Name ____ Test 1, Fall 2020

Part 1: Computational Skills

1) Find the limit below. (4 points)

$$\lim_{x\to 3}\frac{x^2-9}{x-3}=\lim_{x\to 3}\frac{(x-3)(x+3)}{x-3}=\lim_{x\to 3}(x+3)=3+3=6$$

2) Find the limit below. (4 points)

$$\lim_{x\to 3}\frac{x-3}{2x^2-5x-3}=\lim_{x\to 3}\frac{x-3}{(x-3)(2x+1)}=\lim_{x\to 3}\frac{1}{(2x+1)}=\frac{1}{2\cdot 3+1}=\frac{1}{7}$$

3) Find the limit below. (4 points)

$$\lim_{x\to 3}\frac{2x^2+x-4}{x-3}\ \text{DNE}$$

4) Find the limit below. (4 points)

$$\lim_{x\to 3}\frac{\sqrt{x}-\sqrt{3}}{x-3}=\lim_{x\to 3}\frac{\sqrt{x}-\sqrt{3}}{x-3}\cdot\frac{\sqrt{x}+\sqrt{3}}{\sqrt{x}+\sqrt{3}}=\lim_{x\to 3}\frac{x-3}{(x-3)\left(\sqrt{x}+\sqrt{3}\right)}=\frac{1}{\sqrt{3}+\sqrt{3}}=\frac{1}{2\sqrt{3}}$$

5) The limit below comes out to 1. Show every single step and very clear work on how to get there. (14 points)

$$\lim_{x \to 1} \frac{x^2 - 1}{2x - 2} = \lim_{x \to 1} \frac{(x-1)(x+1)}{2(x-1)} = \lim_{x \to 1} \frac{(x+1)}{2} = \frac{1+1}{2} = 1$$

Of particular note for things required for full credit:

- Show each step (4 pts)
- Proper use of equals signs (4pts)
- Proper use of the limit notation (4pts)
- Reserved for unexpected issues (2pts)

6) Find the derivative of the function below. (4 points)

$$f(x) = 3x^4 + 7x^2 - 5$$

$$f'(x) = 12x^3 + 14x$$

7) Find the derivative of the function below. (4 points)

$$f(x) = \sin(x)\tan(x)$$

$$f'(x) = \cos(x)\tan(x) + \sin(x)\sec^2(x)$$

8) Find the derivative of the function below. (4 points)

$$f(x) = \frac{2x^2 + 3x}{6x^7 - 5x^4 + 2}$$

$$f'(x) = \frac{(4x+3)(6x^7 - 5x^4 + 2) - (2x^2 + 3x)(42x^6 - 20x^3)}{(6x^7 - 5x^4 + 2)^2}$$

9) Find the derivative of the function below. (4 points)

$$f(x) = \tan^{-1}\left(e^{5x^4}\right)$$

$$f'(x) = \frac{1}{1 + \left(e^{5x^4}\right)^2} e^{5x^4} 20x^3$$

10) Given the function below, $f'(2) = 63$. Show every single step and very clear work on how to get there. (14 points)

$$f(x) = 3(3x^2 - 5x - 1)^3$$

$$f(x) = 9(3x^2 - 5x - 1)^2 \cdot (6x - 5)$$

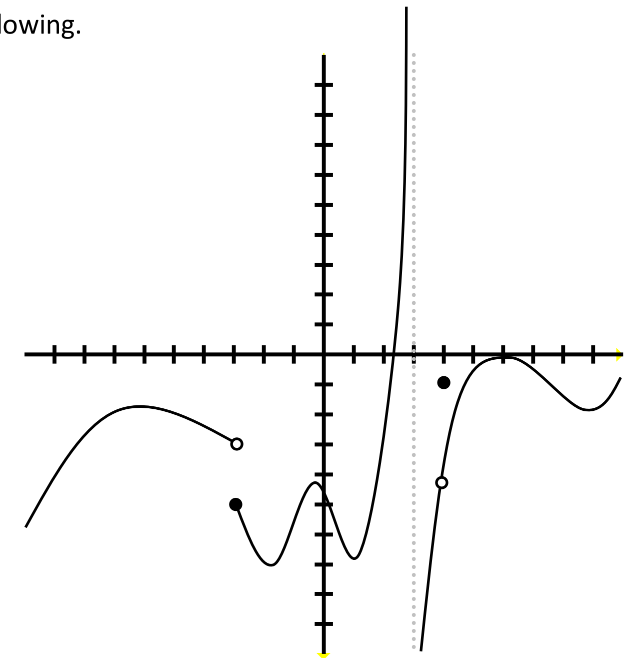
$$f'(2) = 9 \cdot (3 \cdot 2^2 - 5 \cdot 2 - 1)^2 \cdot (6 \cdot 2 - 5) = 9 \cdot (1)^2 \cdot (7) = 63$$

Of particular note for things required for full credit:

- Show each step (4 pts)
- Proper use of equals signs (4pts)
- Proper use of the derivative notation (4pts)
- Reserved for unexpected issues (2pts)

Part 2: Conceptual Understanding

Given the graph of $y = f(x)$ below, find or estimate the following.



11) Find the limit below. (2 points)

$\lim_{x \to -3^-} f(x) = -3$

12) Find the derivative below. (2 points)

$f'(-2) = 0$

13) Find the limit below. (2 points)

$\lim_{x \to 3^+} f(x) = -\infty$

14) Find the derivative below. (2 points)

$f'(5) = 1?$ (Anything in $\left[\frac{1}{2}, 5\right]$ is accepted for full credit)

15) Find the limit below. (2 points)

$\lim_{x \to 4} f(x) = -4$

16) Find the derivative below. (2 points)

$f'(3)$ DNE

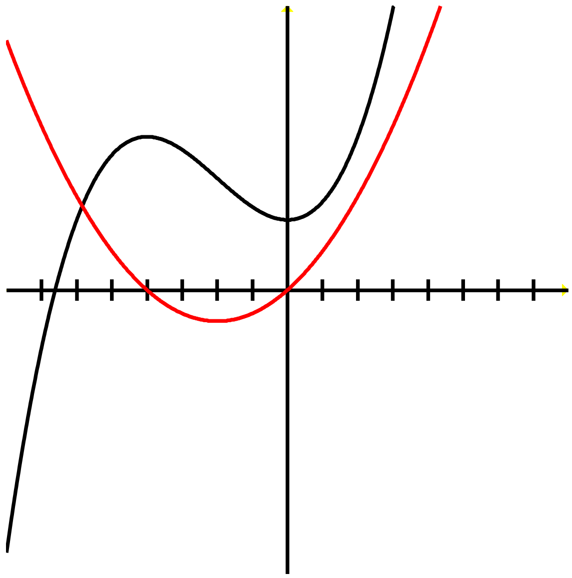
17) What is the average rate of change of $f(x)$ between $x = 0$ and $x = 1$? (2 points)

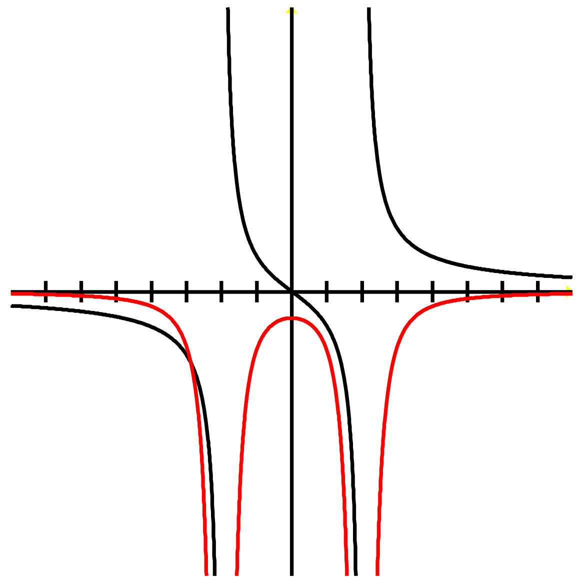
$-2.2?$ (Anything in $[-2, -3]$ is accepted for full credit)

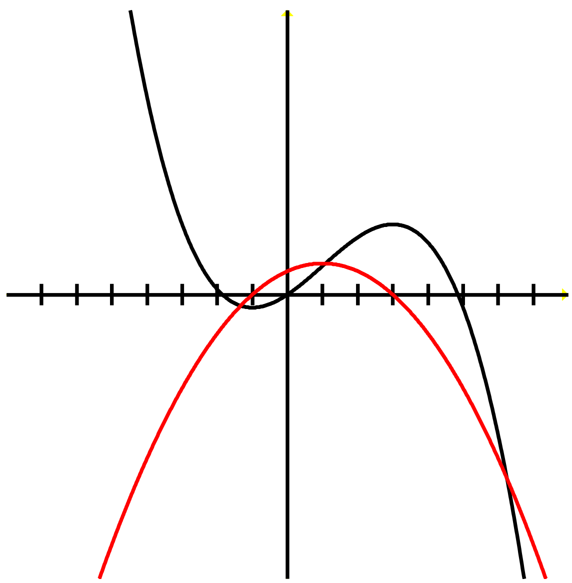
18) What is the instantaneous rate of change of $f(x)$ at $x = -1$? (2 points)

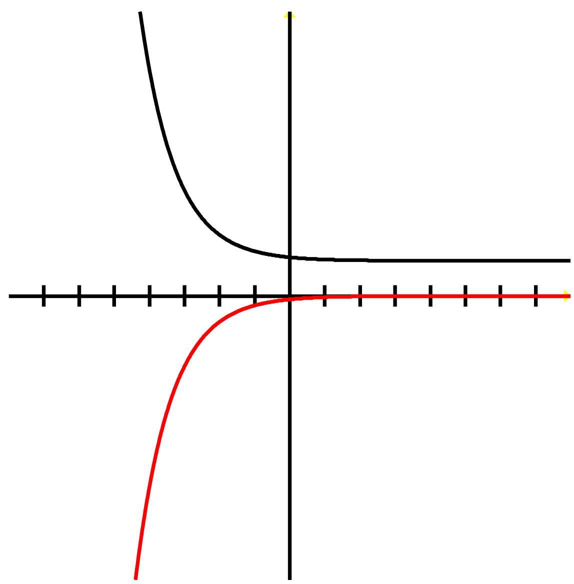
$f'(-1) = 2?$ (Anything in $[1,7]$ is accepted for full credit)

19) For each graph given below, graph the derivative. (3 points each)









Part 3: Applications

20) A spherical rubber bladder is being filled with water. Water is pumped I at a rate of 2 cubic feet per minute. How is the radius changing with respect to time when the radius is equal to 2 feet? (8 points)

Equation:

$V = \frac{4}{3}\pi r^3$

Variables:

$V =$ Unknown but we could solve for it if needed

$V' = 2$

$r = 2$

$r' = ??$

Derivative:

$V' = 4\pi r^2 r'$

Solution:

$2 = 4\pi 2^2 r' = 16\pi r'$

$r' = \frac{2}{16\pi} = \frac{1}{8\pi}$ feet/minute

21) Velocity is defined as the change in position over time. If the relative position of a rodent running away from a cat is given by $p(t) = 3t^2 + 2t$ where p is measured in feet and t is measured in seconds. What is the velocity of the rodent after 2 seconds? (4 points)

$p'(t) = 6t + 2$

$p'(2) = 6 \cdot 2 + 2 = 14$ ft/s