Name ____ Test 2, Fall 2020

Part 1: Computational Skills

1) Find the integral below. (4 points)

$$\int 3x^4 dx = \frac{3x^5}{5} + C$$

2) Find the integral below. (4 points)

$$\int_2^5 3x^2 dx = \frac{3x^3}{3}\bigg|_2^5 = x^3\bigg|_2^5 = 5^3 - 2^3 = 125 - 8 = 117$$

3) Find the integral below. (4 points)

$$\int \sin^2(x)\cos(x)\,dx = \int u^2 \cos(x)\frac{du}{\cos(x)} = \int u^2 du = \frac{u^3}{3} + C = \frac{\sin^3(x)}{3} + C$$

$u = \sin(x)$
$du = \cos(x)\,dx$

$\frac{du}{\cos(x)} = dx$

4) Find the integral below. (4 points)

$$\int_0^\pi \sin(x)\,dx = -\cos(x)\Big|_0^\pi = -\cos(\pi) - (-\cos(0)) = -1 - (-1) = 0$$

5) Find the integral below. (4 points)

$$\int \frac{1}{1+4x^2}\,dx = \int \frac{1}{1+(2x)^2}\,dx = \int \frac{1}{1+u^2}\frac{du}{2} = \frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\tan^{-1}(u) + C = \frac{1}{2}\tan^{-1}(2x) + C$$

$u = 2x$
$du = 2dx$

$\frac{du}{2} = dx$

This problem can also be done intuitively without the need for u-substitution.

6) Find the integral below. (4 points)

$$\int \frac{1}{4+x^2}\,dx = \frac{1}{4}\int \frac{1}{1+\frac{x^2}{4}}\,dx = \frac{1}{4}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}\,dx = \frac{1}{4}\int \frac{1}{1+u^2}\,2du$$

$$= \frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\tan^{-1}(u) + C = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$u = \frac{x}{2}$
$du = \frac{dx}{2}$

$2du = dx$

This problem can also be done intuitively without the need for u-substitution.

7) Find the integral below. (4 points)

$$\int_0^{10} e^{6x}dx = \int_0^{60} e^u \frac{du}{6} = \frac{e^u}{6}\bigg|_0^{60} = \frac{e^{60}}{6} - \frac{e^0}{6} = \frac{e^{60}}{6} - \frac{1}{6}$$

$u = 6x$
$du = 6dx$

$\frac{du}{6} = dx$

When $x = 0, u = 0.$
When $x = 10, u = 60.$

This problem can also be done intuitively without the need for u-substitution.

8) The integral below comes out to $\frac{1}{4}(x^5 + 3)^4 + C$. Show every single step and very clear work on how to get there.
(14 points)

$$\int 5x^4(x^5 + 3)^3 dx = \int 5x^4 u^3 \frac{du}{5x^4} = \int u^3 du = \frac{u^4}{4} + C = \frac{(x^5 + 3)^4}{4} + C$$

$u = x^5 + 3$
$du = 5x^4 dx$

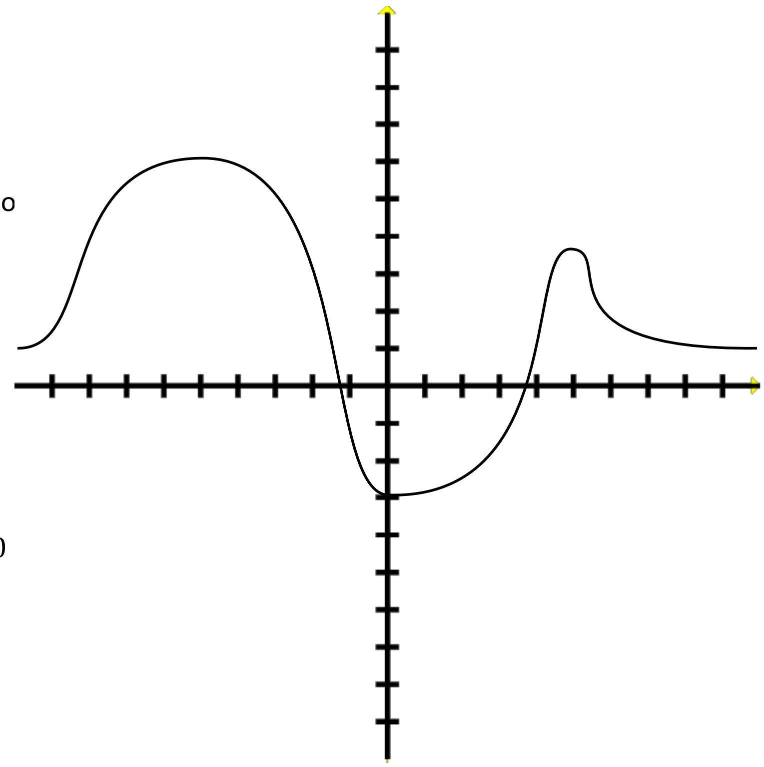
$\frac{du}{5x^4} = dx$

9) The integral below comes out to 72. Show every single step and very clear work on how to get there.
(14 points)

$$\int_{1}^{5} 6x dx = \left.\frac{6x^2}{2}\right|_1^5 = 3x^2\Big|_1^5 = 3 \cdot 25 - 3 \cdot 1 = 75 - 3 = 72$$

Part 2: Conceptual Understanding

Given the graph of $y = f(x)$ below, find or estimate the following.



10) The global maximum of the function. (2 points)

6

11) A local maximum of the function that is not the glo maximum. (2 points)

4

12) Any maximizer. (2 points)

5 or -5

13) The x-value of the inflection point between $x = 0$

4

Anything in $[3.5, 4.5]$ was given full credit.

14) The x-value of the inflection point less than $x = -5$. (2 points)

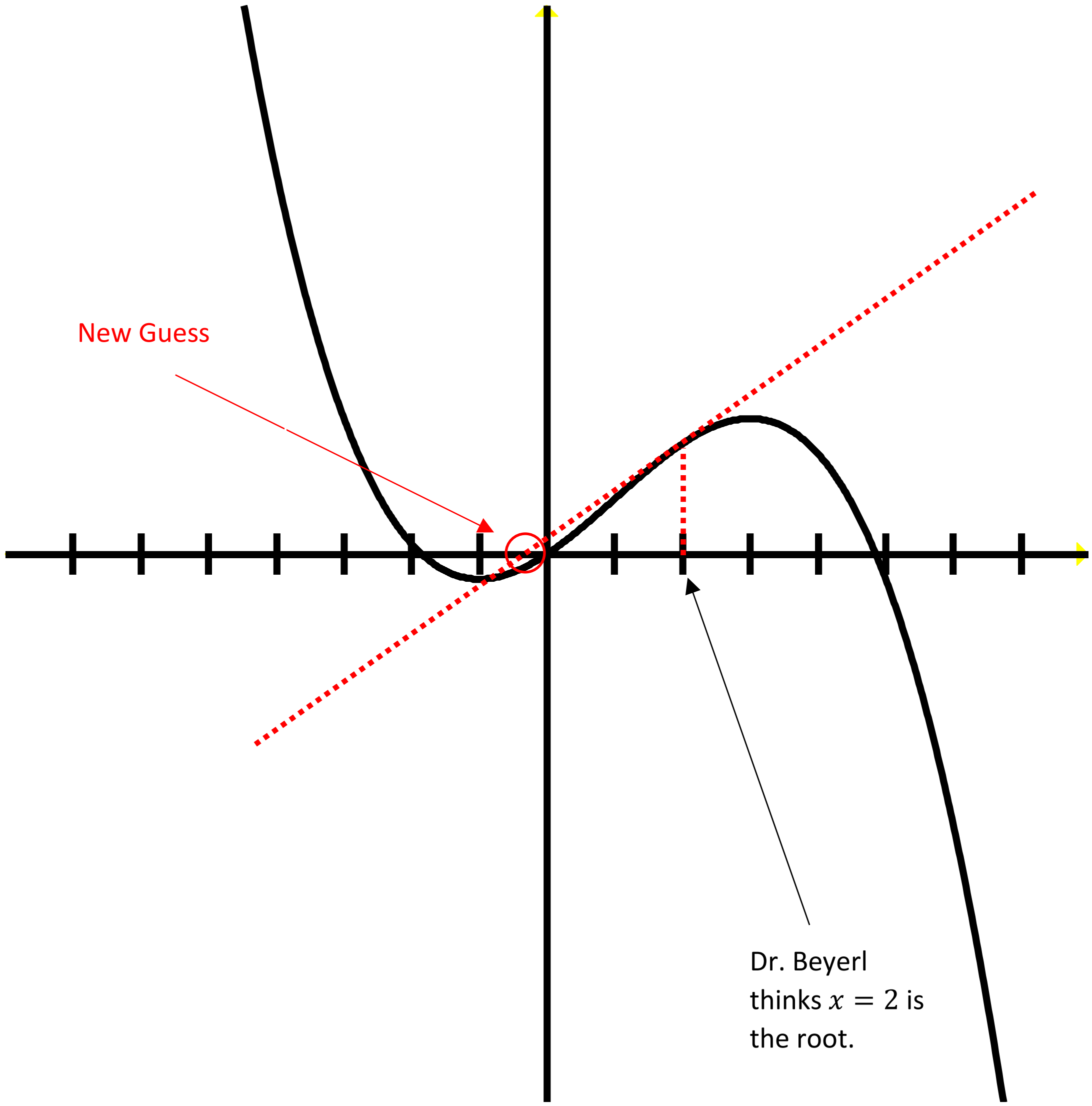
-8

Anything in $[-9, -7]$ was given full credit.

15) limit of $f(x)$ as x goes to infinity. (2 points)

1

16) Below is a graph of a function. We want to know where it has a root. Dr. Beyerl provided the terrible guess as illustrated. Use Newton's method to improve his guess, and illustrate how you got it. (6 points)

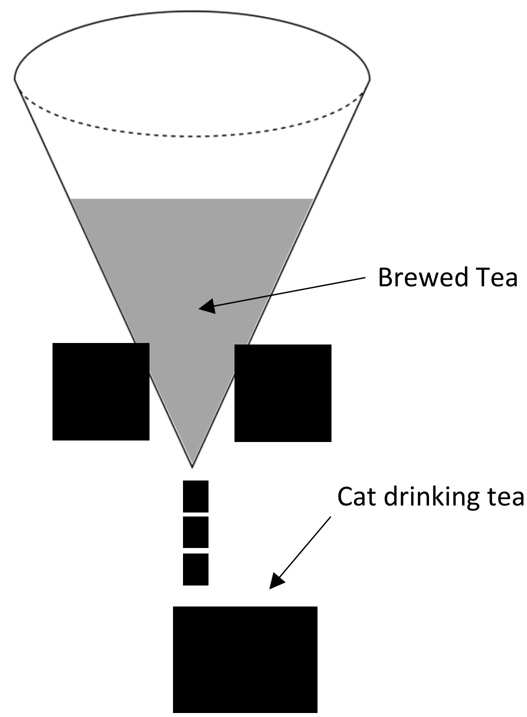


Part 3: Applications

17) The society of Calm-And-Lazy-Cats-Usurping-Lawns-Until-Sundown are holding a secret meeting to decide what lawn to usurp next. Because of the early hour, they require brewed tea. The cat teapot is an inverted cone carefully balanced on its point as illustrated. The height of the cone is four times the radius of the cone. When the height of the tea is 4 centimeters and decreasing at a rate of 1cm/min, how quickly is the tea being poured out the hole in the bottom?

(The volume of a cone is $V = \frac{1}{3}\pi r^2 h$)

(8 points)



Equation

$h = 4r$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h = \frac{\pi h^3}{3 \cdot 4^2}$$

Variables

$h = 24$

$h' = -1$

$V =$ unknown

$V' = ???$

Derivative

$$V' = \frac{3\pi h^2 h'}{3 \cdot 4^2} = \frac{\pi h^2 h'}{4^2}$$

Solution

$$V' = \frac{\pi 4^2(-1)}{4^2} = -\pi\ cm^3/min$$

18) A cat is chasing a mouse. If the mouse starts at the origin and runs away with a velocity function equal to $v(t) = 3t^2 + 2t$ inches per second, how far does the mouse run in the first 2 seconds? (6 points)

$$\int_0^2 3t^2 + 2t\, dt = t^3 + t^2 \Big|_0^2 = 8 + 4 - (0 + 0) = 12 \text{ inches}$$

Part 3: Review

19) Find the limit below. (4 points)

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x - 1} = \lim_{x \to 1} x + 1 = 1 + 1 = 2$$

20) Find the derivaive of the function below. (4 points)

$$f(x) = x^5 \sin(x)$$

$$f'(x) = 5x^4 \sin(x) + x^5 \cos(x)$$

21) Find the derivative of the function below. (4 points)

$$f(x) = \sin(e^{2x} + x^5)$$

$$f'(x) = \cos(e^{2x} + x^5)\,(2e^{2x} + 5x^4)$$