Name _____ Test 1, Spring 2022

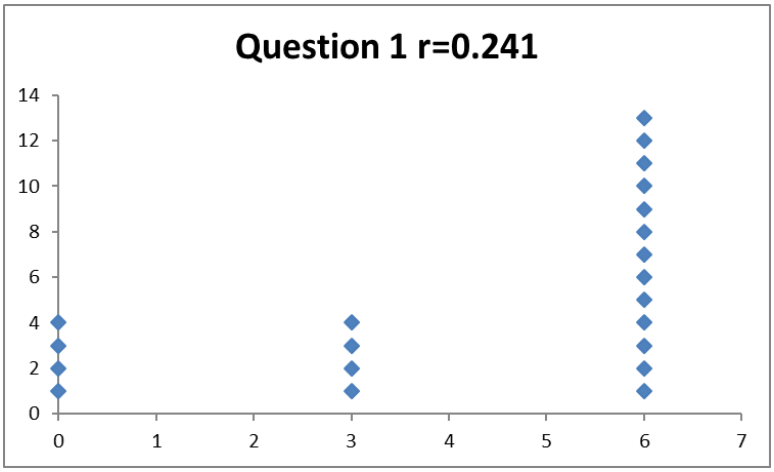
Part 1: Computational Skills

1) Find the limit below. (6 points)

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$$

Answer: $\boxed{\frac{1}{6}}$

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3}=\lim_{x\to 9}\frac{x-9}{(x-9)\left(\sqrt{x}+3\right)}=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{6}$$

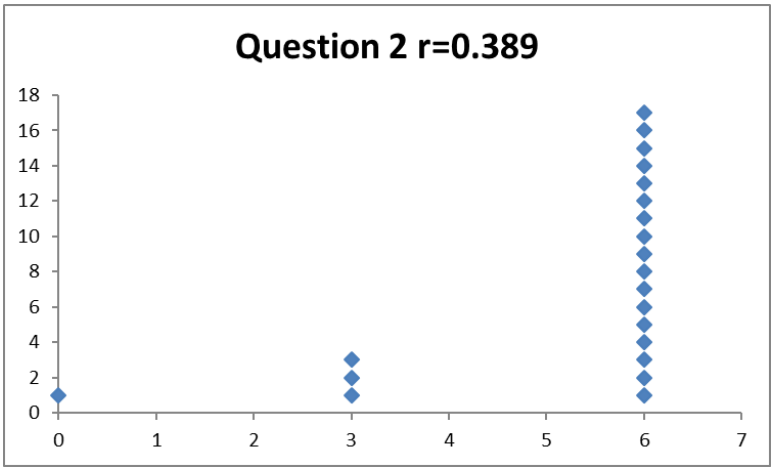


2) Find the limit below. (6 points)

$$\lim_{x\to 3}\frac{2x^2-10x+12}{x-3}$$

Answer: $\boxed{2}$

$$\lim_{x\to 3}\frac{2x^2-10x+12}{x-3}=\lim_{x\to 3}\frac{2(x^2-5x+6)}{x-3}=\lim_{x\to 3}\frac{2(x-3)(x-2)}{x-3}=\lim_{x\to 3}2(x-2)=2\cdot 1=2$$



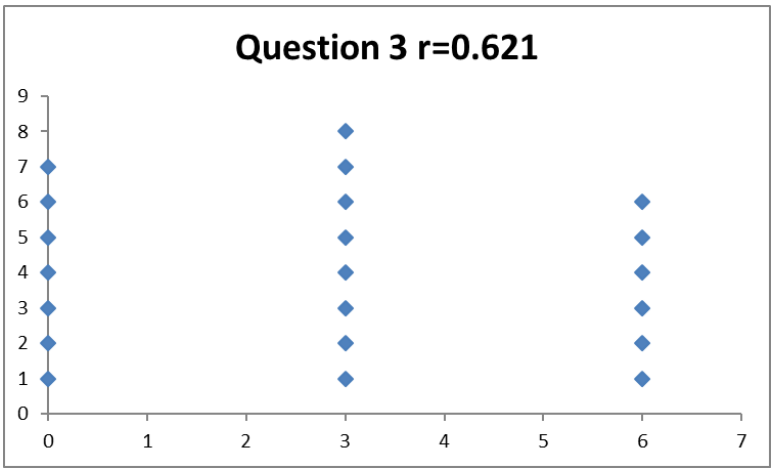
3) Find the limit below. (6 points)

$$\lim_{x\to 1} f(x)$$

$$f(x) = \begin{cases} \dfrac{x^2+x-2}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

Answer: 3

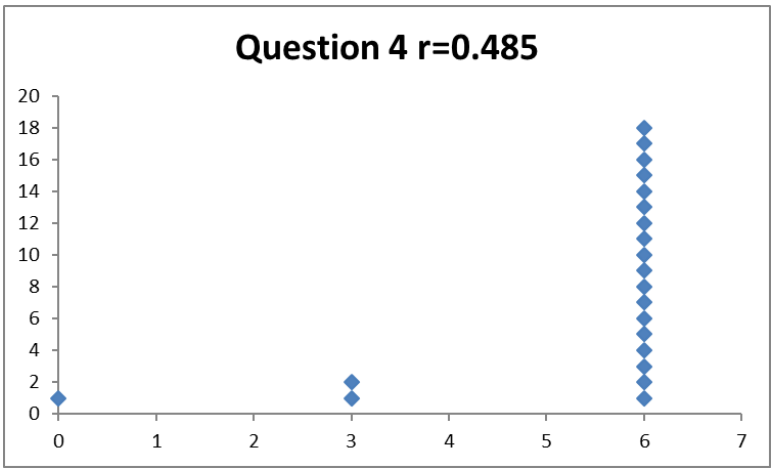
$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2+x-2}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+2)}{x-1} = \lim_{x\to 1} (x+2) = 3$$



4) Find the derivative of the function below. (6 points)

$$f(x) = x^3 e^x$$

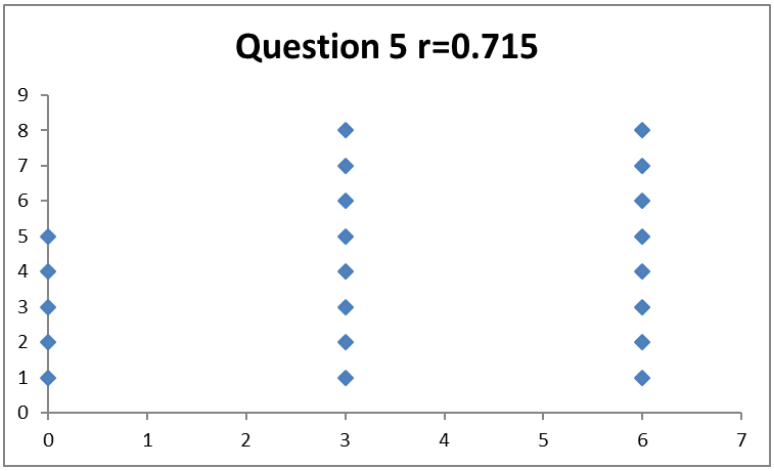
Answer: $\boxed{3x^2e^x + x^3e^x}$



5) Find the derivative of the function below. (6 points)

$$y = |\sin(2^x)|$$

Answer: $\boxed{\frac{|\sin(2^x)|}{\sin(2^x)}\cos(2^x)\,2^x\ln(2)}$

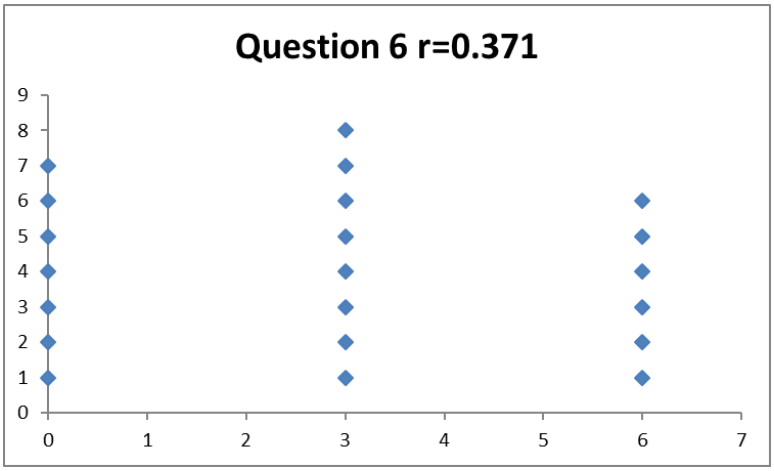


6) Find the derivative of the function below. (6 points)
(Caution! Tricky question)

$$\frac{d}{dx}x^5$$

Answer: $\boxed{20x^3}$

$$\frac{d}{dx}x^5 = 5x^4$$



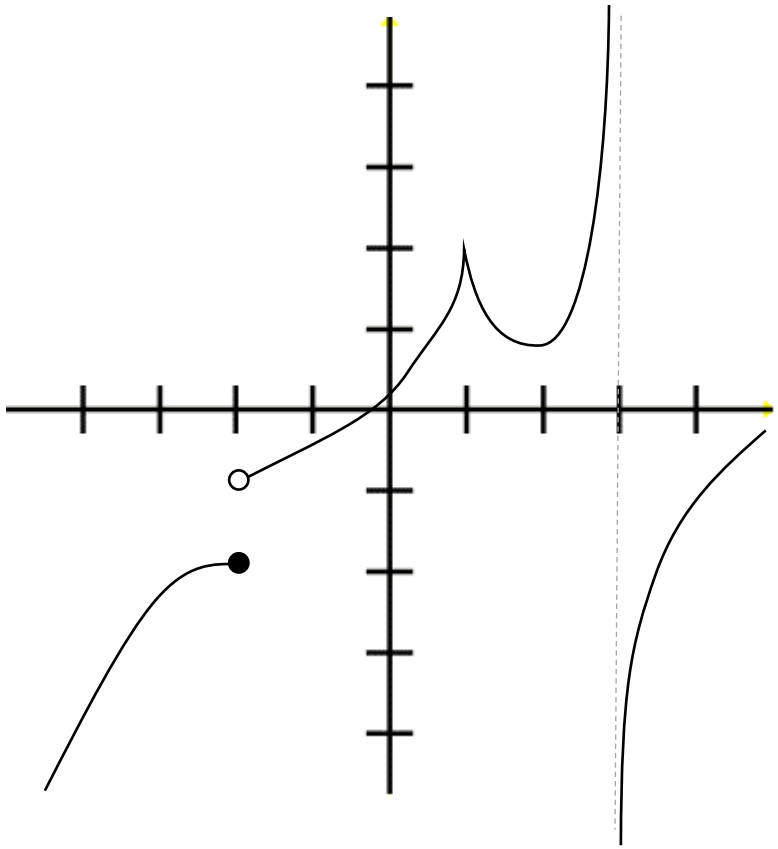
7) Find the limits below based on the graph. (2 points each)

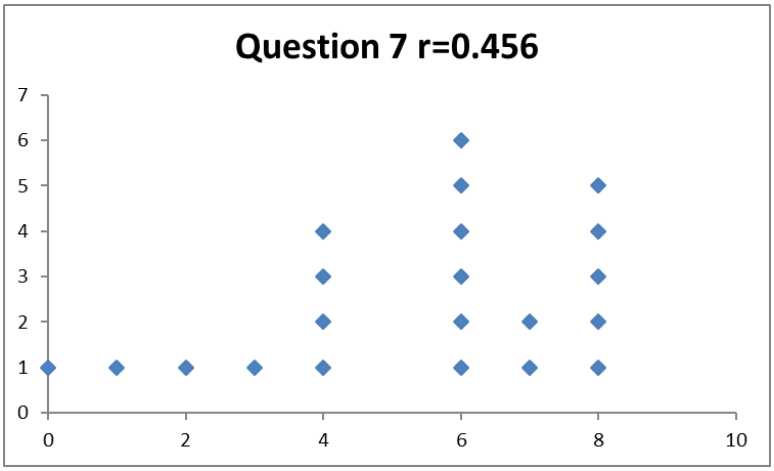
$\lim_{x\to -2^-} f(x) = -2$

$\lim_{x\to 1} f(x) = 2$

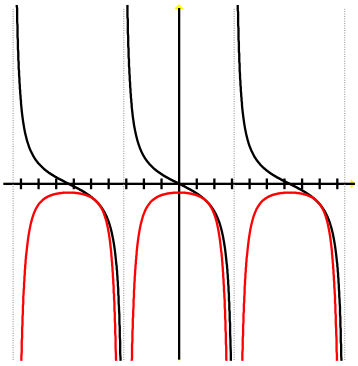
$\lim_{x\to 3^+} f(x) = -\infty$

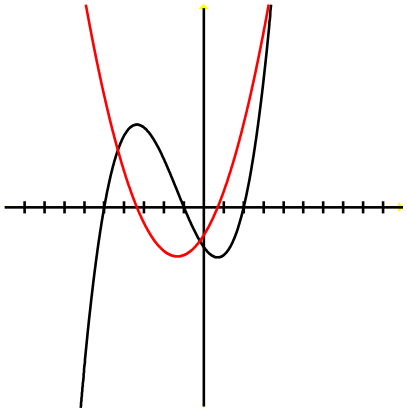
$\lim_{x\to -\infty} f(x) = -\infty$

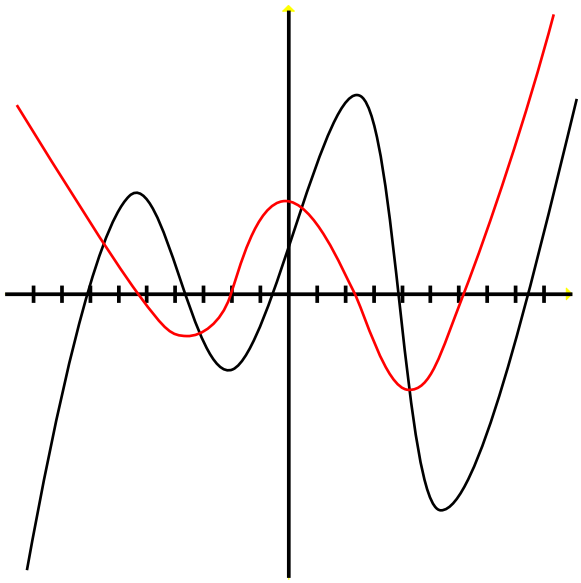


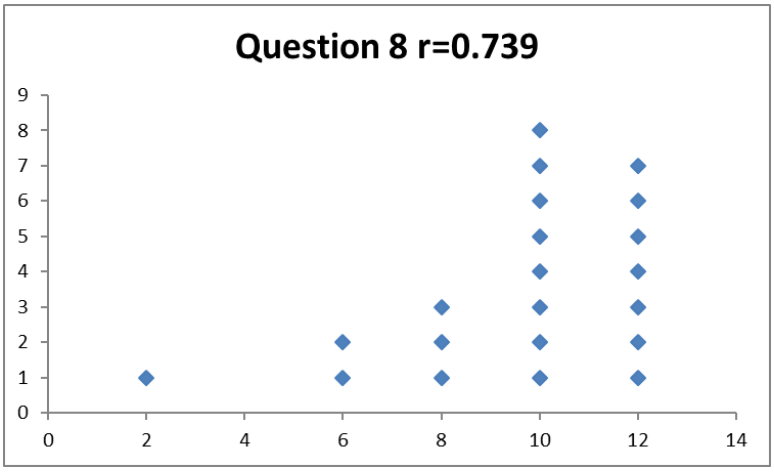


8) Sketch the derivative of each function shown in these graphs. (4 points each)

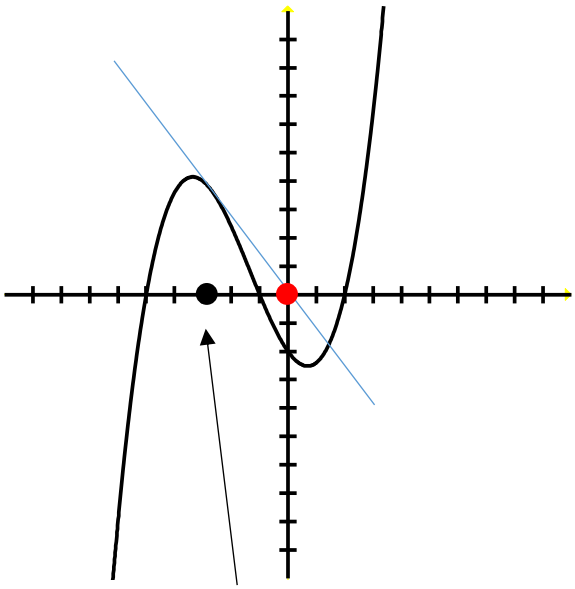




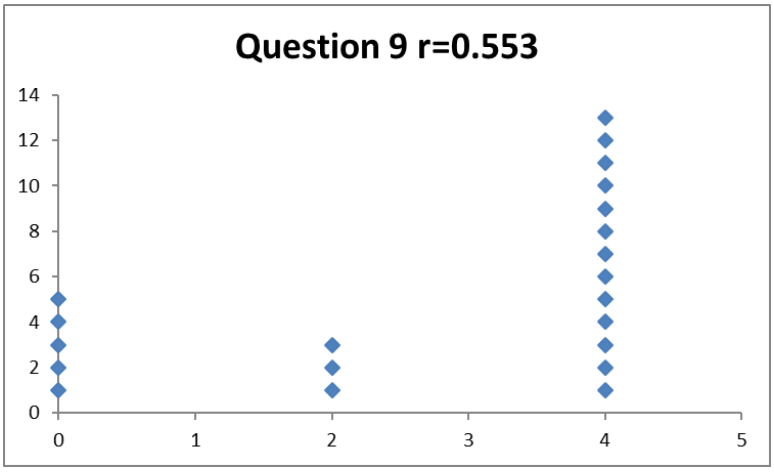




9) Use Newton's method to illustrate an "improved estimate" of a root of the function below. (4 points)



Dr. Beyerl's original guess

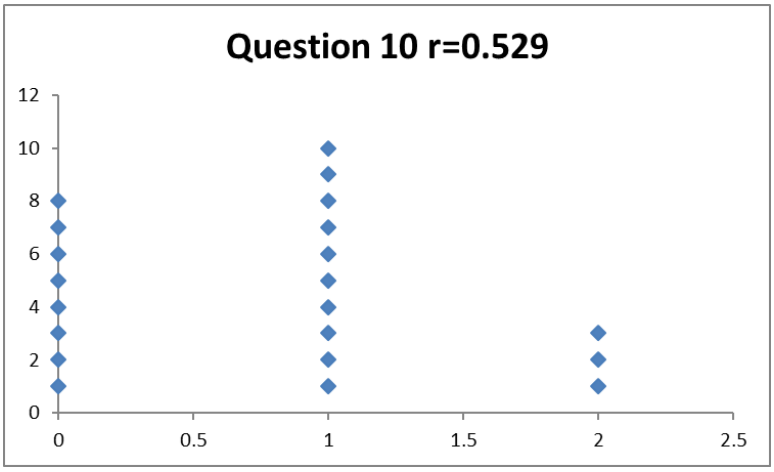


Part 2: Conceptual Understanding

10) Which mathematical concept best describes this scenario? In order to drive from Arkansas to Alaska, you must past through Canada. (2 points)

(A) A Limit
(B) A Derivative
(C) Continuity
(D) Piecewise Functions
(E) A Horizontal Tangent
(F) The Squeeze Theorem
(G) The Intermediate Value Theorem

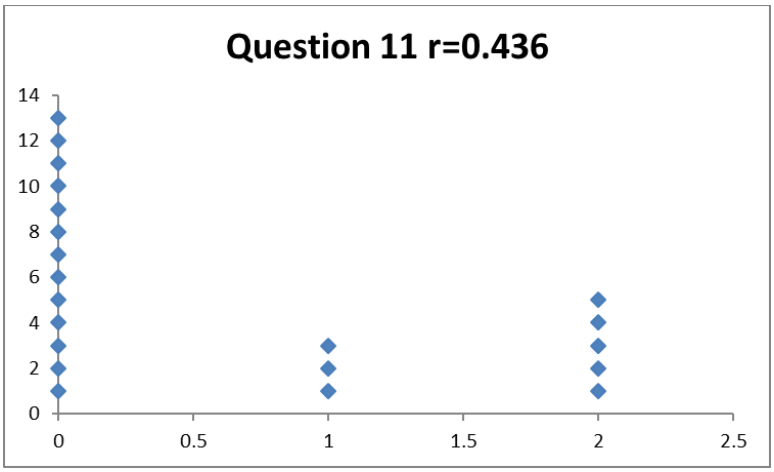
Half credit: Continuity



11) Which mathematical concept best describes this scenario? When you throw a stone in the air, it pauses ever so briefly before changing directions to fall to the ground. (2 points)

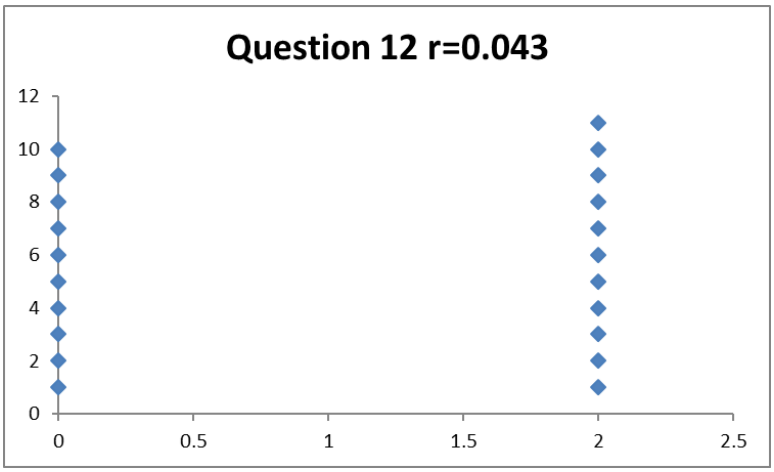
(A) A Limit
(B) A Derivative
(C) Continuity
(D) Piecewise Functions
(E) A Horizontal Tangent
(F) The Squeeze Theorem
(G) The Intermediate Value Theorem

Half credit: A Derivative



12) Which mathematical concept best describes this scenario? If your next electric bill is going to be at least $100, but also simultaneously at most $100, then it is going to be exactly $100. (2 points)

(A) A Limit
(B) A Derivative
(C) Continuity
(D) Piecewise Functions
(E) A Horizontal Tangent
(F) The Squeeze Theorem
(G) The Intermediate Value Theorem



13) Find the value of a that makes the function below continuous. (3 points)

$$f(x) = \begin{cases} 2x + a & \text{if } x < 2 \\ 3x & \text{if } x \geq 2 \end{cases}$$

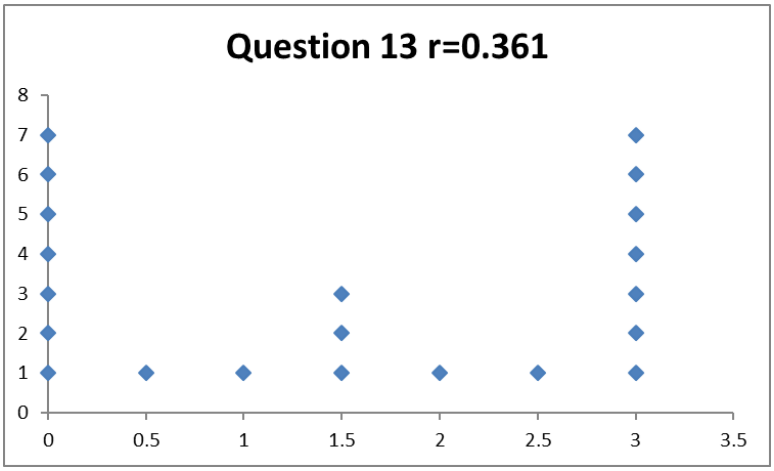
We need:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2^-} 2x + a = \lim_{x \to 2^+} 3x$$

$$4 + a = 6$$

$$a = 2$$



14) Find the value of a that makes the function below differentiable. (3 points)

$$f(x) = \begin{cases} ax^2 + 4 \text{ if } x < 1 \\ 6x + 4 \quad \text{if } x \geq 1 \end{cases}$$

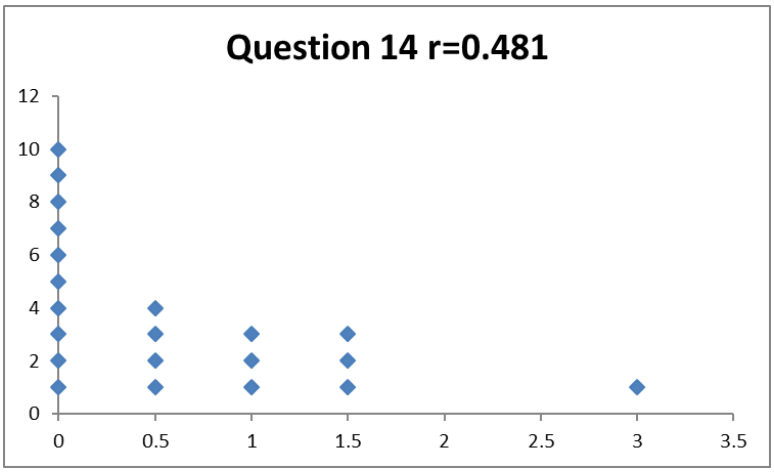
We need:

[Derivative from left] = [derivative from right]

$$\lim_{x \to 1^-} 2ax = \lim_{x \to 1^+} 6$$

$$2a = 6$$

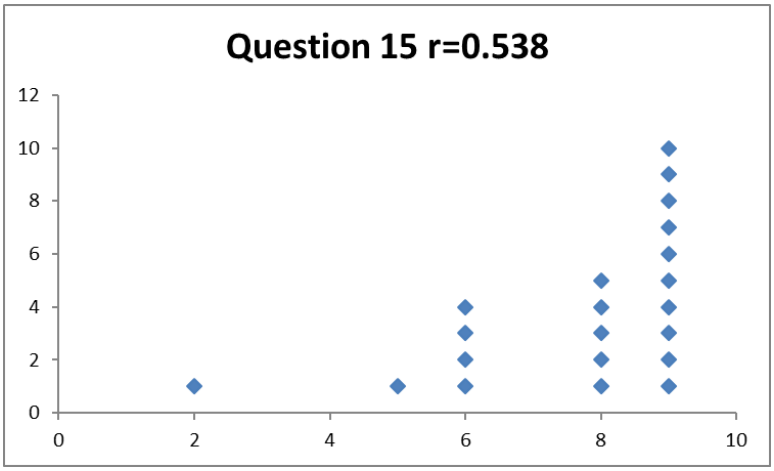
$$a = 3$$



15) Find the limit below and show your work. (9 points)

$$\lim_{x\to 2}\frac{x-2}{x^2-4}$$

$$\lim_{x\to 2}\frac{x-2}{x^2-4}=\lim_{x\to 2}\frac{x-2}{(x-2)(x+2)}=\lim_{x\to 2}\frac{1}{(x+2)}=\frac{1}{4}$$

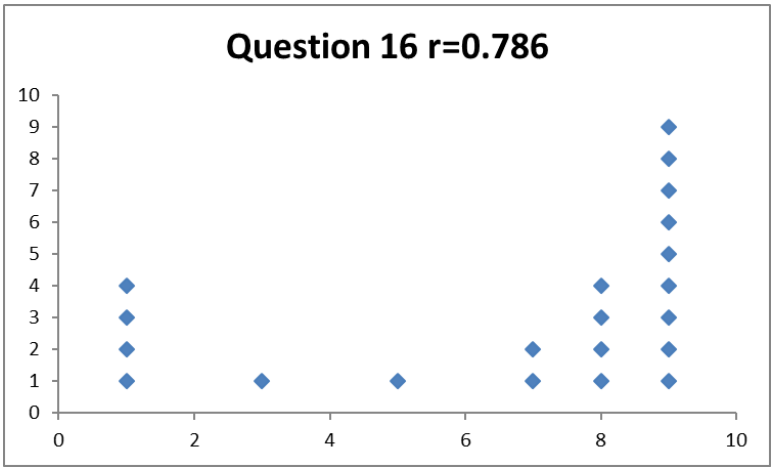


16) Given $f(x)$ below, find $f'(\pi)$, simplify, and show your work. (9 points)

$$f(x) = x\sin(x)$$

$f'(x) = \sin(x) + x\cos(x)$

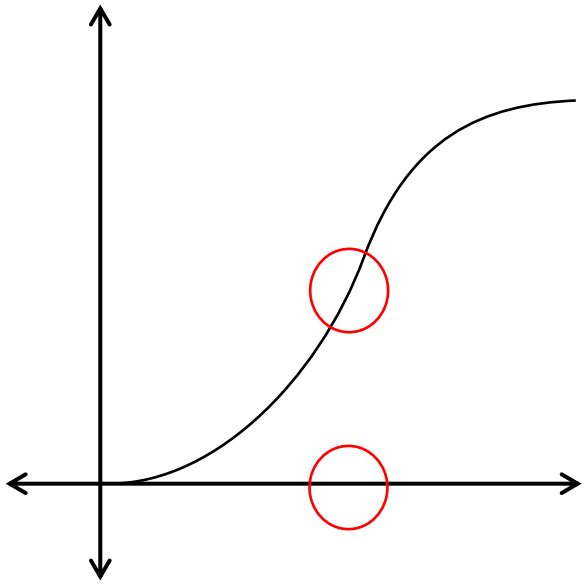
$f'(\pi) = \sin(\pi) + \pi\cos(\pi) = 0 - \pi = -\pi$

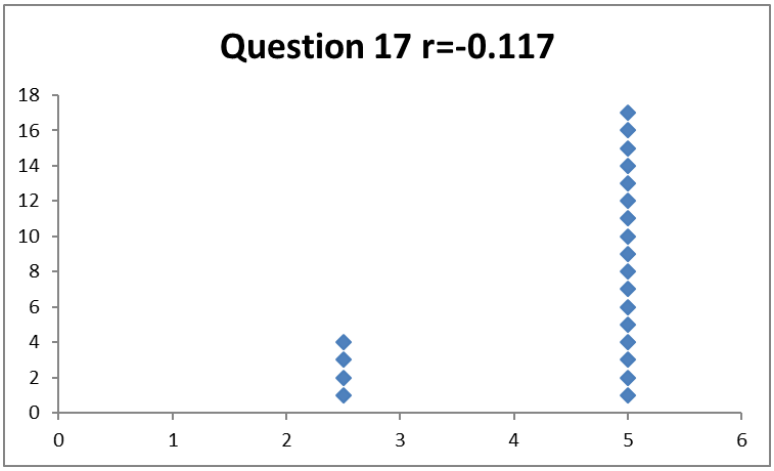


Part 3: Applications

17) A car accelerates from a red light according to the velocity function shown below. At what point do the occupants feel the acceleration the most? Circle that point on the graph. (5 points)

Either the point on the graph, or the x-value would work.



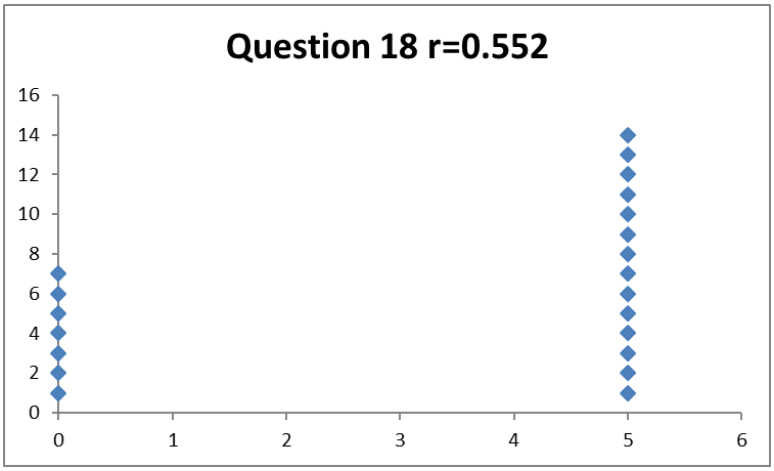


18) Which of the following best describes the growth rate of the population in the year 1985? (5 points)

Year	Global Population (Billions)
1970	3.7
1980	4.5
1990	5.2
2000	6

(A) 4.5
(B) 5.2
(C) $\frac{5.2-4.5}{10}$
(D) $\lim_{h\to 0} \frac{x-4.5}{h}$

Note that (D) makes absolutely no sense. Shame on anyone that answered that just because it had some calculus symbols. Think, think, think!



Study Note: We can't cover everything on a test. I tried to get to the most important parts, but note that this test did skip a few topics that are still fair game that you need to know:

- Discontinuities & their types
- Limits from tables
- Asymptotes
- Limits at infinity
- Infinite limits
- Estimate the derivative at a point
- Identify where a function is not differentiable
- Implicit differentiation
- Derivatives of logs
- Quotient rule
- Derivatives of inverse functions
- Logarithmic differentiation
- Multiple derivatives that follow a pattern
- Others?