

Part 1: Computational Skills

1) Find the limit below. (6 points)

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$$

Answer:

2) Find the limit below. (6 points)

$$\lim_{x \rightarrow 3} \frac{2x^2 - 10x + 12}{x - 3}$$

Answer:

3) Find the limit below. (6 points)

$$\lim_{x \rightarrow 1} f(x)$$

$$f(x) = \begin{cases} \frac{x^2 + x - 2}{x - 1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

Answer:

4) Find the derivative of the function below. (6 points)

$$f(x) = x^3 e^x$$

Answer:

5) Find the derivative of the function below. (6 points)

$$y = |\sin(2^x)|$$

Answer:

6) Find the derivative of the function below. (6 points)

(Caution! Tricky question)

$$\frac{d}{dx} x^5$$

Answer:

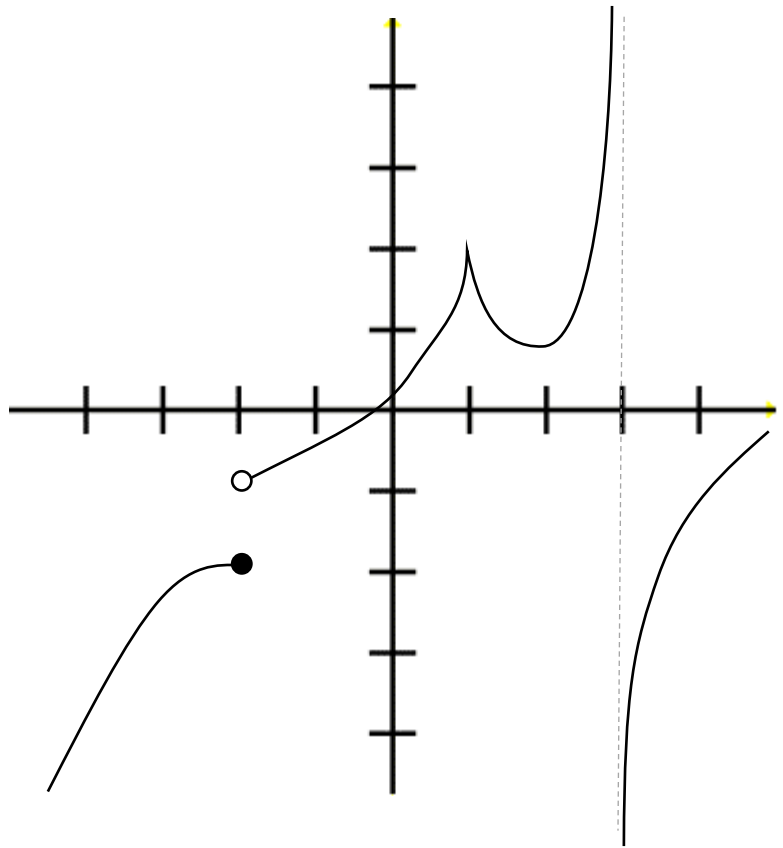
7) Find the limits below based on the graph. (2 points each)

$$\lim_{x \rightarrow -2^-} f(x)$$

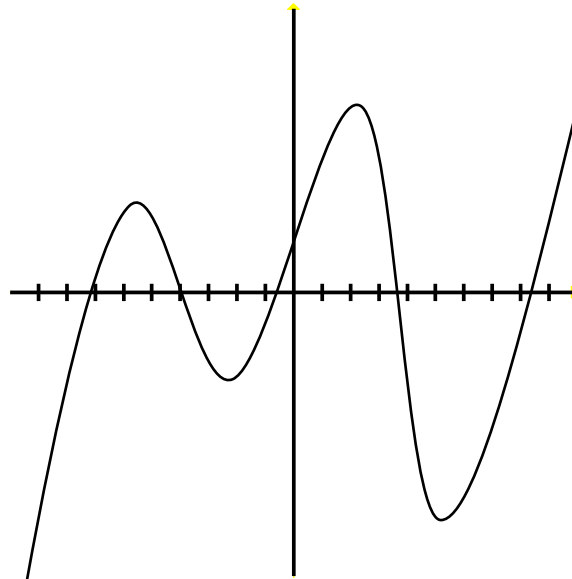
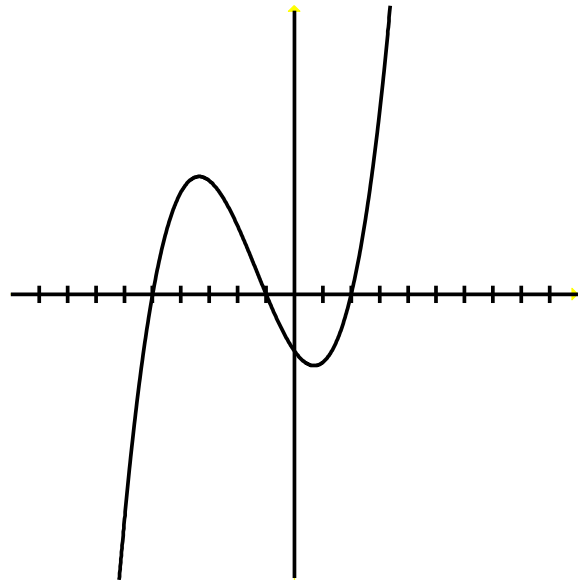
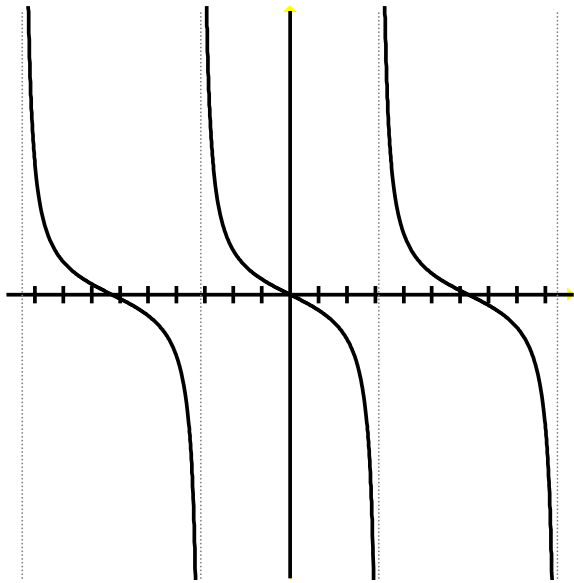
$$\lim_{x \rightarrow 1} f(x)$$

$$\lim_{x \rightarrow 3^+} f(x)$$

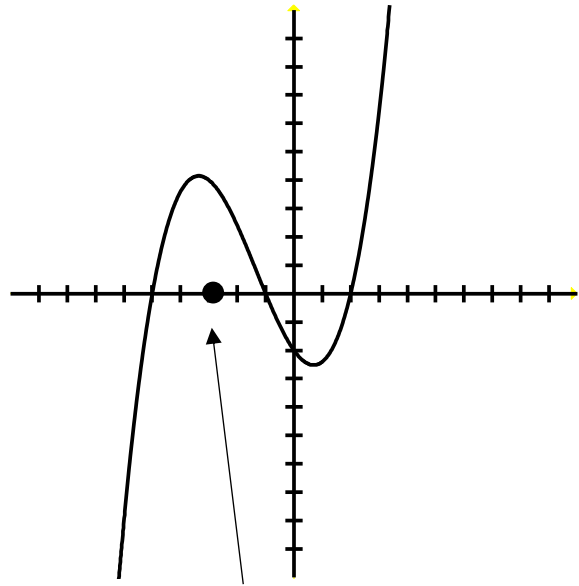
$$\lim_{x \rightarrow -\infty} f(x)$$



8) Sketch the derivative of each function shown in these graphs. (4 points each)



9) Use Newton's method to illustrate an "improved estimate" of a root of the function below. (4 points)



Dr. Beyerl's original guess

Part 2: Conceptual Understanding

10) Which mathematical concept best describes this scenario? In order to drive from Arkansas to Alaska, you must pass through Canada. (2 points)

- (A) A Limit
- (B) A Derivative
- (C) Continuity
- (D) Piecewise Functions
- (E) A Horizontal Tangent
- (F) The Squeeze Theorem
- (G) The Intermediate Value Theorem

11) Which mathematical concept best describes this scenario? When you throw a stone in the air, it pauses ever so briefly before changing directions to fall to the ground. (2 points)

- (A) A Limit
- (B) A Derivative
- (C) Continuity
- (D) Piecewise Functions
- (E) A Horizontal Tangent
- (F) The Squeeze Theorem
- (G) The Intermediate Value Theorem

12) Which mathematical concept best describes this scenario? If your next electric bill is going to be at least \$100, but also simultaneously at most \$100, then it is going to be exactly \$100. (2 points)

- (A) A Limit
- (B) A Derivative
- (C) Continuity
- (D) Piecewise Functions
- (E) A Horizontal Tangent
- (F) The Squeeze Theorem
- (G) The Intermediate Value Theorem

13) Find the value of a that makes the function below continuous. (3 points)

$$f(x) = \begin{cases} 2x + a & \text{if } x < 2 \\ 3x & \text{if } x \geq 2 \end{cases}$$

14) Find the value of a that makes the function below differentiable. (3 points)

$$f(x) = \begin{cases} ax^2 + 4 & \text{if } x < 1 \\ 6x + 4 & \text{if } x \geq 1 \end{cases}$$

15) Find the limit below and show your work. (9 points)

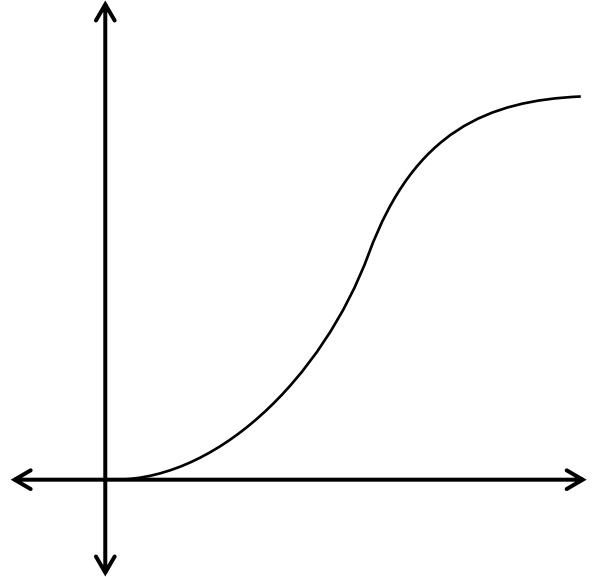
$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}$$

16) Given $f(x)$ below, find $f'(\pi)$, simplify, and show your work. (9 points)

$$f(x) = x \sin(x)$$

Part 3: Applications

17) A car accelerates from a red light according to the velocity function shown below. At what point do the occupants feel the acceleration the most? Circle that point on the graph. (5 points)



18) Which of the following best describes the growth rate of the population in the year 1985? (5 points)

Year	Global Population (Billions)
1970	3.7
1980	4.5
1990	5.2
2000	6

- (A) 4.5
- (B) 5.2
- (C) $\frac{5.2-4.5}{10}$
- (D) $\lim_{h \rightarrow 0} \frac{x-4.5}{h}$