Name ____ Test 2, Spring 2022

Part 1: Computational Skills

1) Given the function below, find all minimizers. (6 points)

$$f(x) = 3x^4 - 20x^3 + 36x^2 + 4$$

Answer: $\boxed{}$

2) Given the function below, find all maximizers. (6 points)

Hint: $\cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta)$

$$f(x) = \sin(x)\cos(x)$$
$$x \in [0, \pi]$$

Answer: $\boxed{}$

3) Find the general antiderivative of $f(x) = 3x^4 - \frac{2}{x^3}$. (6 points)

Answer: $\boxed{}$

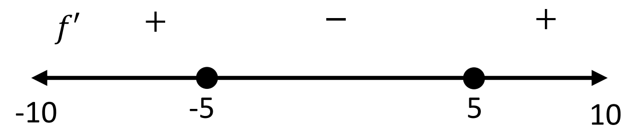
4) Find the antiderivative of $f(x) = e^{2x}$ such that $F(0) = 4$. (6 points)

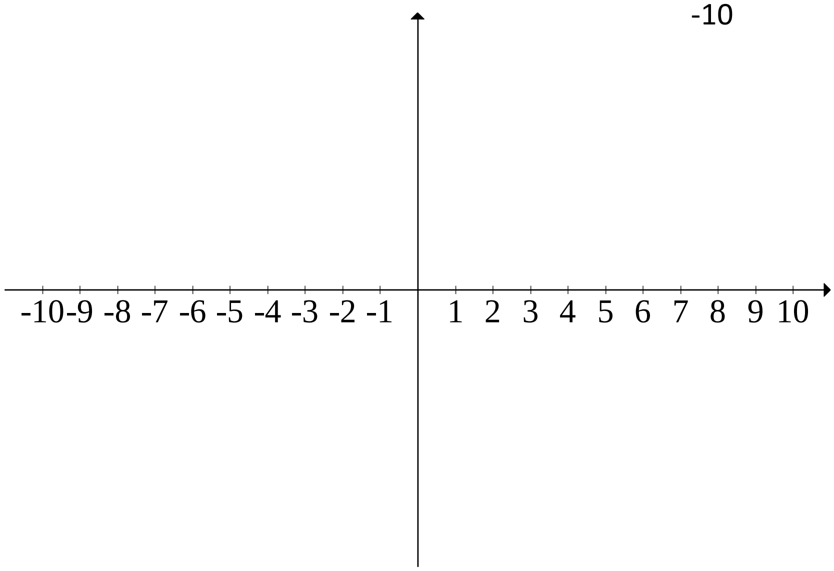
Answer: $\boxed{}$

5) A magic cube is growing at a rate of $3\text{in}^3/\text{s}$. When the cube is $4\text{in} \times 4\text{in} \times 4\text{in}$, how quickly are the side lengths increasing? (6 points)

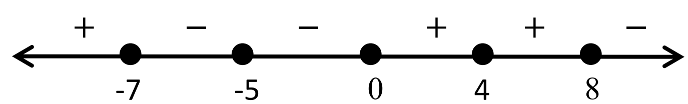
Answer: $\boxed{}$

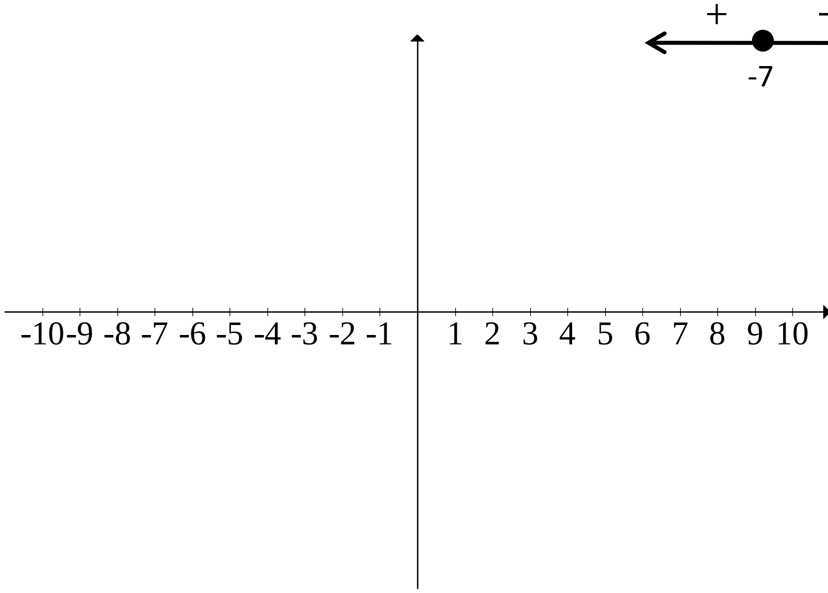
6) Graph a continuous and differentiable function that has the sign chart below with inflection points at $x = 0$ and $x = 7.5$. (6 points





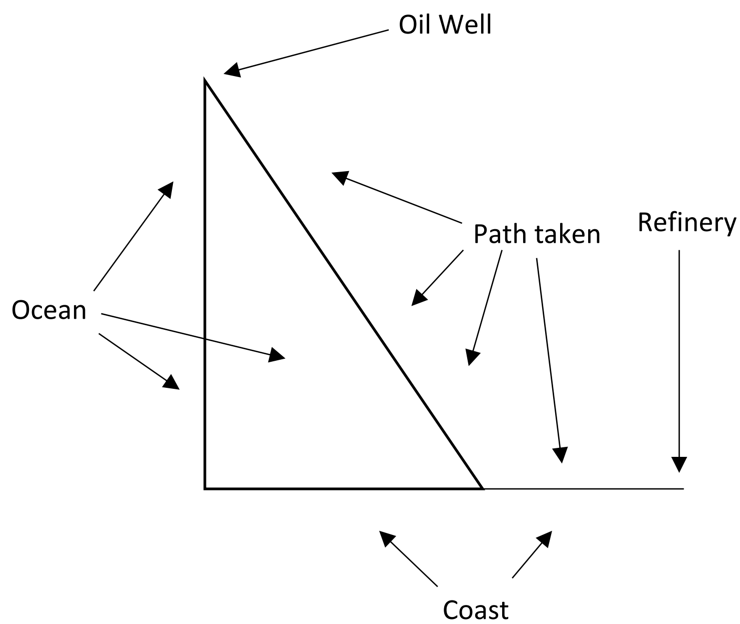
7) Graph a continuous and differentiable function that has the sign chart given below. (6 points)





8) An offshore oil well is 5 kilometers off the coast. The refinery is 6 kilometers down the coast. Laying pipe in the ocean is twice as expensive as on land. How much of the path taken should be on land in order to minimize the cost? (6 points)

Answer:



Part 2: Conceptual Understanding

9) Which theorem or mathematical concept *best* embodies the idea that when considering differentiable functions, instantaneous change must sometimes be equal to average rate of change? (2 points)

(A) A Limit
(B) A Derivative
(C) Piecewise Functions
(D) The Squeeze Theorem
(E) The Intermediate Value Theorem
(F) Rolle's Theorem
(G) The Mean Value Theorem
(H) The Extreme Value Theorem

10) Which theorem or mathematical concept *best* embodies the idea that some functions make a sort of instantaneous jump where they had one type of y-values and suddenly they have much larger or smaller y-values? (2 points)

(A) A Limit
(B) A Derivative
(C) Piecewise Functions
(D) The Squeeze Theorem
(E) The Intermediate Value Theorem
(F) Rolle's Theorem
(G) The Mean Value Theorem
(H) The Extreme Value Theorem

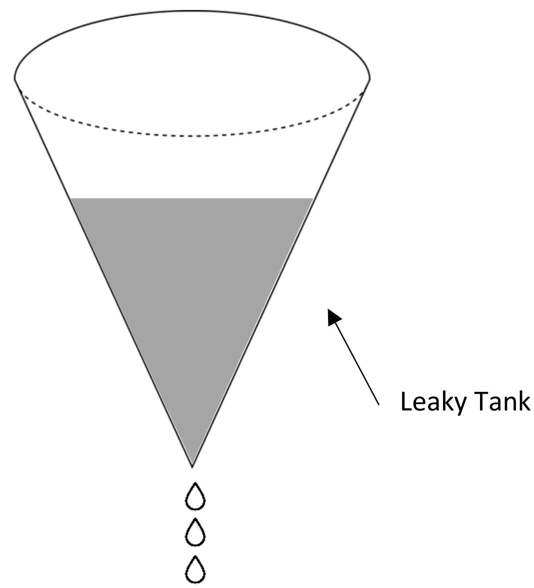
11) Calculus 1 involves three big concepts. We've covered two of them so far, with the third to start after spring break. What are the two big concepts we've covered so far? (1 point)
(Circle two items)

(A) Limit
(B) Infinity
(C) Derivatives
(D) Related Rates

12) Find the antiderivative of $f(x) = 3x^2 + 4x$ such that $F(1) = 5$. Simplify and show your work.
(9 points)

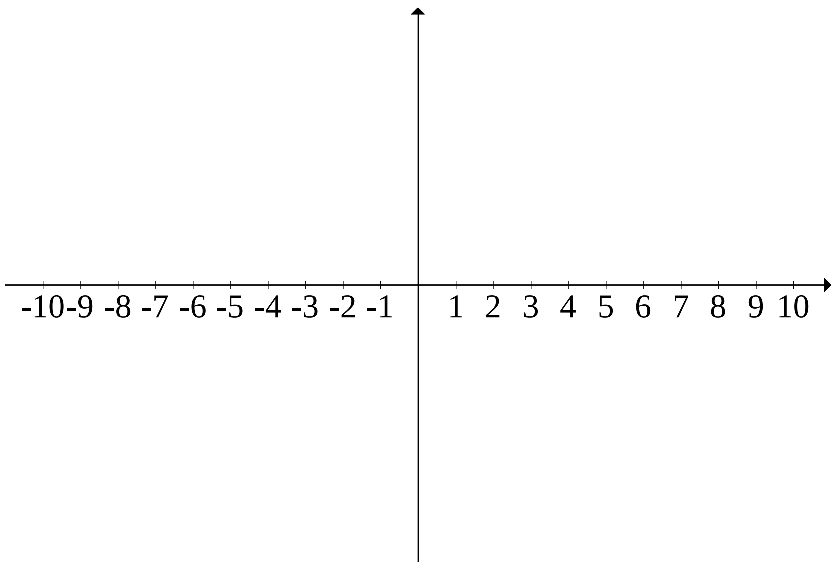
13) Find the maximum of $f(x) = 2x^3 + 3x^2 - 36x$ using the second-derivative test. Show your work.
(9 points)

14) An inverted conical tank is used to hold gasoline. However, unfortunately it leaks. The height of the tank is four times the radius of the cone. When the height of the gas is 4 feet and decreasing at a rate of 1ft/min, how quickly is the gas leaking out the hole in the bottom?
(9 points)



Part 3: Applications

15) Draw an example of a continuous function that has 3 local maxima, but whose derivative is never zero.



16) Let $f(t)$ be a position function. The object it represents is accelerating, but accelerating less quickly over time. For a brief instantaneous moment while accelerating, its position pauses. Which of the following apply at this moment? Circle all that apply. (5 points)
(Circle three items)

(A) $f(t) > 0$	(D) $f'(t) > 0$	(G) $f''(t) > 0$	(J) $f'''(t) > 0$
(B) $f(t) < 0$	(E) $f'(t) < 0$	(H) $f''(t) < 0$	(K) $f'''(t) < 0$
(C) $f(t) = 0$	(F) $f'(t) = 0$	(I) $f''(t) = 0$	(L) $f'''(t) = 0$

Part 4: Review

17) Find the derivative of the function below. (5 points)

$$f(x) = xe^{2x}$$

Answer:

18) Find the limit below. (5 points)

$$\lim_{x\to 4^+} \frac{x-5}{(x-2)(x-4)}$$

Answer: