Name ____ Test 3, Spring 2018

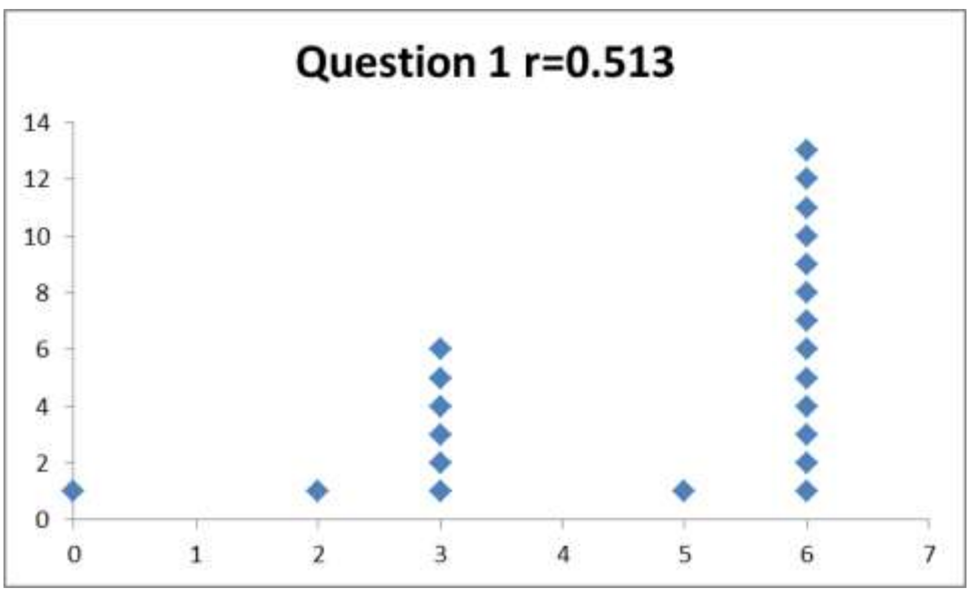
1) Find ONE of the integrals below. (6 points)

$$\int 2x(x^2-1)^{99}dx = \int u^{99}du = \frac{u^{100}}{100} + C = \frac{(x^2-1)^{100}}{100} + C$$

$$u = x^2 - 1$$
$$du = 2xdx$$

$$\int_0^1 2x(4-x^2)dx = \int_{x=0}^{x=1} (-u)du = \left(-\frac{u^2}{2}\right)\Bigg|_{x=0}^{x=1} = \left(-\frac{(4-x^2)^2}{2}\right)\Bigg|_0^1 = -\frac{3^2}{2} - \left(-\frac{4^2}{2}\right) = 8 - \frac{9}{2} = 3.5$$

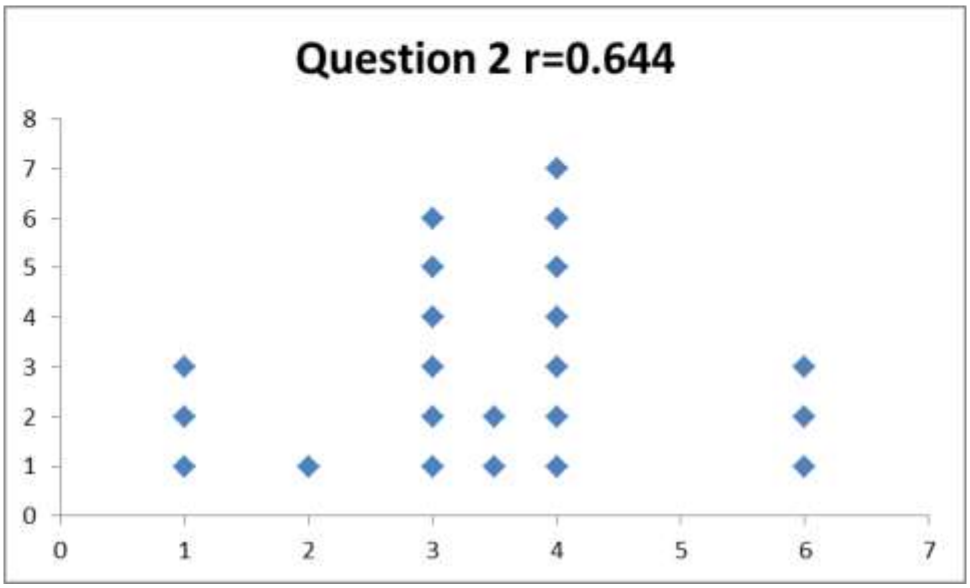
$$u = 4 - x^2$$
$$du = -2xdx$$



2) Find the integral below. (6 points)

$$\int \frac{2x^2}{\sqrt{1-4x^3}}dx = -\frac{1}{6}\int \frac{1}{\sqrt{u}}du = -\frac{1}{6}\int u^{-\frac{1}{2}}du = -\frac{1}{6}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = -\frac{1}{3}(1-4x^3)^{\frac{1}{2}} + C$$

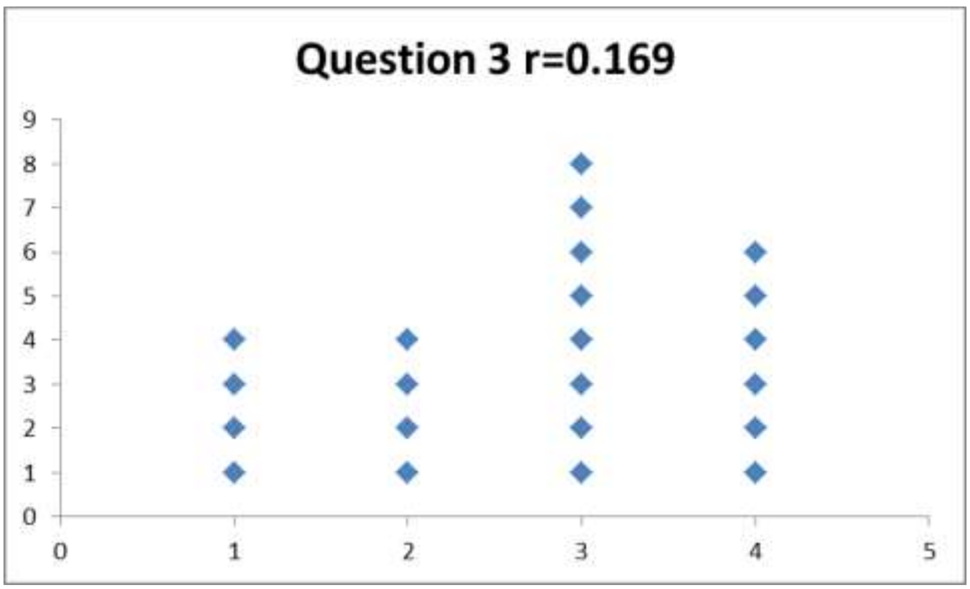
$$u = 1 - 4x^3$$
$$du = -12x^2dx$$



3) Find the integral below. (6 points)

$$\int \frac{x}{\sqrt{x-4}}dx = \int \frac{u+4}{\sqrt{u}}du = \int u^{\frac{1}{2}} + 4u^{-\frac{1}{2}}du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4u^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3}(x-4)^{\frac{3}{2}} + 8(x-4)^{\frac{1}{2}} + C$$

$$u = x - 4$$
$$du = dx$$
$$u + 4 = x$$



4) Find ONE of the integrals below. (6 points)

$$\int_0^{\ln(4)} \frac{e^x}{3+2e^x}dx = \frac{1}{2}\int_5^{11}\frac{1}{u}du = \frac{\ln|u|}{2}\Bigg|_5^{11} = \frac{\ln(11)}{2} - \frac{\ln(5)}{2}$$

$$u = 3 + 2e^x$$
$$du = 2e^x dx$$

When $x = 0, u = 3 + 2 = 5$

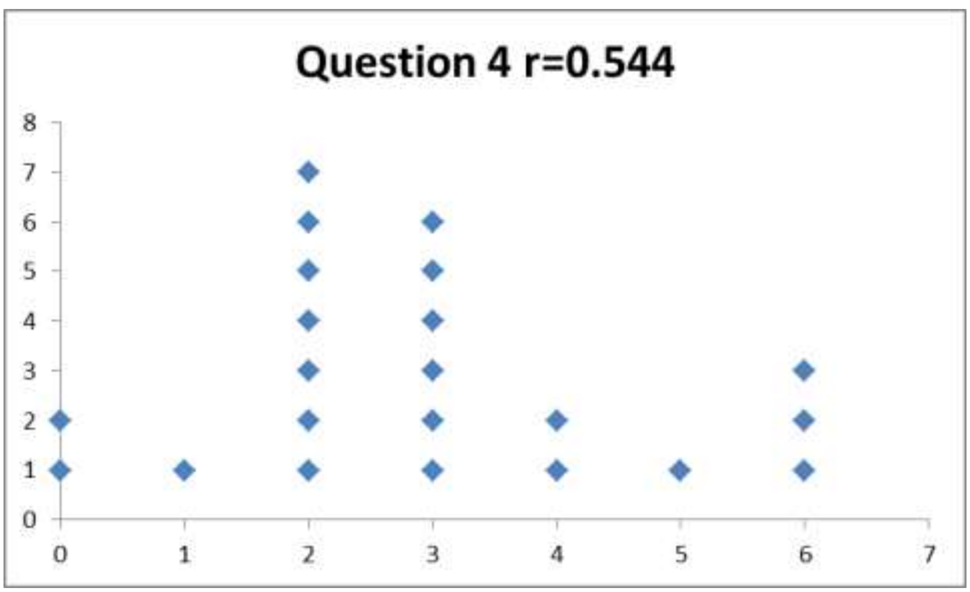
When $x = \ln(4), u = 3 + 2e^{\ln(4)} = 3 + 8 = 11$

$$\int_0^{\frac{\pi}{2}} \sin^2(\theta)\cos(\theta)\,d\theta = \int_0^1 u^2 du = \frac{u^3}{3}\Bigg|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

$$u = \sin(\theta)$$
$$du = \cos(\theta)$$

When $\theta = 0, u = \sin(0) = 0$

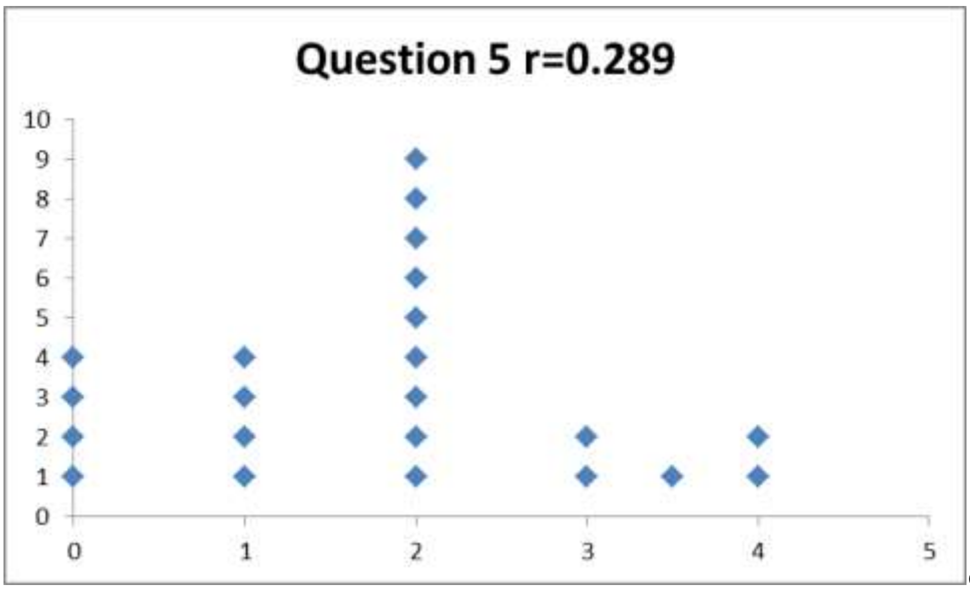
When $\theta = \frac{\pi}{2}, u = \sin\left(\frac{\pi}{2}\right) = 1$



5) Consider the function $f(x) = x^2$ on the interval $[1,4]$. The mean value theorem guarantees that there is a point $x = c$ such that $f'(c) = m$. In this context, what is m?
(4 points)

m is the average rate of change on the interval, which is:

$$\frac{4^2 - 1^1}{4 - 1} = \frac{15}{3} = 5$$



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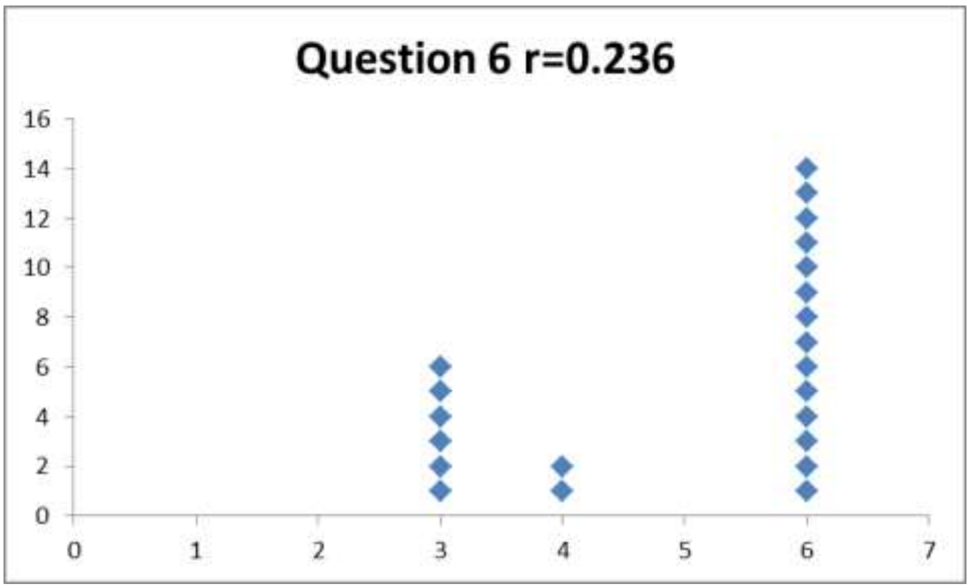
6) Consider the function f with derivative $f'(x) = 3x^2 + 3$ and initial value $f(1) = 8$. Find $f(x)$.
(6 points)

$$\int f'(x)dx = \int (3x^2 + 3)dx = x^3 + 3x + C$$

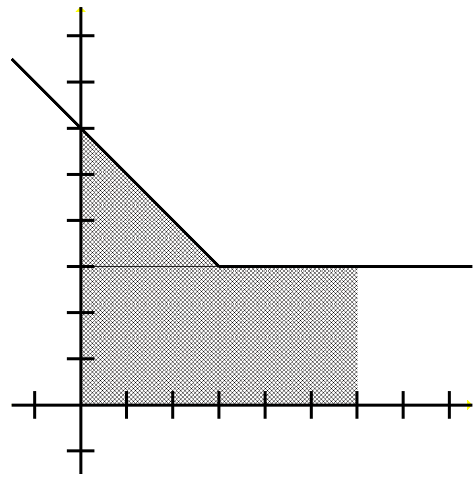
$$1^3 + 3 \cdot 1 + C = 8$$
$$\therefore C = 4$$

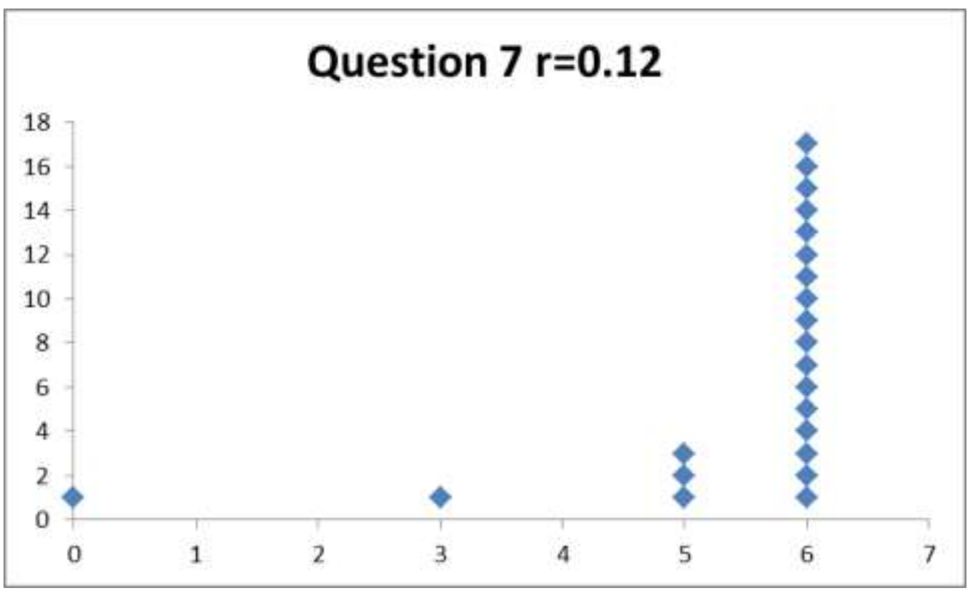
$$f(x) = x^3 + 3x + 4$$



7) Use geometry to find the **actual** area between $f(x)$ and the x-axis between $x = 0$ and $x = 6$.
(6 points)

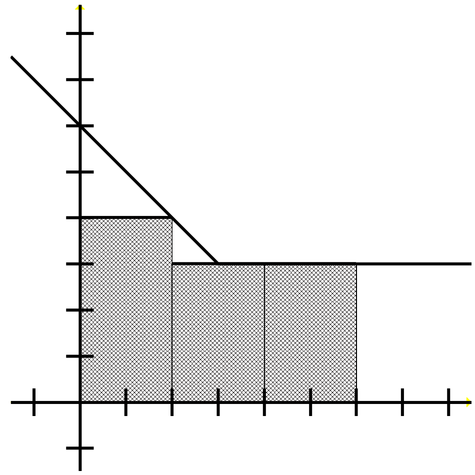
$\frac{1}{2}3 \cdot 3 + 3 \cdot 6 = \frac{9}{2} + 18 = 22.5$

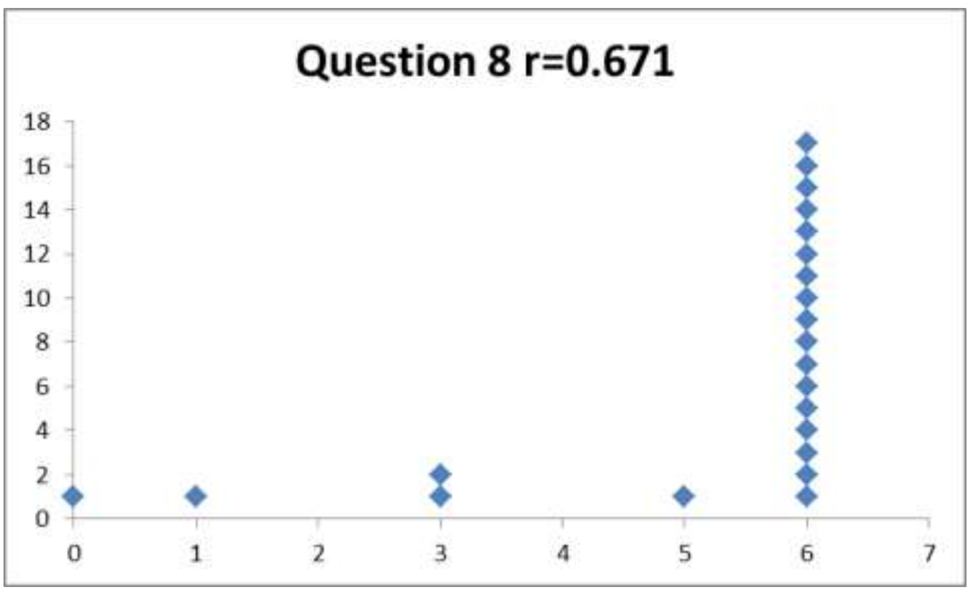




8) Use a **right** Riemann Sum with 3 rectangles to **estimate** the area between $f(x)$ and the x-axis between $x = 0$ and $x = 6$. (Illustrate your rectangles and find the area)
(6 points)

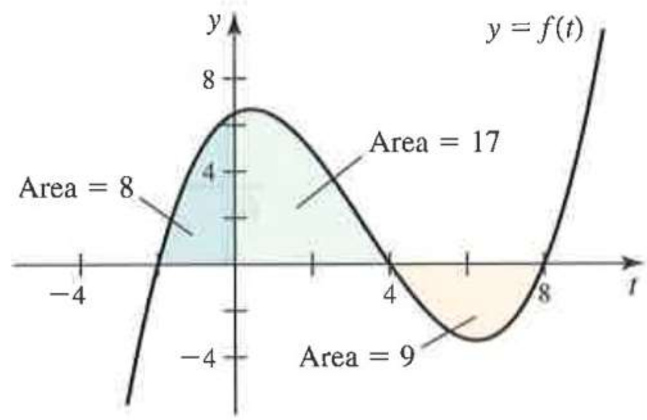
$4 \cdot 2 + 3 \cdot 2 + 3 \cdot 2 = 20$

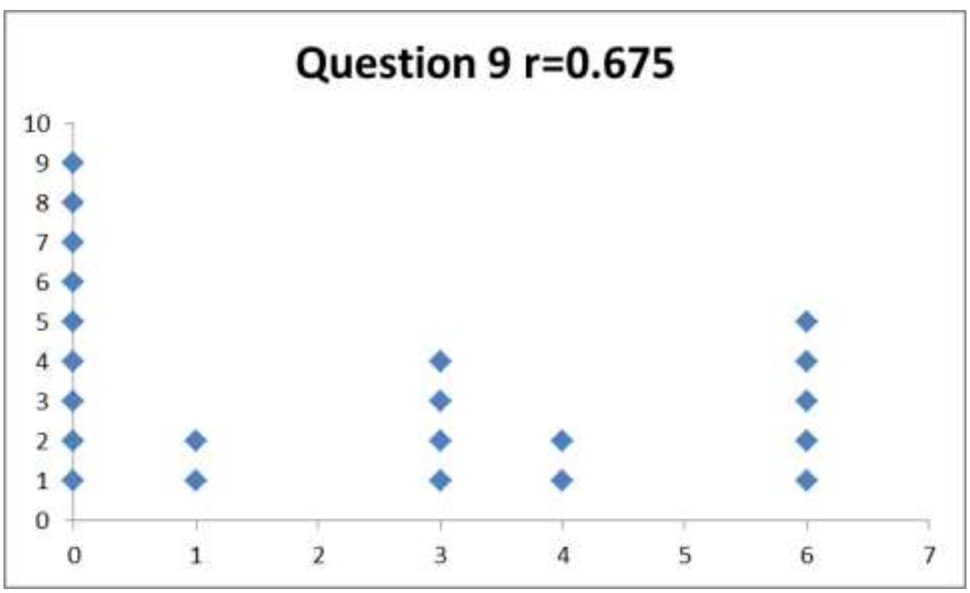




9) Below is a graph of a function with some associated areas. Use the graph to find the integral below.
(6 points)

$$\int_0^8 f(x)dx = 17 - 9 = 8$$



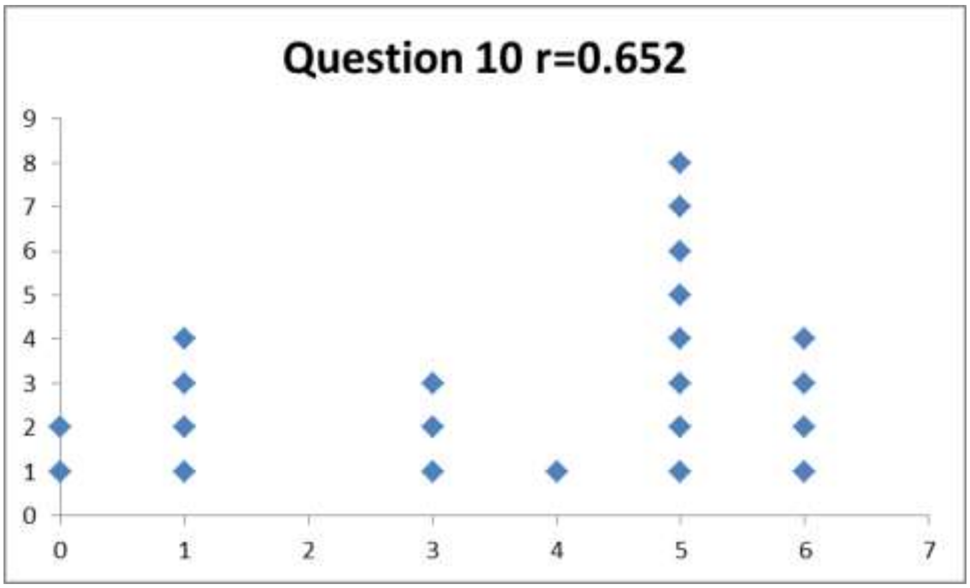


10) A velocity function is given below. Find the displacement over the interval $0 \leq t \leq 8$

$$v(t) = t^2 - 6t + 8$$

(Set up, but do not integrate.)
(6 points)

$$\int_0^8 (t^2 - 6t + 8)dt$$

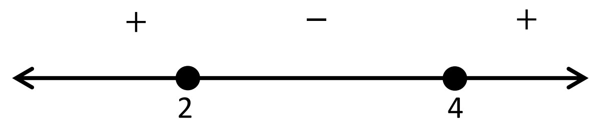


11) A velocity function is given below. Find the distance travelled over the interval $0 \le t \le 8$

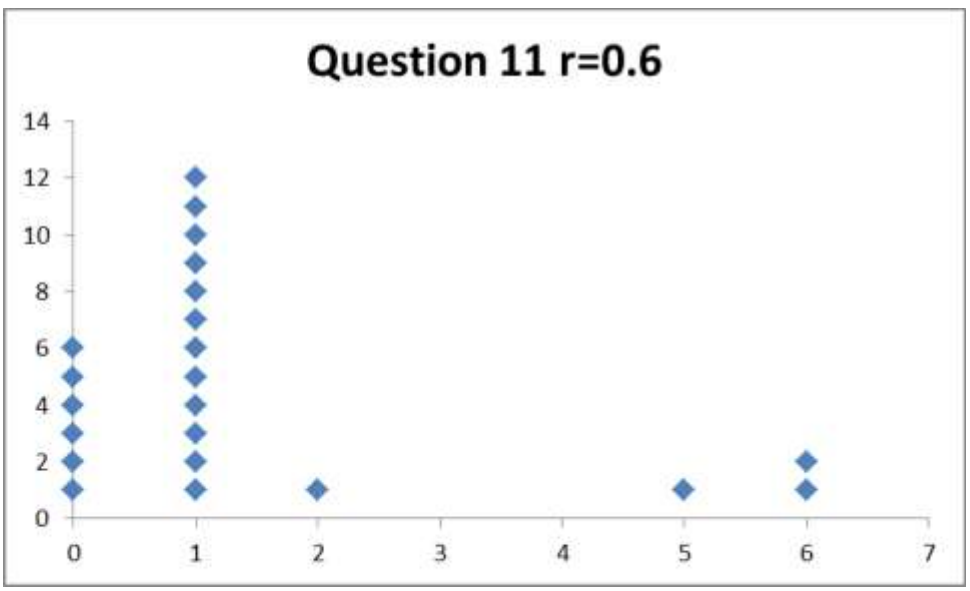
$$v(t) = t^2 - 6t + 8$$

(Set up, but do not integrate.)
(6 points)

$$t^2 - 6t + 8 = 0$$
$$(t-2)(t-4) = 0$$



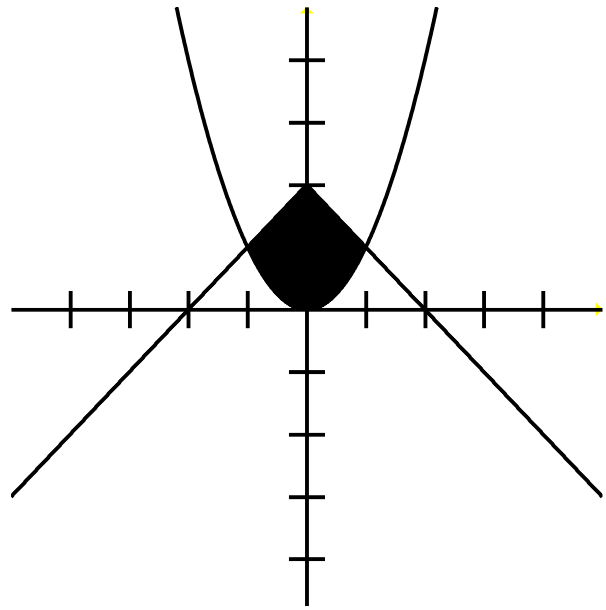
$$\int_0^2 (t^2 - 6t + 8)dt - \int_2^4 (t^2 - 6t + 8)dt + \int_4^8 (t^2 - 6t + 8)dt$$

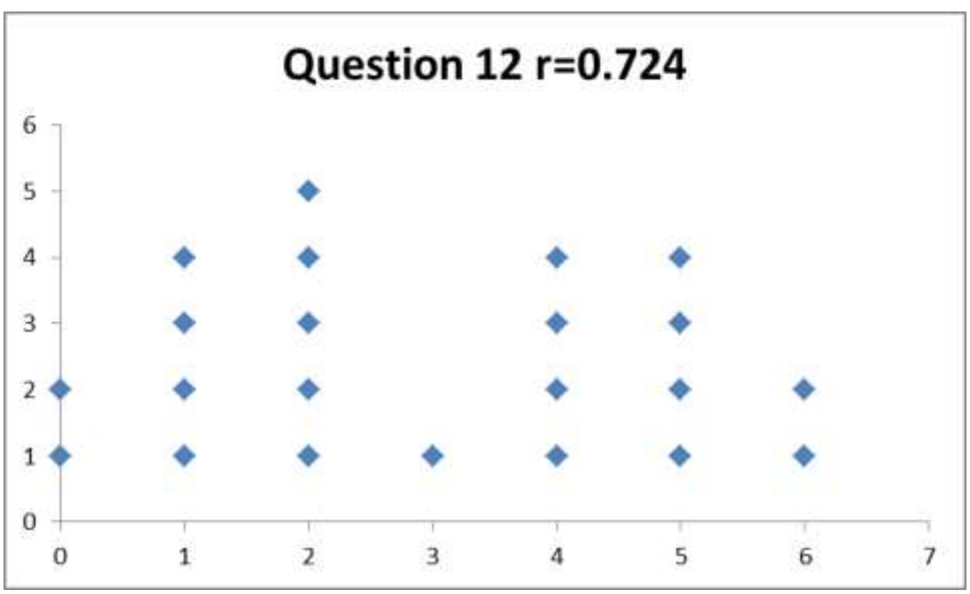


On problems 12-17 illustrate the problem using a Cartesian plane, then solve it.

12) Find the area of the region bounded by $y = 2 - |x|$ and $y = x^2$.
(Set up, but do not integrate.)
(6 points)

$$\int_{-1}^{1} (2 - |x|) - x^2 dx$$





13) The region bounded by $y = \sqrt{25 - x^2}$ and $y = 0$ is rotated around the x-axis. Find the volume of the resulting 3-D solid. (Set up, but do not integrate. Circle which method you're using)

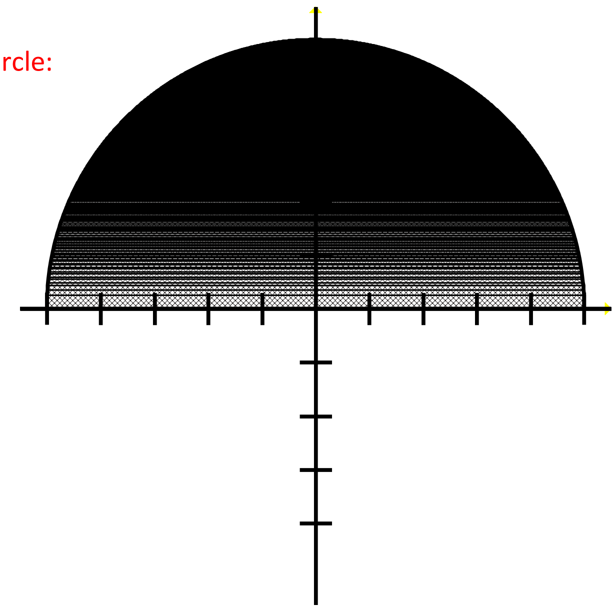
(Disk/washer) (Cylindrical Shell)

(6 points)

The shape created is a sphere.

You might need to do some algebra to see the equation of the circle:

$$y^2 + x^2 = 25$$

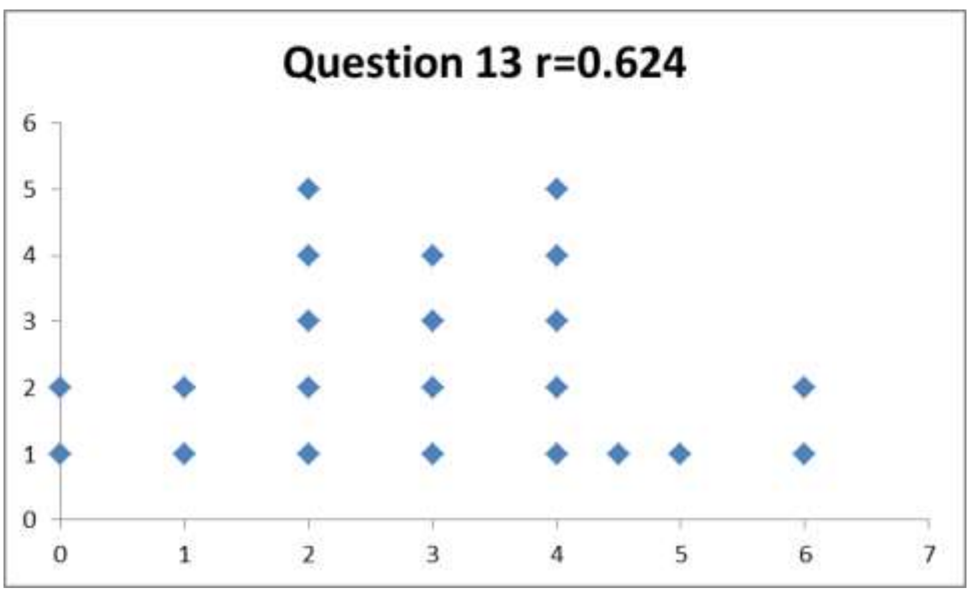


Disk Method:

$$\int_{-5}^{5} \pi r^2 dx = \int_{-5}^{5} \pi \left(\sqrt{25 - x^2}\right)^2 dx$$

Shell Method:

$$\int_{0}^{5} 2\pi rh dy = \int_{0}^{5} 2\pi y \left(\sqrt{25 - y^2} - \left(-\sqrt{25 - y^2}\right)\right) dy$$



14) The region bounded by $y = x$ and $y = \sqrt{x}$ is rotated around the x-axis. Find the volume of the resulting 3-D solid. (Set up, but do not integrate. Circle which method you're using)
(Disk/washer) (Cylindrical Shell)
(6 points)

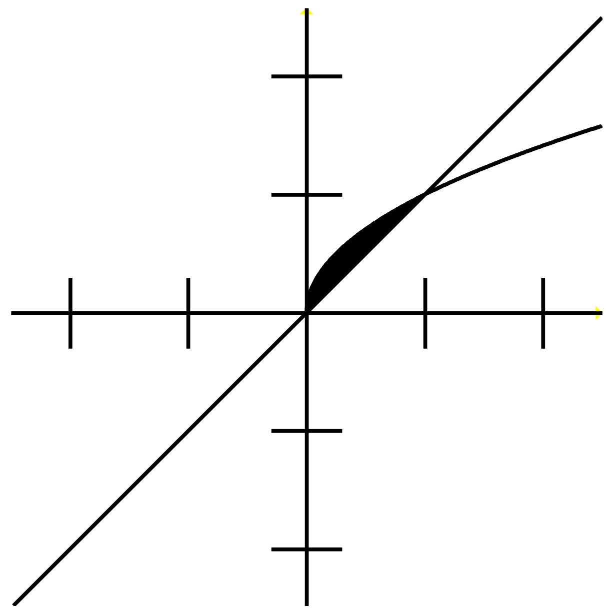
It's some kind of bowl on its side. The inside is straight, while the outside is rounded.

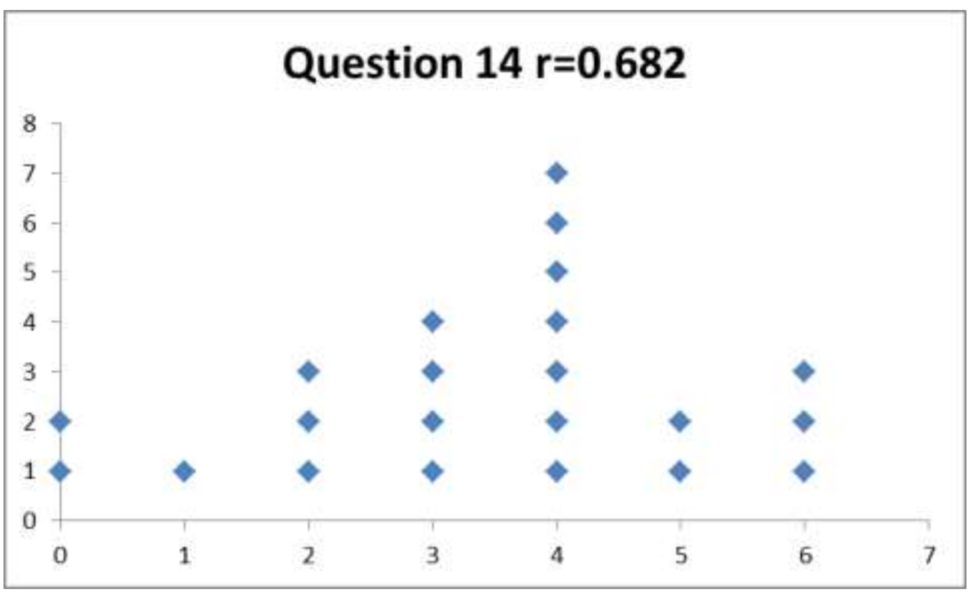
Washer Method:

$$\int_0^1 \pi r_{\text{outer}}^2 dx - \int_0^1 \pi r_{\text{inner}}^2 dx = \int_0^1 \pi\left(\sqrt{x}\right)^2 dx - \int_0^1 \pi x^2 dx$$

Shell Method:

$$\int_0^1 2\pi rh dy = \int_0^1 2\pi y(y - y^2) dy$$





15) The region bounded by $y = x^3 - x^8 + 1$ and $y = 1$ is rotated around the x-axis. Find the volume of the resulting 3-D solid. (Set up, but do not integrate. Circle which method you're using)
(Disk/washer)   (Cylindrical Shell)

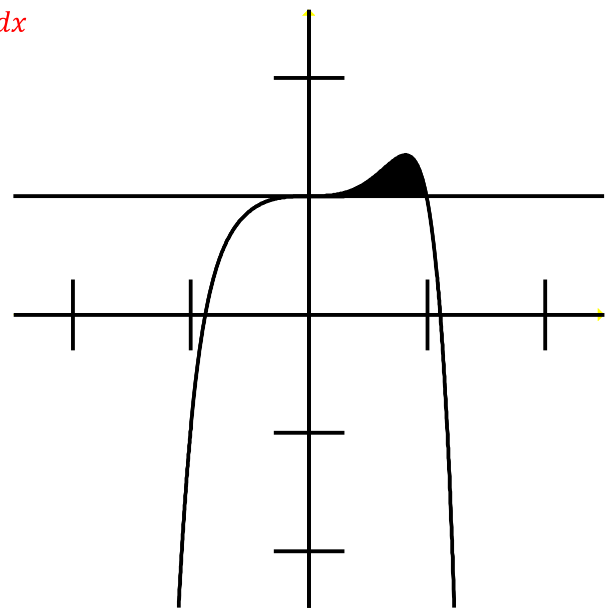
What does that curve look like?? You can't know exactly what it looks like, but you can figure out a few key pieces of information:
When $x = 0, y = 1$
When $x = 1, y = 1$

So the region is bounded between $x = 0$ and $x = 1$. The only question now is, is it above or below the line $y = 1$? To determine this, note that $\left(\frac{1}{2}\right)^3 > \left(\frac{1}{2}\right)^8$, or in general on this interval:

$$x^3 > x^8$$

Hence it is above $y = 1$.

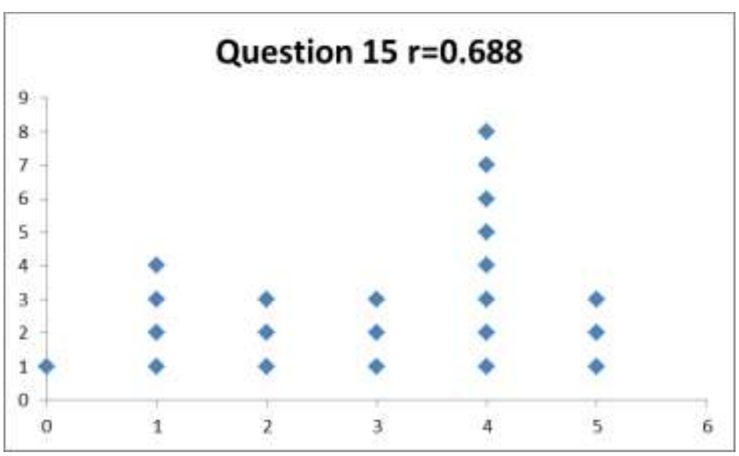
Washer Method:
$\int_0^1 \pi r_{\text{outer}}^2 dx - \int_0^1 \pi r_{\text{inner}}^2 dx = \int_0^1 \pi \left(x^3 - x^8 + 1\right)^2 dx - \int_0^1 \pi (1)^2 dx$



Shell Method:

$$\int_0^1 2\pi r h dy = \int_0^1 2\pi y(???)dy$$

(We can't easily determine h, because we can't easily solve $y = x^3 - x^8 + 1$ for x)

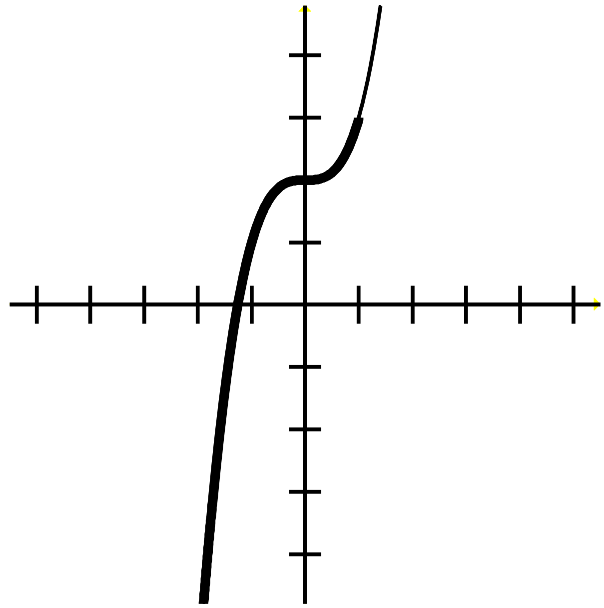


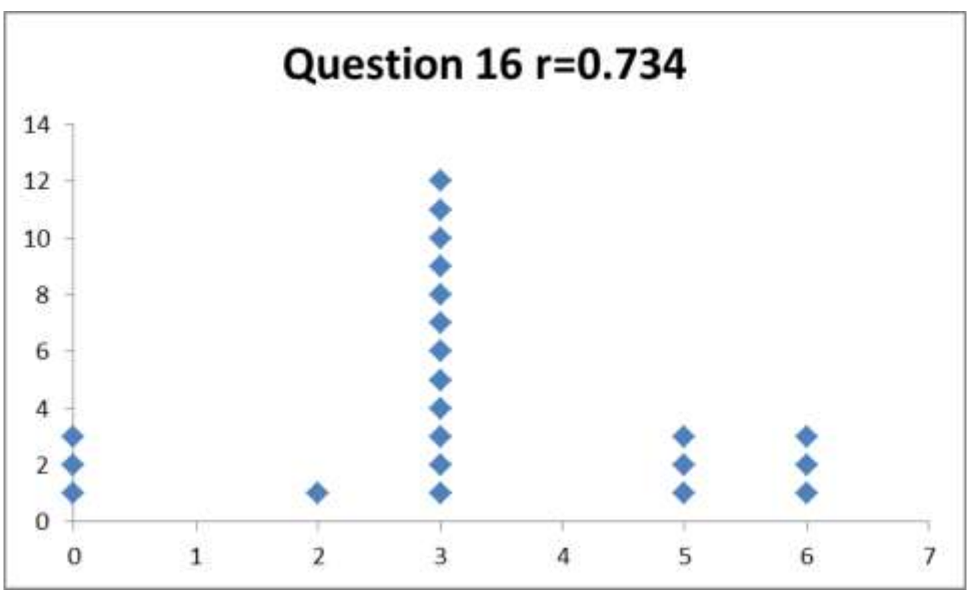
16) Find the length of the curve $y = x^3 + 2$ between $x = -2$ and $x = 1$.

(Set up, but do not integrate.)

(6 points)

$$\int_{-2}^{1} \sqrt{1 + \left(f'(x)\right)^2}\, dx = \int_{-2}^{1} \sqrt{1 + (3x^2)^2}\, dx$$





17) Find the area of the surface generated when the curve $y = 8\sqrt{x}$ between $x = 9$ and $x = 20$ is rotated around the x-axis.

(Set up, but do not integrate.)

(6 points)

$$\int_9^{20} 2\pi r \sqrt{1 + \left(f'(x)\right)^2}\, dx = \int_9^{20} 2\pi 8\sqrt{x} \sqrt{1 + \left(\frac{4}{\sqrt{x}}\right)^2}\, dx$$