Name ____ Test 3, Fall 2020

Part 1: Computational Skills

1) Find the limit below. (4 points)

$$\lim_{x\to 0}\frac{\sin(4x)}{6}=\frac{\sin(0)}{6}=0$$

2) Find the limit below. (4 points)

$$\lim_{x\to 0}\frac{\sin(4x)}{6x}=\lim_{x\to 0}\frac{4\cos(4x)}{6}=\frac{4}{6}\cos(0)=\frac{4}{6}=\frac{2}{3}$$

3) Find the limit below. (4 points)

$$\lim_{x\to 0}\frac{\sin(4x)}{6x^2}=\lim_{x\to 0}\frac{4\cos(4x)}{12x}\ \text{DNE}$$

Note that from the right it goes to $+\infty$, but from the left it goes to $-\infty$. Hence the limit does not exist.

4) Find the integral below. (4 points)

$$\int \frac{1}{3x+6}dx = \int \frac{1}{u}\frac{du}{3} = \frac{1}{3}\int \frac{1}{u}du = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|3x+6| + C$$

$u = 3x + 6$
$du = 3dx$
$\frac{du}{3} = dx$

5) Find the integral below. (4 points)

$$\int \frac{2x}{x^2+7}dx = \int \frac{2x}{u}\frac{du}{2x} = \int \frac{1}{u}du = \ln|u| + C = \ln|x^2+7| + C$$

$u = x^2 + 7$
$du = 2xdx$
$\frac{du}{2x} = dx$

6) Find the integral below. (4 points)

$$\int \frac{1}{1+4x^2}dx = \int \frac{1}{1+(2x)^2}dx = \int \frac{1}{1+u^2}\frac{du}{2} = \frac{1}{2}\int \frac{1}{1+u^2}du = \frac{1}{2}\tan^{-1}(u) + C = \frac{1}{2}\tan^{-1}(2x) + C$$

$u = 2x$
$du = 2dx$
$\frac{du}{2} = dx$

7) Find the integral below. (4 points)

$$\int \frac{6}{9+x^2}dx = 6\int \frac{1}{9\left(1+\frac{x^2}{9}\right)} = \frac{6}{9}\int \frac{1}{1+\left(\frac{x}{3}\right)^2}dx = \frac{6}{9}\int \frac{1}{1+u^2}\, 3du = \frac{6}{3}\int \frac{1}{1+u^2}du$$
$$= 2\tan^{-1}(u) + C = 2\tan^{-1}\left(\frac{x}{3}\right) + C$$

$u = \frac{x}{3}$
$du = \frac{1}{3}dx$
$3du = dx$

Part 2: Conceptual Understanding

8) Let $f(x)$ be an odd function: that is, it satisfies $f(-x) = -f(x)$. Use this fact to find the integral below (4 points)

$$3\int_{-5}^{5} f(x)dx = 0$$

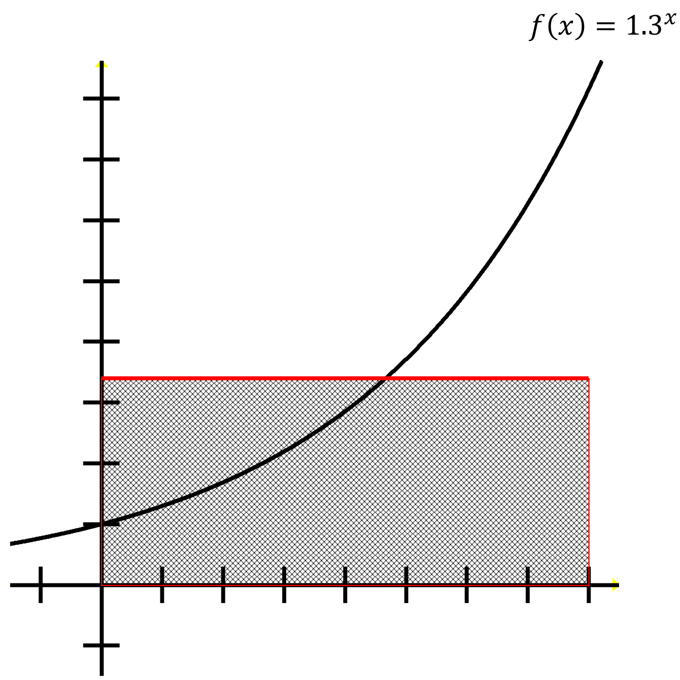
Every odd function on a symmetric domain around the origin has integral 0.

9) Given the table shown, find the limit below. (4 points)

x	$f(x)$
1	43.2
10	36.5
100	35.2
1000	35.08
10000	35.007

$\lim_{x \to \infty} f(x) = 35$

10) The average value of the function $f(x) = 1.3^x$ on the interval $[0,8]$ is 3.4. Illustrate this on the graph below. (4 points)

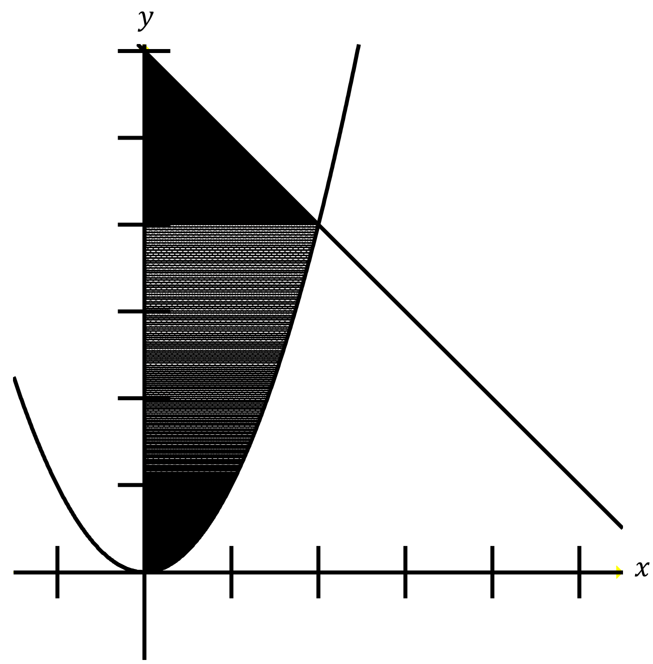


Part 3: Applications

Problems 11-19: for all the integrals in this section, set up but do not evaluate them. *Seriously*, you will not have enough time if you try to compute all these integrals and I don't even know if you'll be able to evaluate them without skills from calculus II. Set them up and do not integrate.

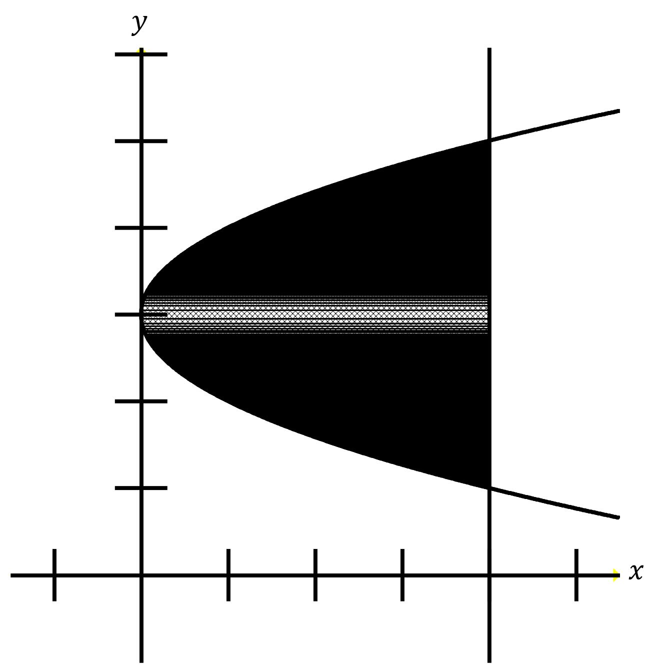
11) Find the area between the two curves $y = 6 - x$ and $y = x^2$, shown here. (4 points)

$\int_0^2 6 - x - x^2 dx$

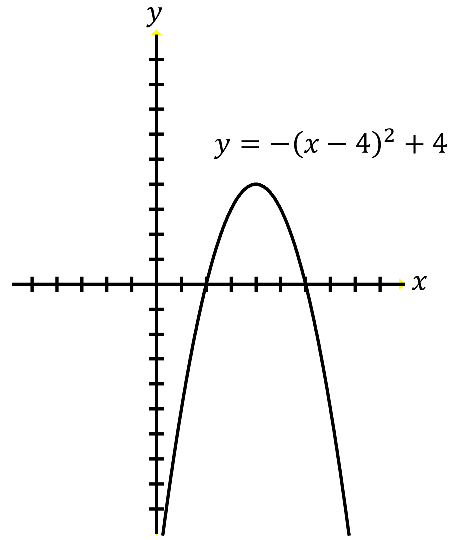


12) Find the area between the two curves $x = (y-3)^2$ and $x = 4$, shown here. (4 points)

$$\int_1^5 4 - (y-3)^2 dy$$



13) The velocity of a bird is given by the function $f(x) = -(x-4)^2 + 4$ shown here. After 6 seconds, how far is it from where it started? (4 points)



$$\int_0^6 -(x-4)^2 + 4dx$$

14) The velocity of a bird is given by the function $f(x) = -(x-4)^2 + 4$ as shown in the previous problem. After 6 seconds, how far did the bird end up traveling? (4 points)

$$-\int_0^2 -(x-4)^2 + 4dx + \int_2^6 -(x-4)^2 + 4dx$$

15) The region illustrated here is rotated around the x-axis. (4 points)

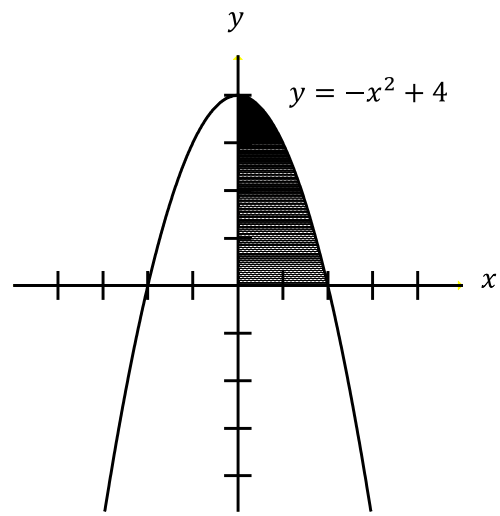
a) Circle the descriptor that best describes the shape.

I. A bump with a rounded top.

II. A bump with a slighlty sharp point.

III. A ring with some flat side(s)

IV. The top of a volcano



b) Find the volume of the shape created.

$$\int_0^2 \pi(-x^2+4)^2 dx$$

(Disk)

$$\int_0^4 2\pi y\sqrt{4-y}dy$$

(Shell)

16) The region illustrated here is rotated around the y-axis. (4 points)

a) Circle the descriptor that best describes the shape.

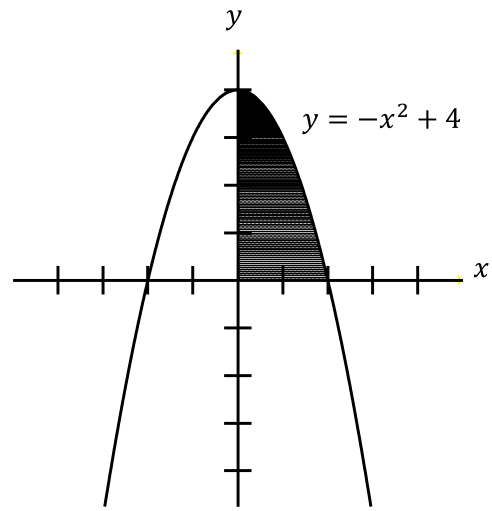
  I. A bump with a rounded top.

  i. A bump with a slighlty sharp point

  ii. A ring with some flat side(s)

  iii. The top of a volcano

b) Find the volume of the shape created.



$$\int_0^2 2\pi x(-x^2+4)dy$$

(Shell)

$$\int_0^4 \pi\left(\sqrt{y-4}\right)^2 dy$$

(Disk)

17) The region illustrated here is rotated around the line $x = -3$. (4 points)

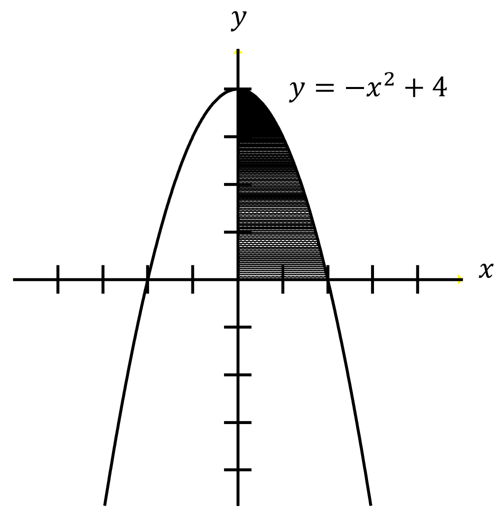
a) Circle the descriptor that best describes the shape.

II. A bump with a rounded top.

i. A bump with a slighlty sharp point.

ii. A ring with some flat side(s)

iii. The top of a volcano



b) Find the volume of the shape created.

$$\int_0^2 2\pi(x+3)(-x^2+4)dx$$

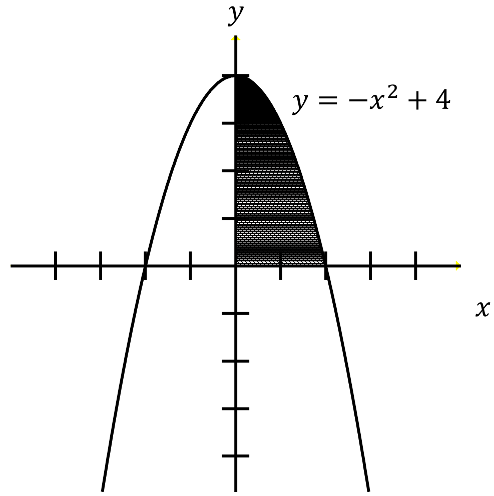
(Shell)

$$\int_0^4 \pi\left(\sqrt{4-y}+3\right)^2 dy - \int_0^4 \pi 3^2 dy$$

(Washer: Two disks subtracted)

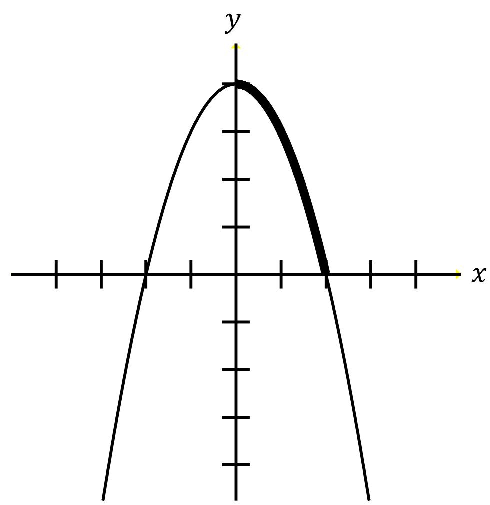
18) The region illustrated here is rotated around the y-axis. (4 points)

a) Find the surface area of the curvy part of the shape created. You may ignore any flat side(s).



$$\int_0^2 2\pi x\sqrt{1+(-2x)^2}\,dx$$

19) Find the arc length of $f(x) = -x^2 + 4$ between $x = 0$ and $x = 2$, as shown here. (4 points)



$$\int_0^2 \sqrt{1+(-2x)^2}\,dx$$

Part 3: Review

20) Find the limit below. (4 points)

$$\lim_{x \to 2^+} \frac{x-4}{x-2} = -\infty$$

21) Find the limit below. (4 points)

$$\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x \to 3} x+3 = 6$$

22) Find the derivaive of the function below. (4 points)

$$f(x) = (x^2+2)^5$$

$$f'(x) = 5(x^2+2)^4 \cdot 2x$$

23) Find the derivaive of the function below. (4 points)

$$f(x) = \sin(x)\, e^x$$

$$\cos(x)\, e^x + \sin(x)\, e^x$$

24) Find the integral below. (4 points)

$$\int_0^2 6x^2 dx = 2x^3\Big|_0^2 = 2 \cdot 2^3 - 0 = 16$$

25) Find the integral below. (4 points)

$$\int_0^{\pi/2} \sin(x)\, dx = -\cos(x)\Big|_0^{\frac{\pi}{2}} = -(0) - (-1) = 1$$

Part 4: Extra Credit Problem

26) Who discovered calculus? (4 bonus points)

Either Newton or Leibniz are acceptable answers.