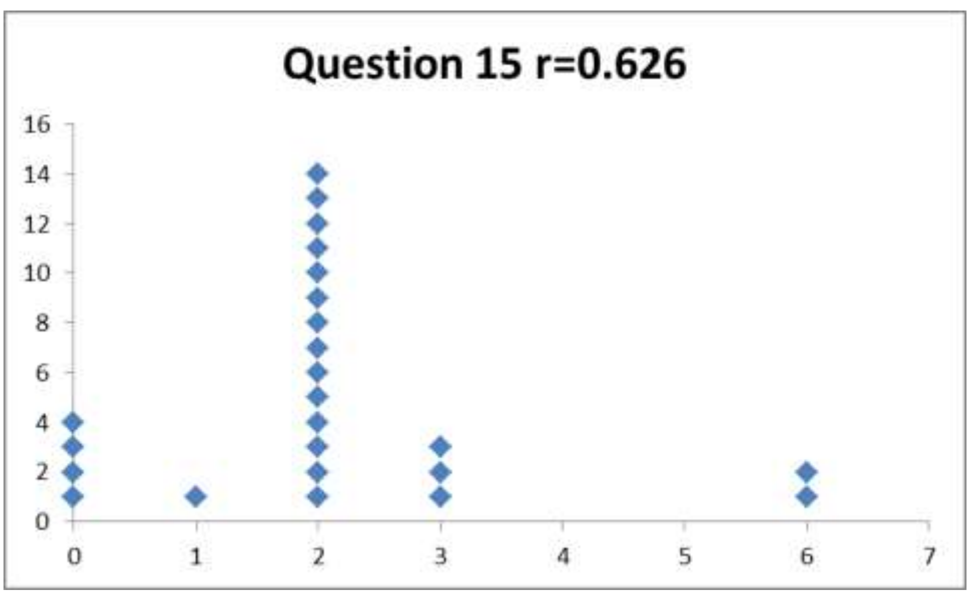
$$f'(x) = 2 - \frac{12}{x^2}$$

CVs: $2 - \frac{12}{x^2} = 0 \Rightarrow x = \pm\sqrt{6}$. Keep only $x = \sqrt{6}$ because x must be positive.

Sign chart:



We see that the maximum value is when $x = \sqrt{6}$ and $y = \frac{12}{\sqrt{6}}$



16) Evaluate the limit below.

$$\lim_{x \to \pi} \frac{\cos(x) + 1}{(x - \pi)^2}$$

$$\lim_{x \to \pi} \frac{\cos(x) + 1}{(x - \pi)^2} \overset{h}{=} \lim_{x \to \pi} \frac{-\sin(x)}{2(x - \pi)} \overset{h}{=} \lim_{x \to \pi} \frac{-\cos(x)}{2} = \frac{1}{2}$$