Name ____ Test 2, Fall 2019

Part 1: Computational Skills

1) Find derivative of the function below. (6 points)

$$f(x) = \frac{x^2}{3x^2 + 2}$$

$$f'(x) = \frac{(2x)(3x^2 + 2) - x^2(6x)}{(3x^2 + 2)^2}$$

2) Find derivative of the function below. (6 points)

$$f(x) = \cos(3x + 2)$$

$$f'(x) = -\sin(3x + 2) \cdot 3$$

3) Find derivative of the function below. (6 points)

$f(x) = e^{x^5+4x^3}$

$f'(x) = e^{x^5+4x^3}(5x^4 + 12x^2)$

4) Find derivative of the function below. (6 points)

$f(x) = \ln(sin(x))$

$f'(x) = \frac{\cos(x)}{\sin(x)}$

5) Find derivative of the function below. (6 points)

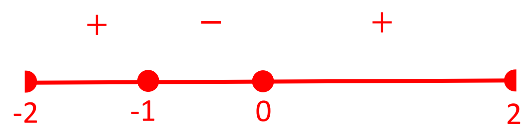
$f(x) = |\sin(x)|$

$f'(x) = \frac{\sin(x)}{|\sin(x)|}\cos(x)$

Use the function $f(x) = 2x^3 + 3x^2 + 1$ on the domain $[-2, 2]$ for these problems.

6) Where are the local maxima of f? (4 points)

$f'(x) = 6x^2 + 6x$
$6x^2 + 6x = 0$
$6x(x+1) = 0$
$x = -1, 0$ (Critical values!)



$x = -1$ and $x = 2$ are the maxima.

7) What are the local maxima of f? (4 points)

$$f(-1) = 2(-1)^3 + 3(-1)^2 + 1 = -2 + 3 + 1 = 2$$
$$f(2) = 2(2)^3 + 3(2^2) + 1 = 16 + 12 + 1 = 29$$

$$y = 2, 29$$

8) What and where is the global minimum of f, if it exists? (4 points)
(Two part answer)

$f(0) = 1$ which probably isn't the global min, so let's try the other possible place, $x = -2$ and calculate $f(-2)$

$$f(-2) = 2(-2)^3 + 3(-2)^2 + 1 = -16 + 12 + 1 = -3$$

Global min/min value is then: $(-2, -3)$

9) If the domain were $(-2,2)$ instead of $[-2,2]$, does your answer to #the previous question change? How does it change?
(Answer should be a sentence) (4 points)

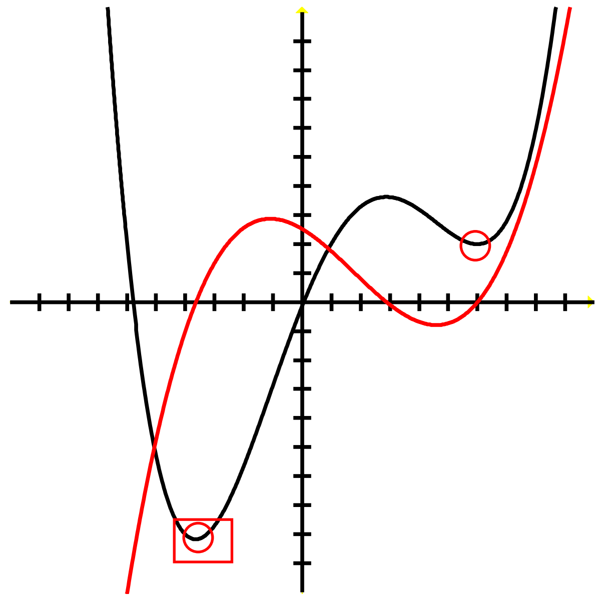
Yes it would change, to "There is no global minimum."

Part 2: Conceptual Understanding

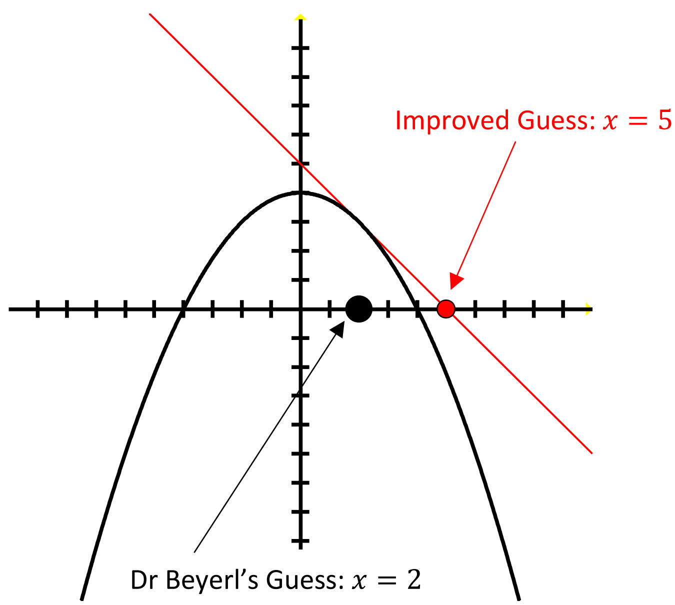
10) To the right is a graph of a function. Draw its derivative. (6 points)

11) To the right is a graph of a function. Circle ALL local minima. That means draw a circle around it/them.
(4 points)

12) To the right is a graph of a function. Box ALL global minima. That means draw a box with flat sides and corners around it/them.
(4 points)



13) On this problem we want to find a root of the function given below. Dr. Beyerl's has a guess, and he was kind enough to label it for us. Use Newton's Method to improve that guess! Illustrate what you do by graphically showing one iteration of Newton's Method.
(6 points)



Part 3: Applications

14) All the edges of a cube are expanding at a rate of 3cm/s. How fast is the surface area changing when the cube's size is $4 \times 4 \times 4$? (10 points)

Equation:

$S = 6x^2$

Variables:

$S = ??$

$S' = ??$

$x = 4$

$x' = 3$

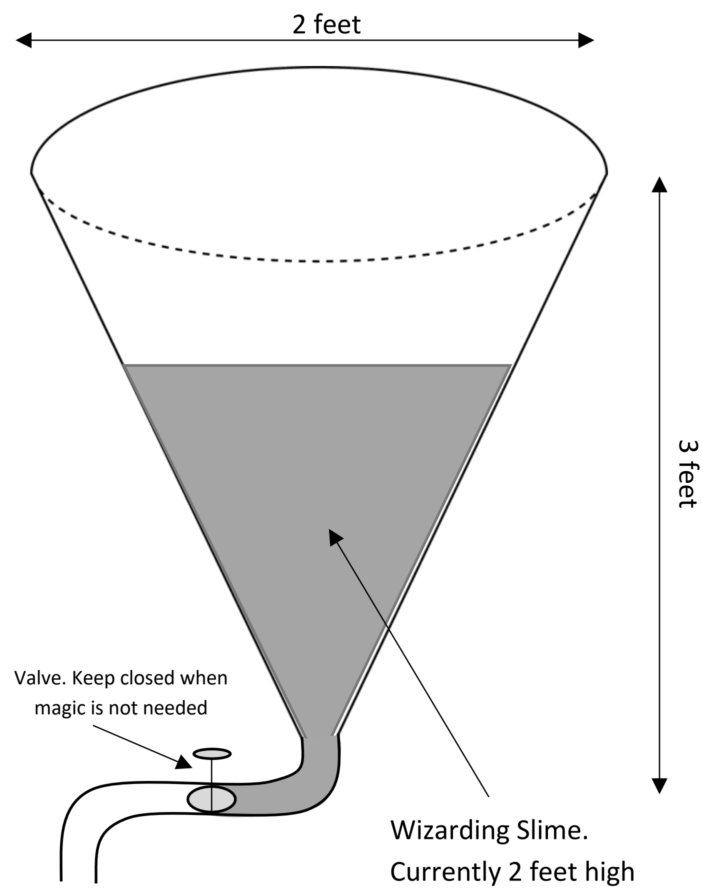
Derivative:

$S' = 12xx'$

Solution:

$S' = 12(4)(3) = 144cm^2/s$

15) A wizard uses “Wizarding Slime” to perform his magic. He keeps his slime in a tank whose shape is an inverted circular cone. The conical tank measures 3 feet tall and 2 feet across at the top. The wizard taps the tank by removing 1/9th of a cubic foot of wizarding slime per month. How quickly is the height of the slime in the tank decreasing when the level of the slime measures 2 feet from the bottom?
(10 points)



Equation:

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = \frac{1}{27}\pi h^3$

Variables:

$\boldsymbol{V} = ??$

$\boldsymbol{V'} = -\frac{1}{9}$

$\boldsymbol{h} = \mathbf{2}$

$\boldsymbol{h'} = ??$

$\boldsymbol{r} = ??$

$\boldsymbol{r'} = ??$

Derivative:

$V' = \frac{3}{27}\pi h^2 h' = \frac{1}{9}\pi h^2 h'$

Solution:

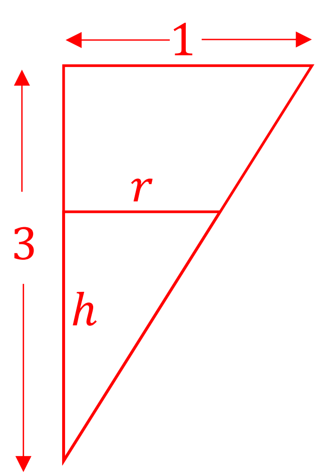
$-\frac{1}{9} = \frac{1}{9}\pi(2)^2 h'$

$-1 = 4\pi h'$

$h' = -\frac{1}{4\pi}$ feet/month

Relation between r and h used above:

$\frac{h}{3} = \frac{r}{1}$



16) There are two "hyperbolic trig functions" called "sinh" and "cosh". They're related to "sin" and "cos", but are not identical. In particular they're derivatives are nearly what we would expect:

$$\frac{d}{dx}\sinh(x) = \cosh(x)$$
$$\frac{d}{dx}\cosh(x) = \sinh(x)$$

Use this information to find the 488th derivative of $\cosh(2x)$.
(10 points)

$y = \cosh(2x)$
$y' = 2\sinh(2x)$
$y'' = 4\cosh(2x) = 2^2\cosh(2x)$
$y''' = 8\sinh(2x) = 2^3\sinh(2x)$
$y^{(4)} = 2^4\cosh(2x)$
…
$y^{(488)} = 2^{488}\cosh(2x)$

Part 4: Review Problems

17) Find the limit below. (4 points)

$$\lim_{x\to 4}\frac{x^2-16}{x-4} = \lim_{x\to 4}\frac{(x-4)(x+4)}{x-4} = \lim_{x\to 4}(x+4) = 4+4 = 8$$