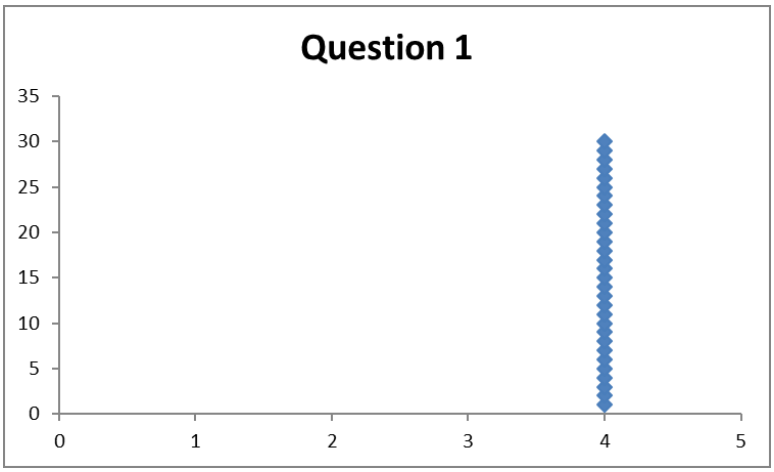
Name _____ Test 1, Fall 2019

Part 1: Computational Skills

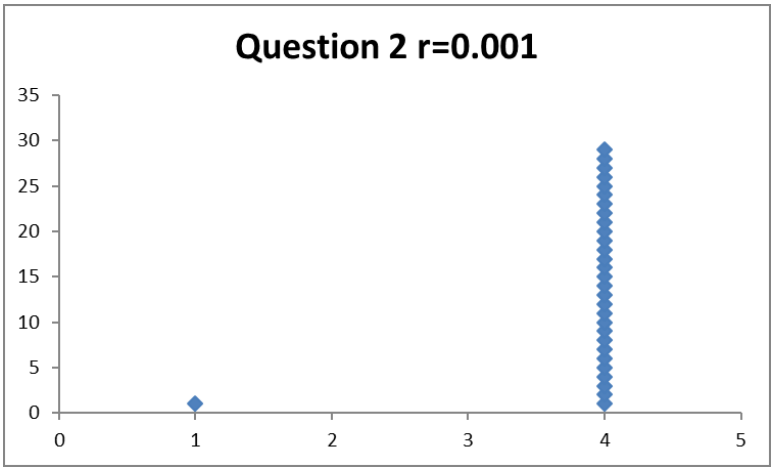
1) Find the limit below. (4 points)

$\lim_{x \to 4} 3x + 7 = 3 \cdot 4 + 7 = 19$



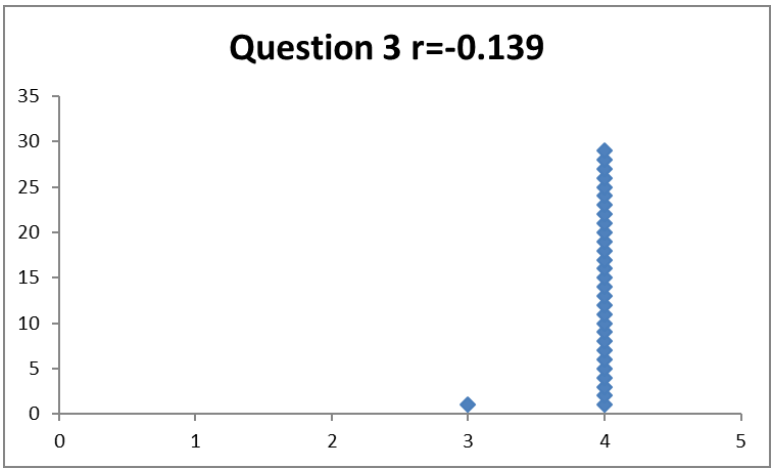
2) Find the limit below. (4 points)

$$\lim_{x \to 4} \sqrt{2x+1} = \sqrt{2 \cdot 4 + 1} = \sqrt{9} = 3$$



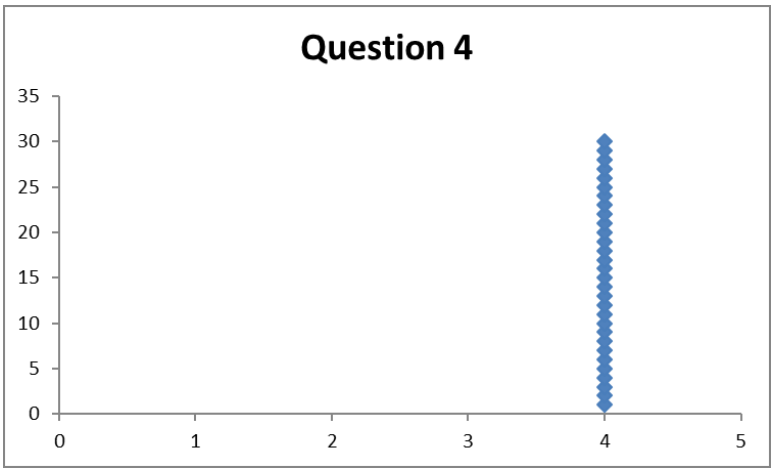
3) Find the limit below. (4 points)

$$\lim_{x\to 4}\frac{x^2-16}{x-4}=\lim_{x\to 4}\frac{(x-4)(x+4)}{x-4}=\lim_{x\to 4}x+4=4+4=8$$



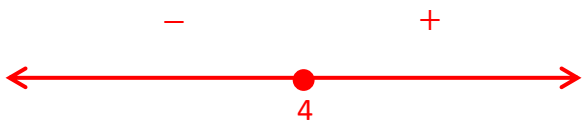
4) Find the limit below. (4 points)

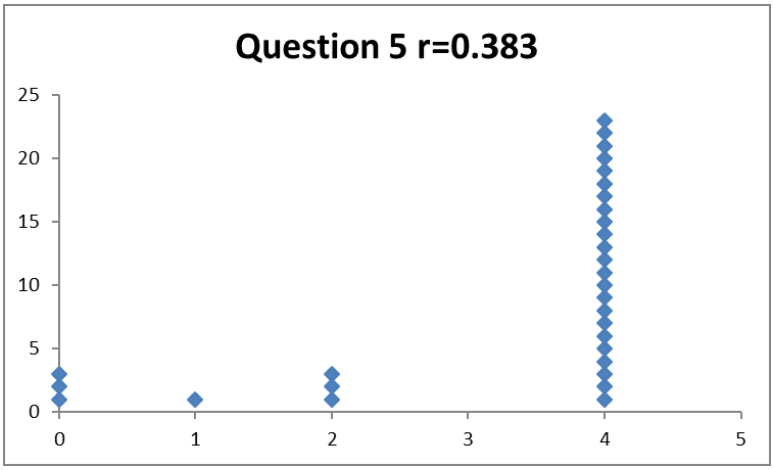
$$\lim_{x\to 4}\frac{x^2-3x-4}{x-4}=\lim_{x\to 4}\frac{(x-4)(x+1)}{x-4}=\lim_{x\to 4}(x+1)=4+1=5$$



5) Find the limit below. (4 points)

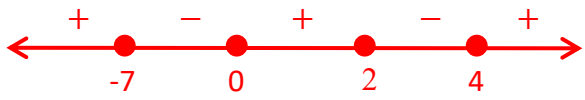
$$\lim_{x \to 4^-} \frac{1}{x-4} = -\infty$$

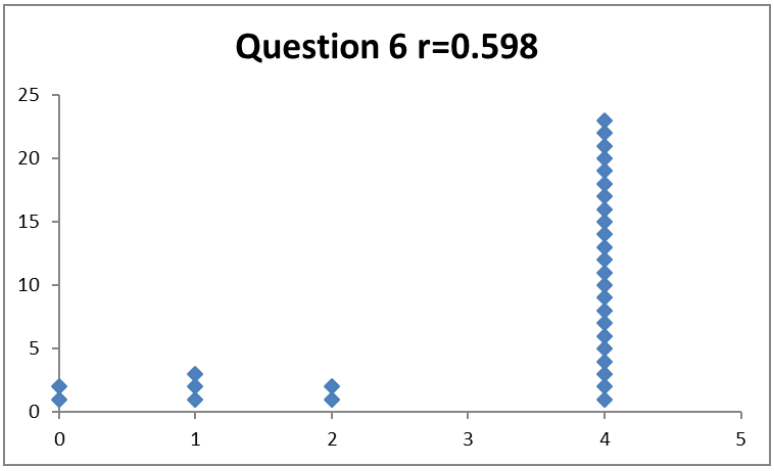




6) Find the limit below. (4 points)

$$\lim_{x\to 4^+} \frac{x(x+6)}{(x-2)(x-4)} = \infty$$

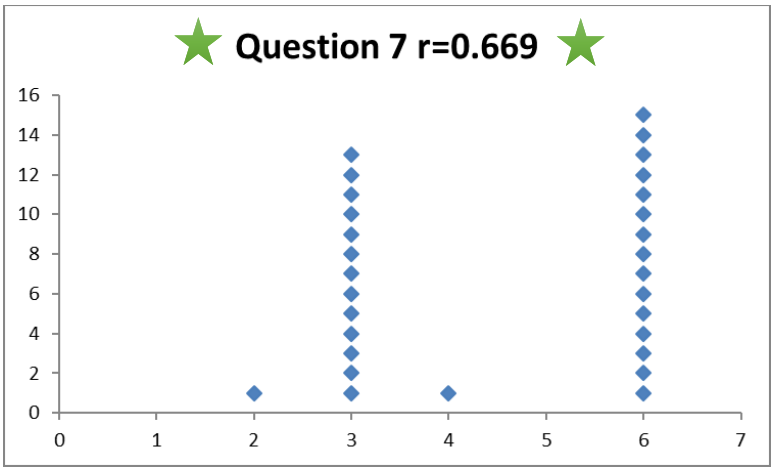




7) Find derivative of the function below. (6 points)

$f(x) = x^2t^5$

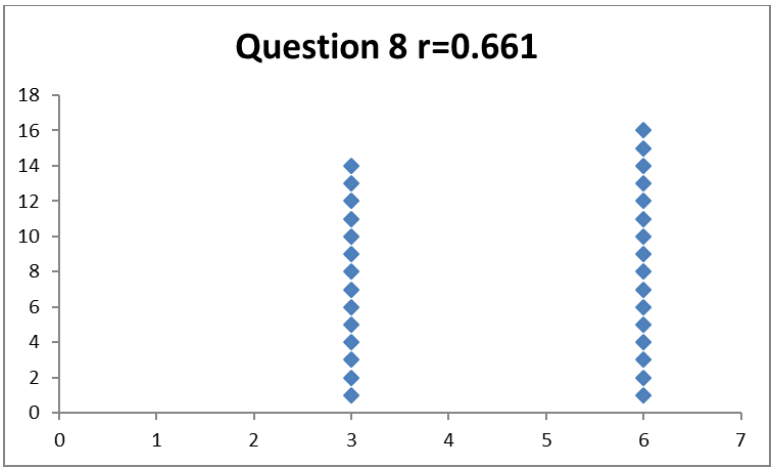
$f'(x) = 2xt^5$



8) Find derivative of the function below. (6 points)

$f(t) = x^2t^5$

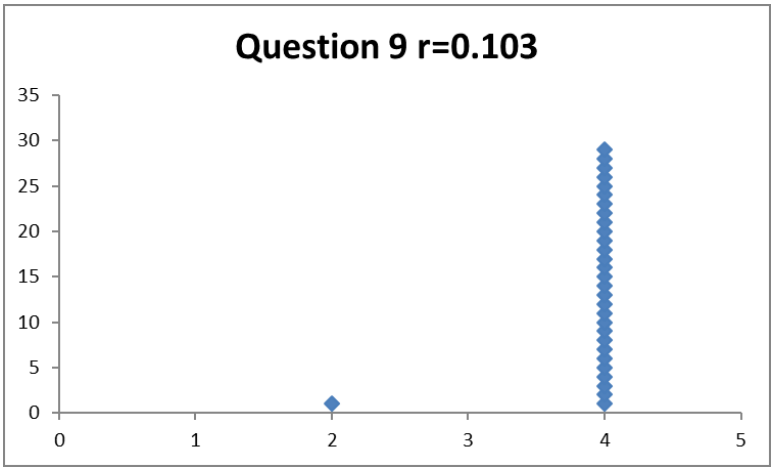
$f(t) = 5x^2t^4$



9) Find derivative of the function below. (4 points)

$f(x) = 2x^2 + 3x + 1$

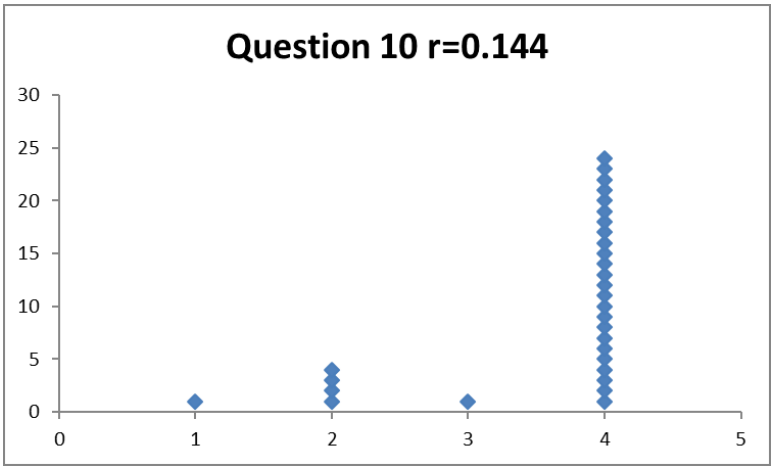
$f'(x) = 4x + 3$



10) Find derivative of the function below. (4 points)

$f(x) = 3e^x$

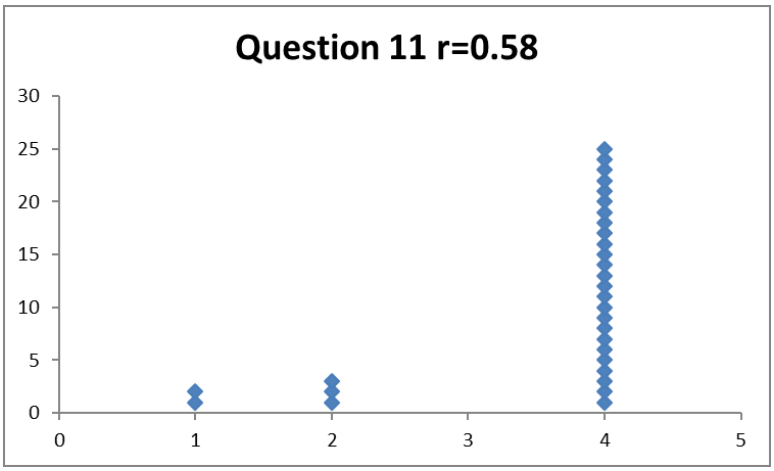
$f'(x) = 3e^x$



11) Find derivative of the function below. (4 points)

$f(x) = 7^x$

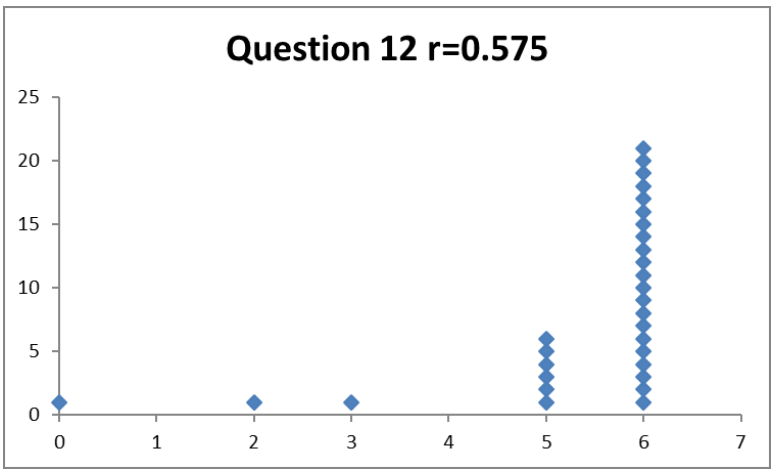
$f'(x) = 7^x \ln(7)$



12) Find derivative of the function below. (6 points)

$$f(x) = \frac{x^2 + 2x + 1}{x^3 + 5x + 2}$$

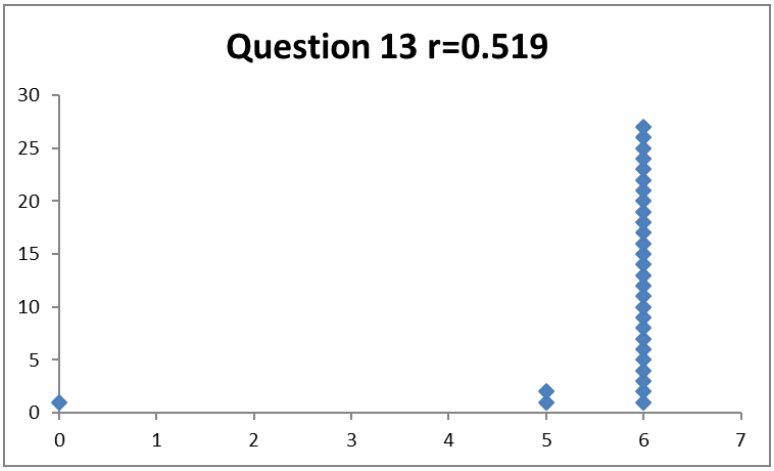
$$f'(x) = \frac{(2x + 2)(x^3 + 5x + 2) - (x^2 + 2x + 1)(3x^2 + 5)}{(x^3 + 5x + 2)^2}$$



13) Find derivative of the function below. (6 points)

$$f(x) = (x^2 + 2x + 1)(x^3 + 5x + 2)$$

$$f'(x) = (2x + 2)(x^3 + 5x + 2) + (x^2 + 2x + 1)(3x^2 + 5)$$



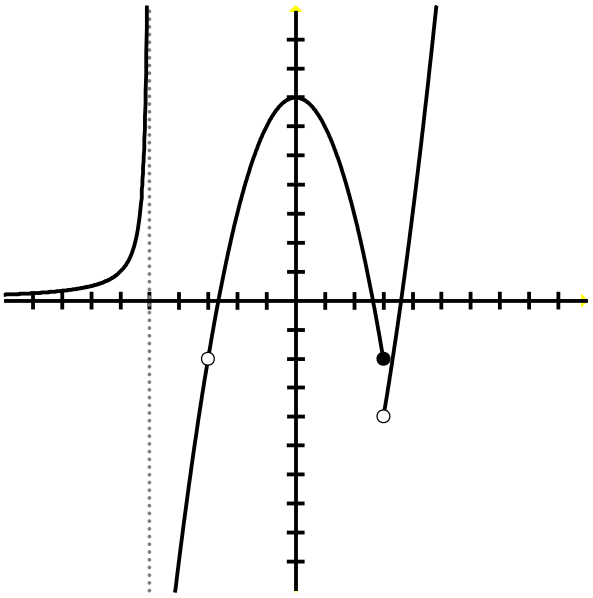
Part 2: Conceptual Understanding

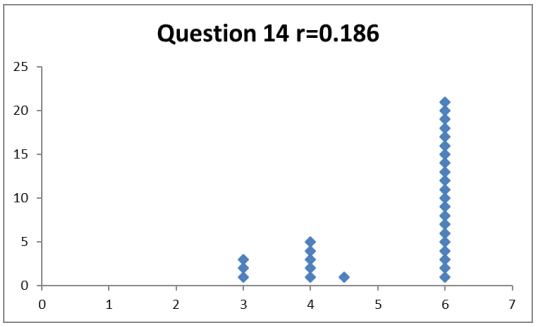
Use the graph for the problems on this page.

14) Estimate each of the following limits. (6 points)

$\lim_{x \to -3^+} f(x) = -2$

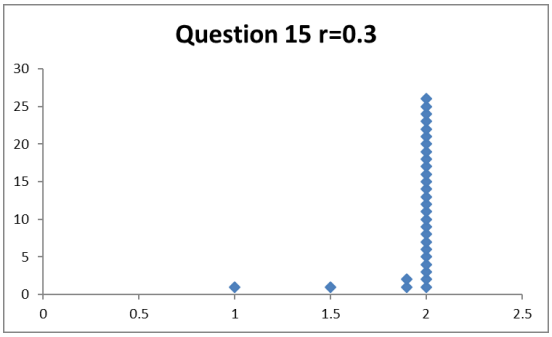
$\lim_{x \to -5^-} f(x) = \infty$

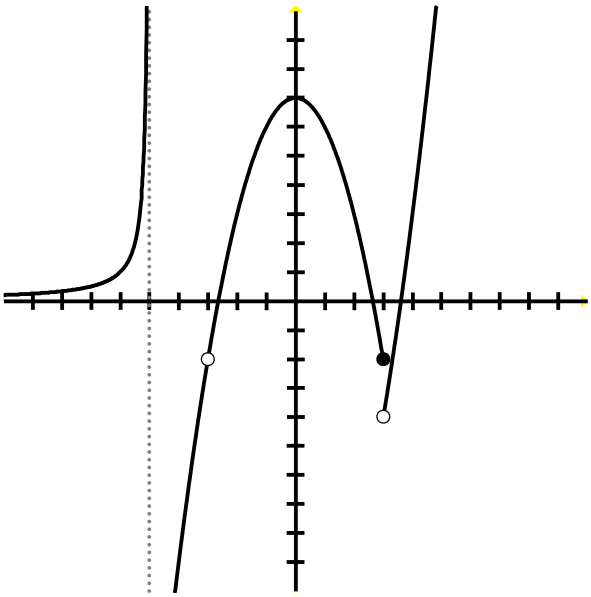




15) Identify two discontinuities and the type of discontinuity. (2 points)

$x = -5, x = -3$ and $x = 3$ are all discontinuities. They are, in order, infinite, removable, and a jump discontinuity.

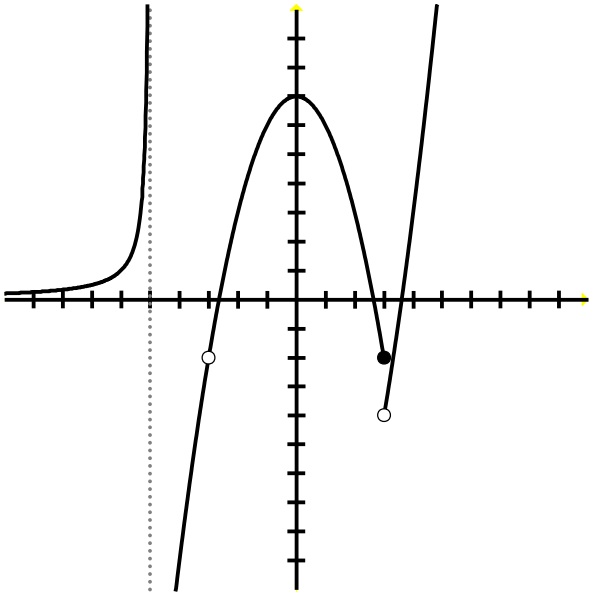


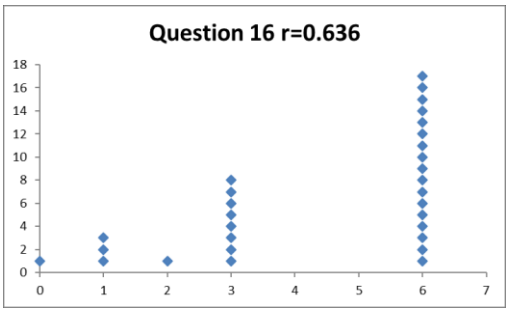


16) Estimate each of the following derivatives. (6 points)

$f'(0) = 0$

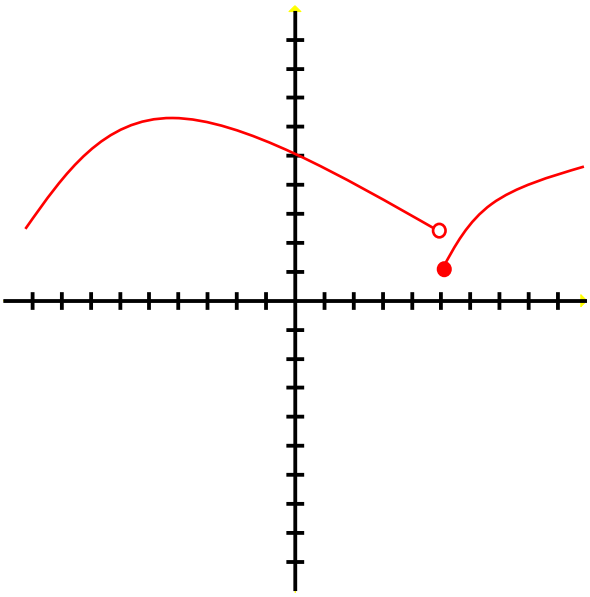
$f'(2) = -2\text{ish}$

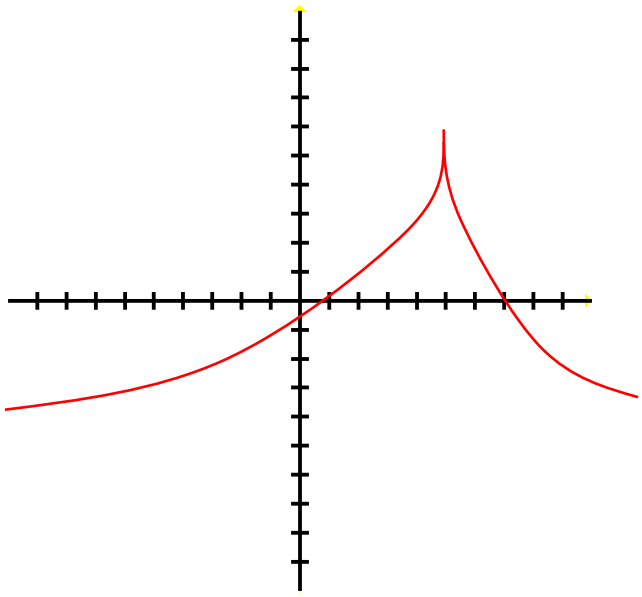


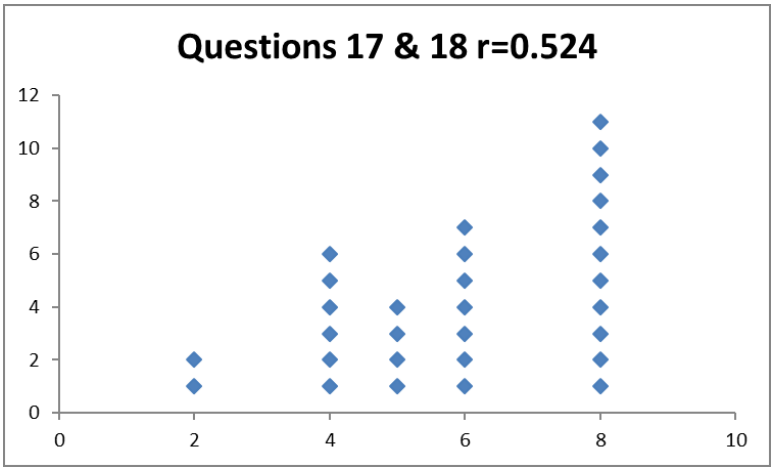


17) On the axes below to the RIGHT, construct a function that is continuous everywhere, but not differentiable at the position $x = 5$. (4 points)

18) On the axes below to the LEFT, construct a function that is defined everywhere, but does not have a limit at the position $x = 5$. (4 points)





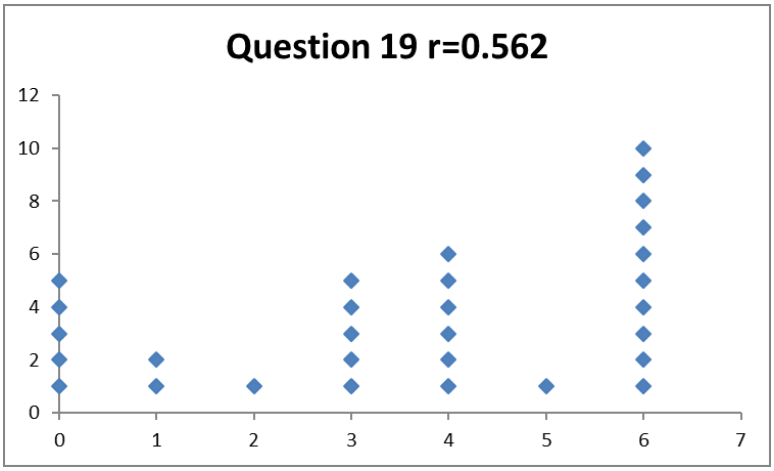


19) Below is a table of values of a function $f(x)$. Use it to estimate $f'(2)$. (6 points)

x	$f(x)$
0	2
1	3
2	5
3	8
4	11
5	15

There are many potential estimates. However the best estimates will find the slope of a line that is a good approximation of the tangent line. Such as:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 5}{3 - 2} = 3$$



Part 3: Applications

20) The equation $2y' + y = x$ is called a differential equation because it involves an unknown function y, and its derivative y'. This type of equation is often used in engineering and physics. Find constants a and b such that the function $y = ax + b$ satisfies the equation given. (6 points)

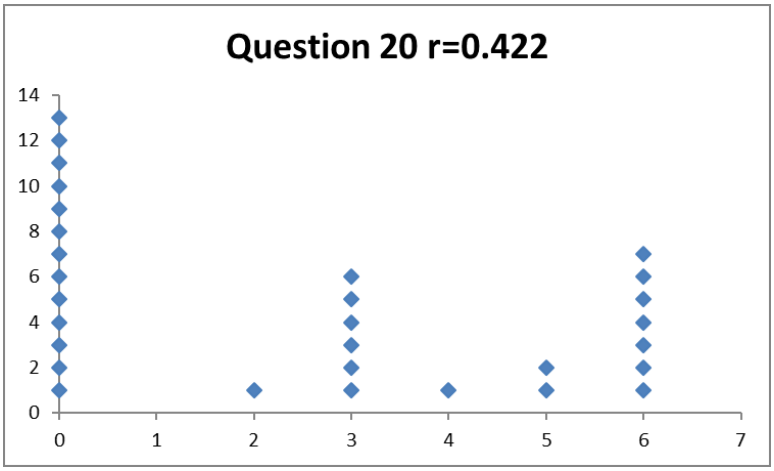
$$y = ax + b$$
$$y' = a$$

$$2a + ax + b = x$$
$$ax + (2a + b) = x + 0$$

$$a = 1$$
$$2a + b = 0$$
$$b = -2$$

$$y = a - 2$$

Half credit if you did something meaningful:
Plugging y' into the equation with y' in it.
Plugging $2y' + y$ into the equation with x in it. (Ultimately useless. But good try!)



21) The position of a beetle is given by $p(t) = 4t^2 + 3t + 1$. Here t is measured in seconds and p is measured in feet. How fast is the beetle moving after 2 seconds? (6 points)

Velocity is the derivative of position. Hence $v(t) = p'(t) = 8t + 3$.

$v(2) = 8 \cdot 2 + 3 = 16 + 3 = 19$

19 feet per second.