

Name _____ Quiz 3

In the land of Fictoria, the great King Omen has a magic orb. This orb is a sphere in shape and contains a magic liquid that powers King Omen's tremendous magic. The orb has a radius of 10 feet, and the magic liquid is being used (drained) at a rate of 3 ft^3 per year.

The orb is nearly empty, with the magic liquid reaching only a height of 2 feet above the bottom. See the picture on the projector for reference and note that the volume of this magic liquid can be modeled as a spherical cap with volume given by $V = \pi r h^2 - \frac{\pi}{3} h^3$. In this formula r is the radius of the orb, and h is the height of the liquid within the orb.

How quickly is the height of the magic liquid decreasing?

Equation:

$$V = \pi r h^2 - \frac{\pi}{3} h^3$$

$$V = 10\pi h^2 - \frac{\pi}{3} h^3$$

Variables:

$$V = ??$$

$$V' = -3$$

$$r = 10$$

$$r' = 0$$

$$h = 2$$

$$h' = ??$$

Derivative:

$$V' = 20\pi h h' - \pi h^2 h'$$

Solution:

$$-3 = 20\pi h h' - \pi h^2 h' = 40\pi h' - 4\pi h' = 36\pi h'$$

$$h' = -\frac{3}{36\pi} = -\frac{1}{12\pi} \text{ feet per year}$$

