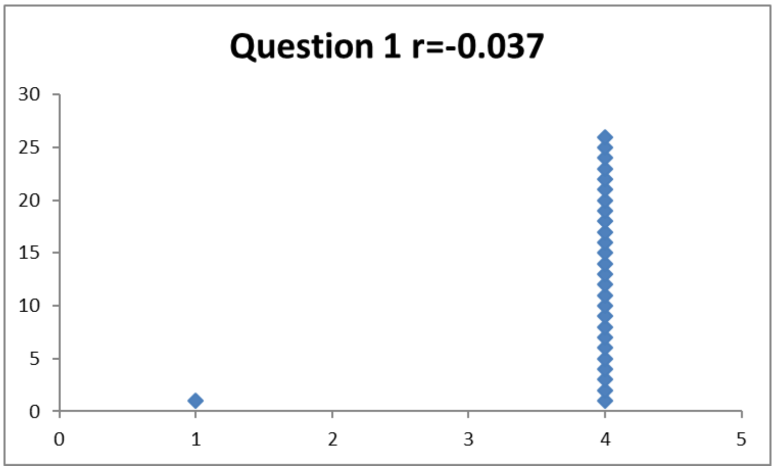
Name ____ Test 3, Fall 2019

Part 1: Computational Skills

1) Find the integral below. (4 points)

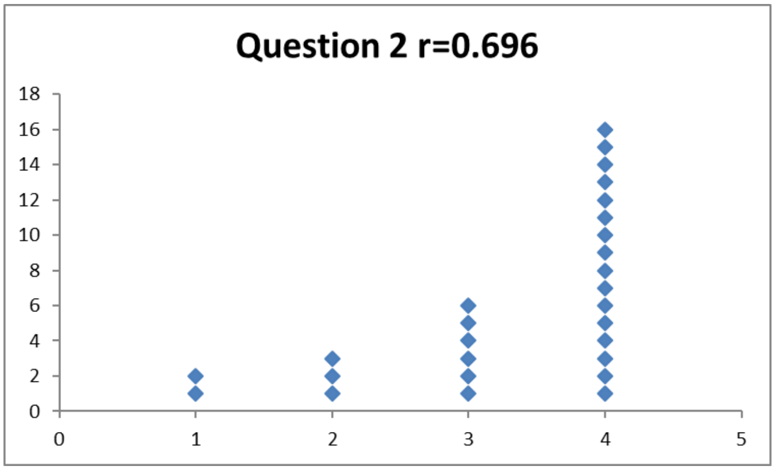
$$\int 3x^4 dx = \frac{3x^5}{5} + C$$



2) Let $f(x) = 3x^4$. Find the specific antiderivative $F(x)$ such that $F(1) = 4$. (4 points)

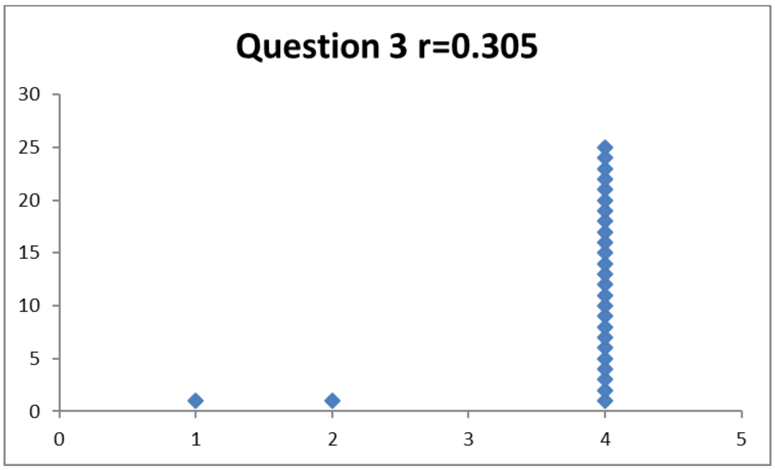
$$\frac{3 \cdot 1^5}{5} + C = 4$$

$$C = 4 - \frac{3}{5} = \frac{17}{5}$$



3) Find the limit below. (4 points)

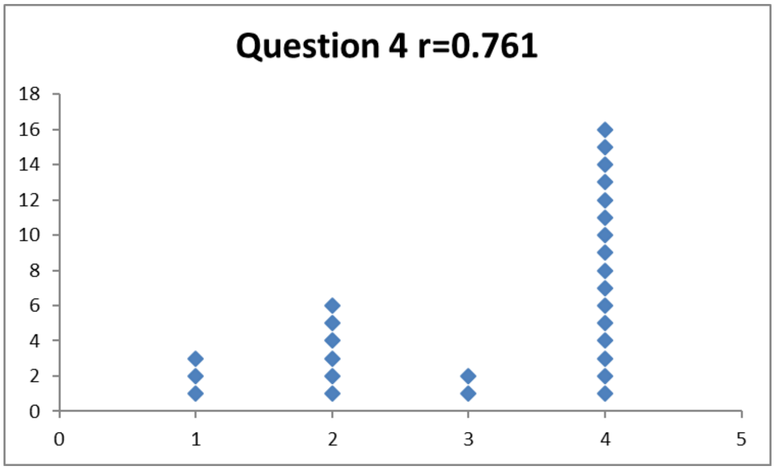
$$\lim_{x\to\infty}\frac{5x^6-4x^3+2x+1}{8x^6+5x^4+x^2+6}=\frac{5}{8}$$



4) Find the integral below. (4 points)

$$\int \cos(4x)\,dx = \frac{\sin(4x)}{4} + C$$

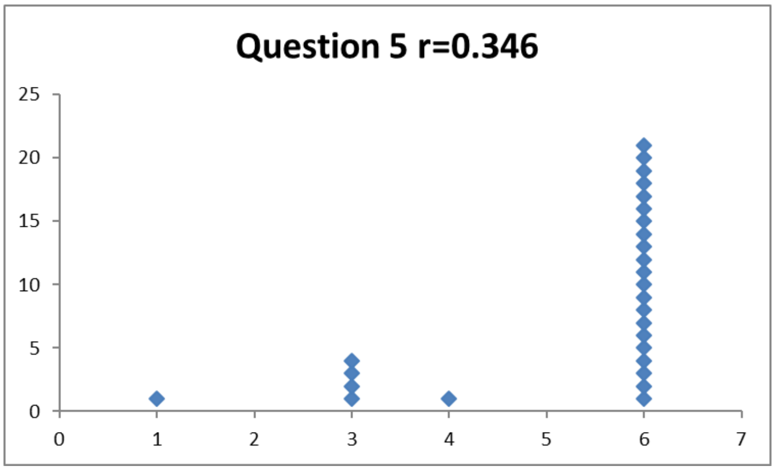
$u = 4x$, or use your intuition.



5) Find the integral below. (6 points)

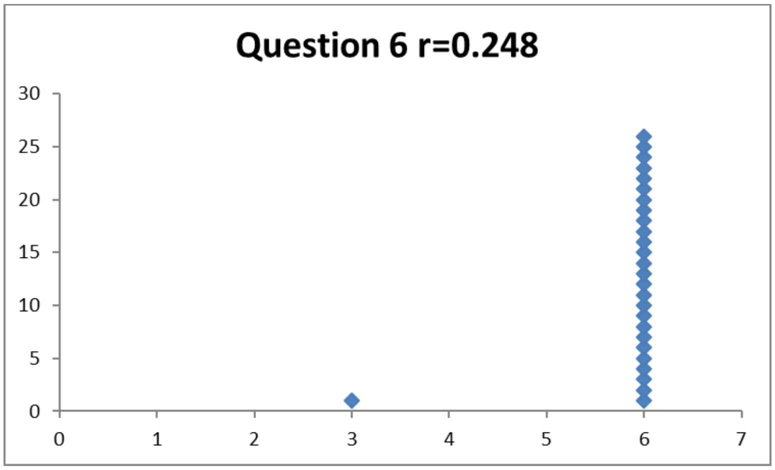
$$\int \frac{1}{x-5}\,dx = \ln|x-5| + C$$

$u = x - 5$, or use your intuition.



6) Find the integral below. (6 points)

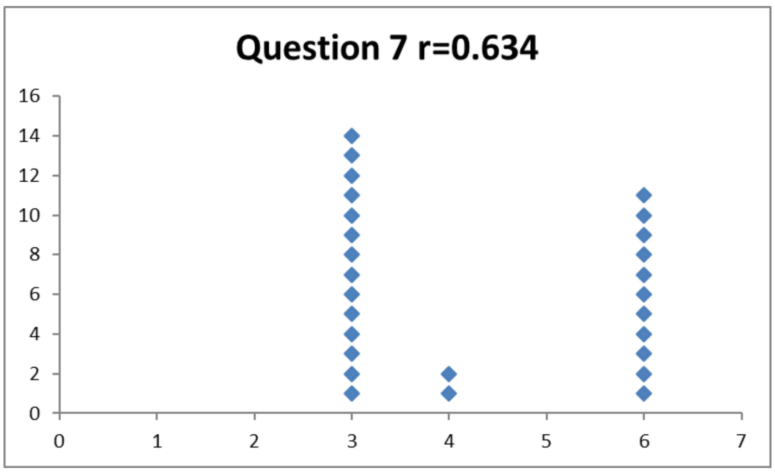
$$\int_3^6 1dx = \frac{x}{1}\Big|_3^6 = 6 - 3 = 3$$



7) Let $b \neq 1$ be a constant. Find the integral below. (6 points)

$$\int_0^1 x^{4b}\,dx = \left.\frac{x^{4b+1}}{4b+1}\right|_0^1 = \frac{1^{4b+1}}{4b+1} = \frac{1}{4b+1}$$

Note that this only works if the exponent $4b \neq -1$, otherwise we would get a logarithm. hence why we had the domain restriction $b \neq 1$....wait, we need $4b \neq -1$, so $b \neq -\frac{1}{4}$. Anyone that recognized this gained 5 bonus points.



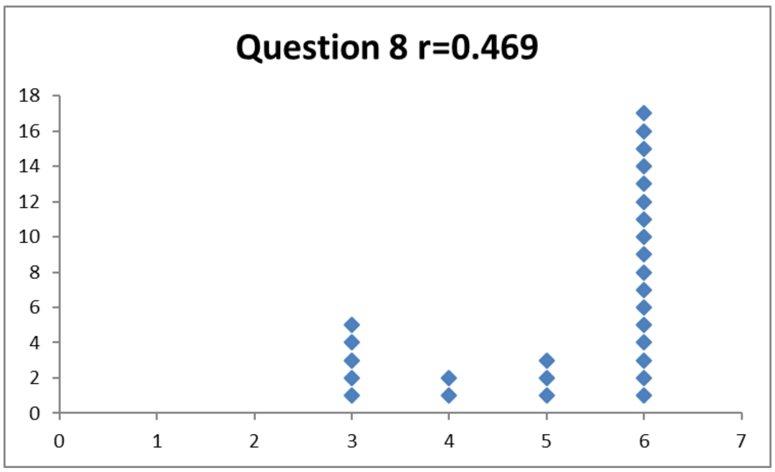
8) Find the integral below. (6 points)

$$\int x^3(x^4-5)^3dx = \frac{1}{4}\int u^3du = \frac{1}{16}u^4 + C = \frac{1}{16}(x^4-5)^4 + C$$

$u = x^4 - 5$

$du = 4x^3dx$

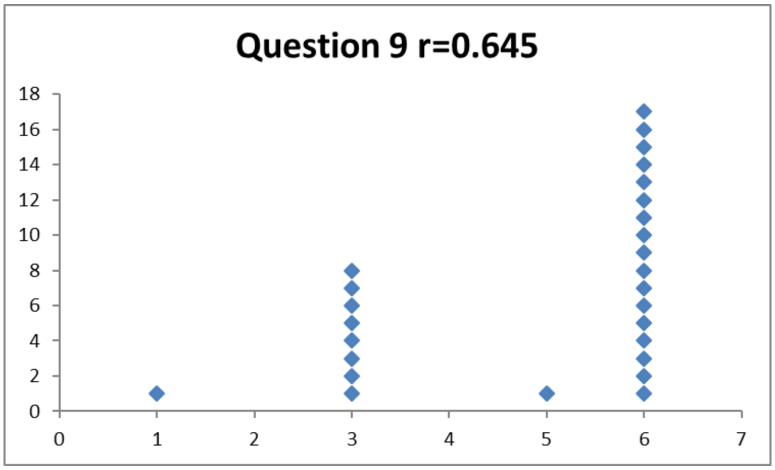
$\frac{du}{4} = x^3dx$



9) Find the integral below. (6 points)

$$\int \cos(x)\sin^5(x)\,dx = \int u^5 du = \frac{u^6}{6} + C = \frac{\sin^6(x)}{6} + C$$

$u = \sin(x)$
$du = \cos(x)\,dx$



10) Find the integral below. (6 points)

$$\int \frac{1}{9+x^2}dx = \int \frac{1}{9\left(1+\left(\frac{x}{3}\right)^2\right)}dx = \frac{1}{9}\int \frac{1}{1+\left(\frac{x}{3}\right)^2}dx = \frac{3}{9}\int \frac{1}{1+(u)^2}du = \frac{1}{3}\tan^{-1}(u) + C$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$u = \frac{x}{3}$

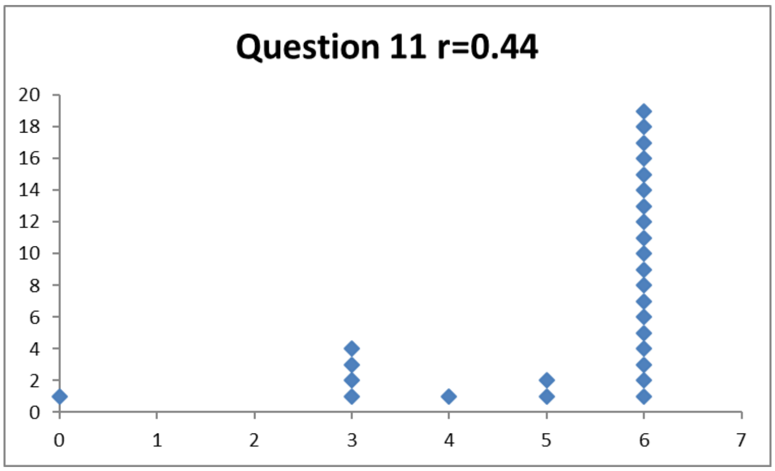
$du = \frac{1}{3}dx$

$3du = dx$

11) Find the integral below. (6 points)

$$\int (e^x + x)(e^x + 1)\,dx = \int u\,du = \frac{u^2}{2} + C = \frac{(e^x + x)^2}{2} + C$$

$u = e^x + x$
$du = (e^x + 1)dx$

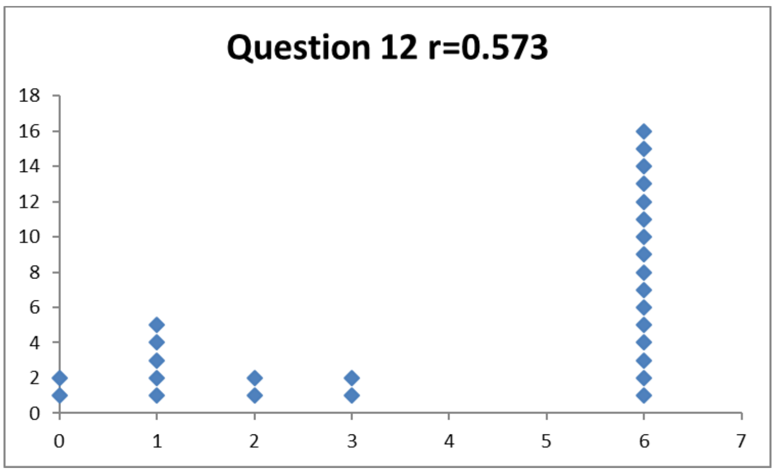


Part 2: Conceptual Understanding

12) Assume $f(x)$ and $g(x)$ are continuous functions such that $f(5) = 8, f'(5) = 3, g(5) = 8,$ $g'(5) = 2$. Use this information to find the limit below. (6 points)

$$\lim_{x \to 5} \frac{f(x) - g(x)}{x^2 - 25} = \lim_{x \to 5} \frac{f'(x) - g'(x)}{2x} = \frac{3 - 2}{10} = \frac{1}{10}$$

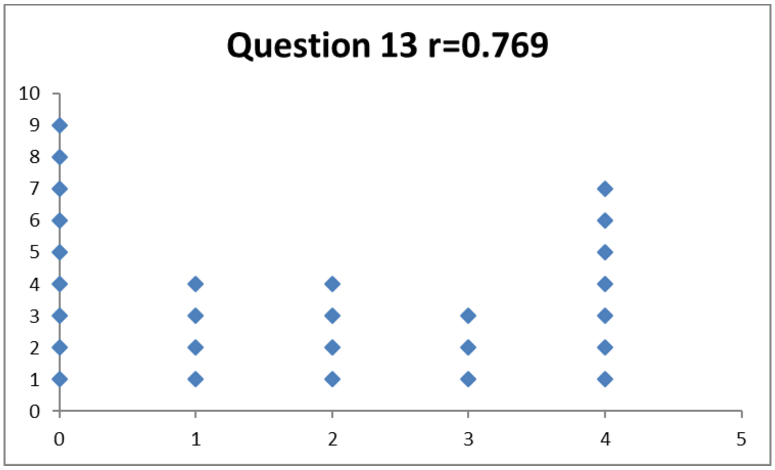
(It's L'Hospital's rule!)



13) Find the value of the limit of the summation below. (4 points)

$$\lim_{n\to\infty} 3\sum_{k=1}^{n} \frac{2}{n}\left(\frac{2k}{n}\right)^2$$

$$3\int_0^2 x^2 dx = \frac{3x^3}{3}\Big|_0^2 = 2^3 - 0 = 8$$



14) To the right is a table of values of a function. Create three estimates for the integral of $\int_2^{10} f(x)dx$ Each estimate must be better than the previous, so don't start with an estimate that is too good. (8 points)
Make sure to show your work! If I don't understand where you answer came from, you will not receive full credit.

x	$f(x)$
1	13
2	4
3	3
4	8
5	9
6	10
7	7
8	5
9	3
10	6
11	11
12	12

Using the Right Riemann Sum approach I'll use 1, 2, and 4 rectangles.

Estimate 1:

One rectangle: $8 \cdot 6 = 48$

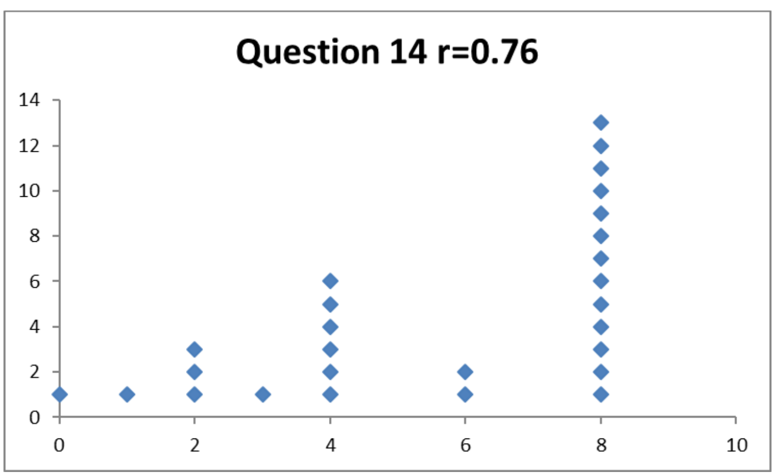
Estimate 2:

Two rectangles: $4 \cdot 10 + 4 \cdot 6 = 64$

Estimate 3:

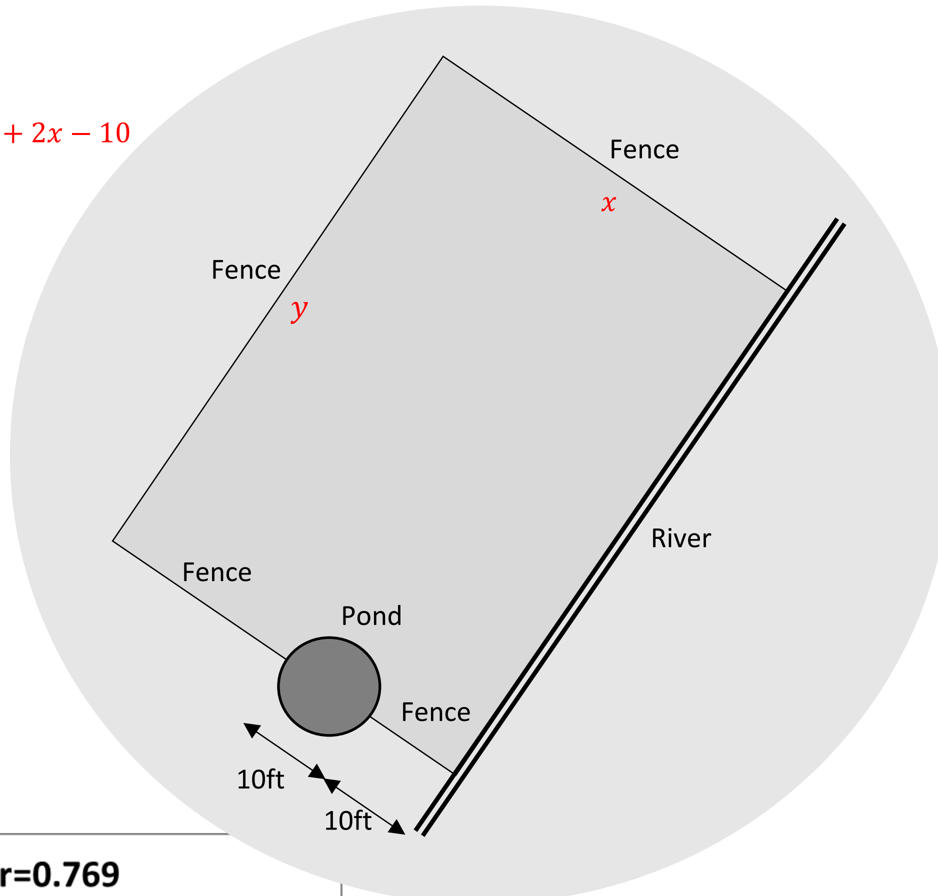
Four rectangles. Oh, screw it, just do 8 rectangles so I don't have to multiply:

$$3 + 8 + 9 + 10 + 7 + 5 + 3 + 6 = 46$$



Part 3: Applications

15) A farmer's pasture consists of a field with a circular pond and a straight river. The farmer would like to create a pasture as large as possible using only the 390 feet of fencing he has available. Hence he will create a 3-sided pasture, using the pond to save some fencing as shown. What is the dry-land-area of the largest such pasture he can make? (6 points)



Label x and y as shown.

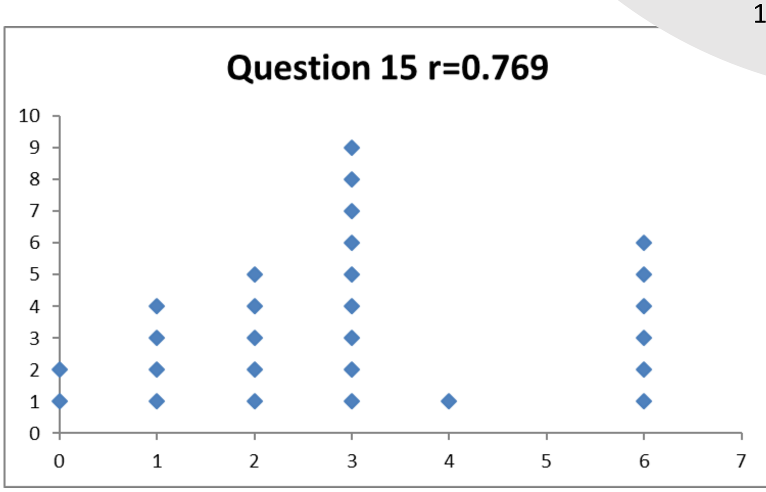
$A = xy - 25\pi$
$390 = x + y + (x - 20) + 10 = y + 2x - 10$
$y = 390 - 2x + 10 = 400 - 2x$

$A = x(400 - 2x) - 25\pi$
$\quad = 400x - 2x^2 - 25\pi$

$A' = 400 - 4x = 0$
$x = 100$

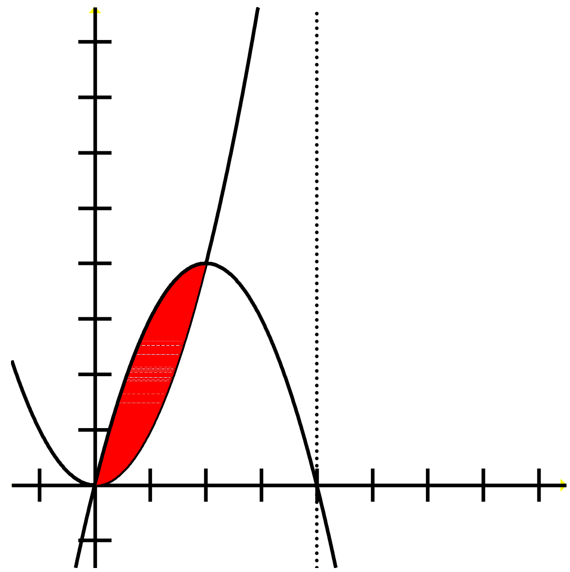
(Check it's a maximum somehow)
(Otherwise -1 point)

$A = 100 \cdot 200 - \frac{25}{2}\pi$
$\quad = 20{,}000 - \frac{25}{2}\pi ft$



16) Set up the integral(s) for volume of the following object. The region bounded by $y = x^2$ and the curve $y = 4x - x^2$ is rotated around the line $x = 4$. (6 points)

The diagram here illustrates this region. Do not ask which curve is which, you should be able to figure that out.

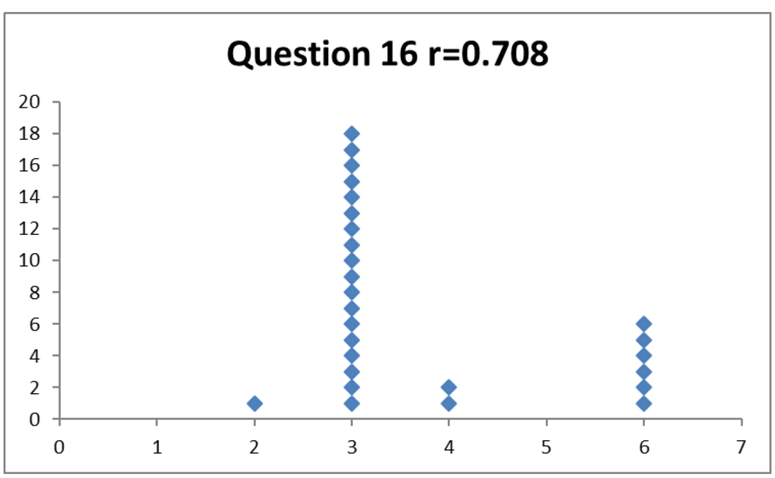


Washer method:

$$\int_0^4 \pi r_{\text{outer}}^2 - \pi_{\text{inner}}^2 dy = \int_0^4 \pi(4 - x_{\text{outer}})^2 - \pi(4 - x_{\text{inner}})^2 dy = \int_0^4 \pi\left(4 - \left(-\sqrt{4-y} + 2\right)\right)^2 - \pi\left(4 - \sqrt{y}\right)^2 dy$$

Shell method:

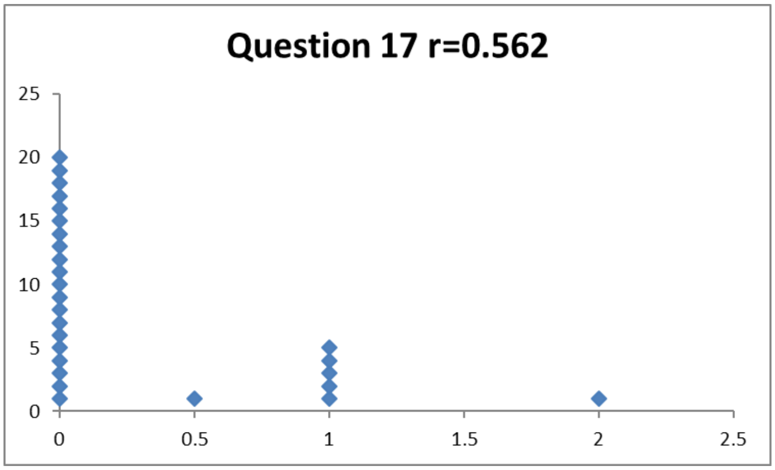
$$\int_0^2 2\pi rh\ dx = \int_0^2 2\pi(4 - x)\left(y_{top} - y_{\text{bottom}}\right)dx = \int_0^2 2\pi(4 - x)(4x - x^2 - x^2)dx$$



Part 4b: Extra Credit Problems

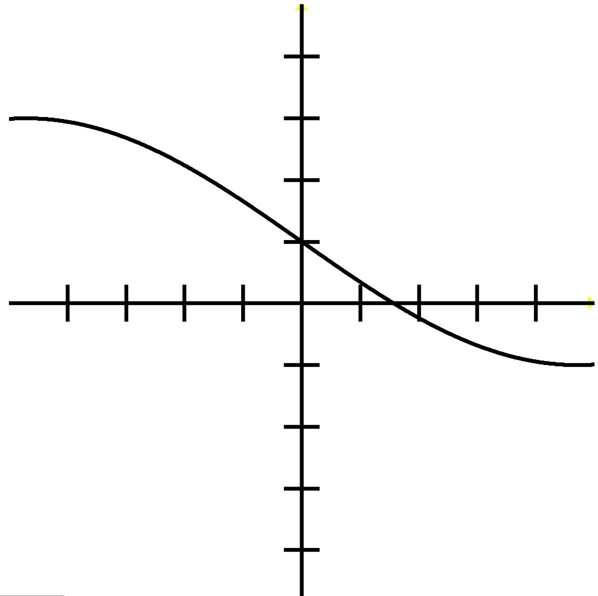
17) Set up the integral(s) for surface area of the following object from the previous problem. (2 bonus points)

$$\int_0^2 2\pi(4x - x^2)\sqrt{1 + (4 - 2x)^2}dx + \int_0^2 2\pi(x^2)\sqrt{1 + (2x)^2}dx$$

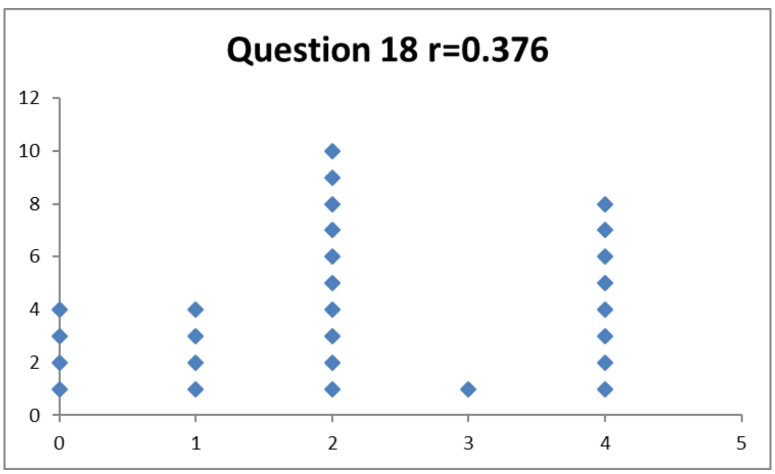


Part 4: Review Problems

18) Estimate the largest value of the derivative of the function shown to the right (4 points)

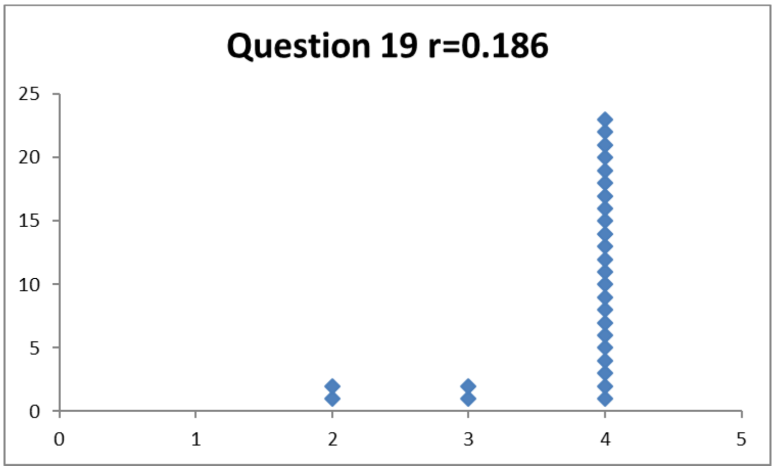


It’s close to zero at either end of the graph.



19) Find the derivative of the function $f(x) = 4x^3 - 3x$ at the point $x = 2$. (4 points)

$f'(x) = 12x^2 - 3$
$f'(2) = 12 \cdot 2^2 - 3 = 48 - 3 = 45$



20) Find the following: (4 points)

$$\frac{d}{dx}\left(\frac{d}{dx}(x\sin(5x))\right) = \frac{d}{dx}(\sin(5x) + 5x\cos(5x)) = 5\cos(5x) + 5\cos(5x) - 25x\sin(5x)$$