Name ____ Test 3, Spring 2022

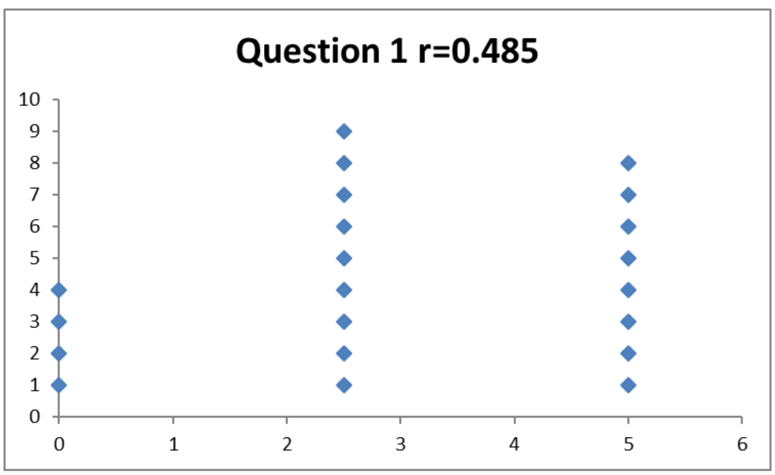
Part 1: Computational Skills

1) Evaluate. (5 points)

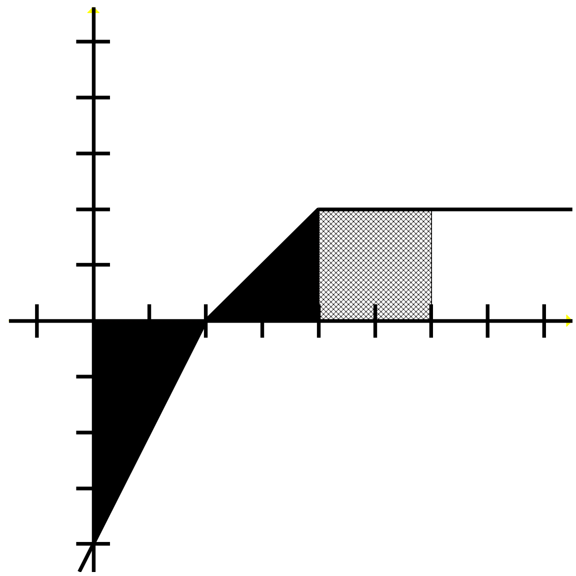
$$\sum_{j=2}^{5} 2j$$

Answer: $\boxed{28}$

$$\sum_{j=2}^{5} 2j = 4 + 6 + 8 + 10 = 28$$

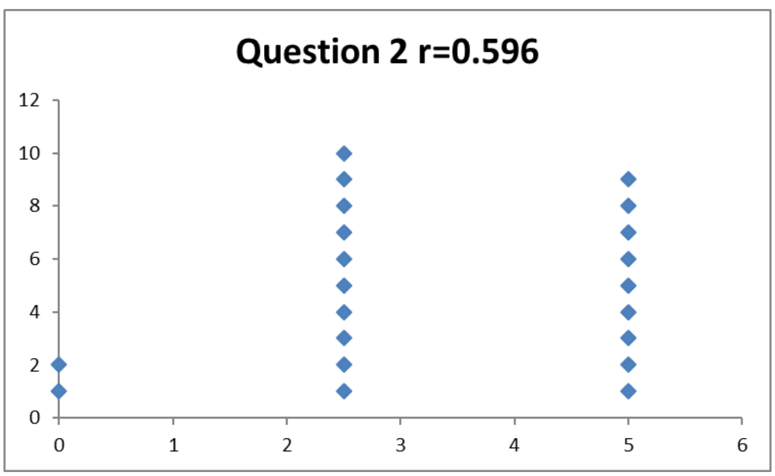


2) Given the graph below, find $\int_0^6 f(x)dx$. (5 points)



Answer: $\boxed{2}$

$-\frac{1}{2}\cdot 4\cdot 2+\frac{1}{2}\cdot 2\cdot 2+2\cdot 2=-4+2+4=2$



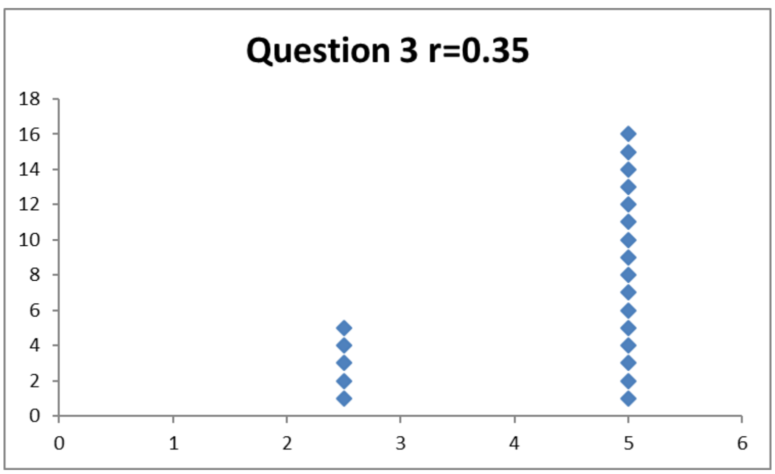
3) Evaluate. (5 points)

$$\int 2x(x^2+4)^3 dx$$

Answer: $\boxed{\frac{(x^2+4)^4}{4}+C}$

$u = x^2 + 4$
$du = 2xdx$

$$\int 2x(x^2+4)^3 dx = \int u^3 du = \frac{u^4}{4} + C = \frac{(x^2+4)^4}{4} + C$$



4) Evaluate. (5 points)

$$\int \frac{1}{2x+4}\,dx$$

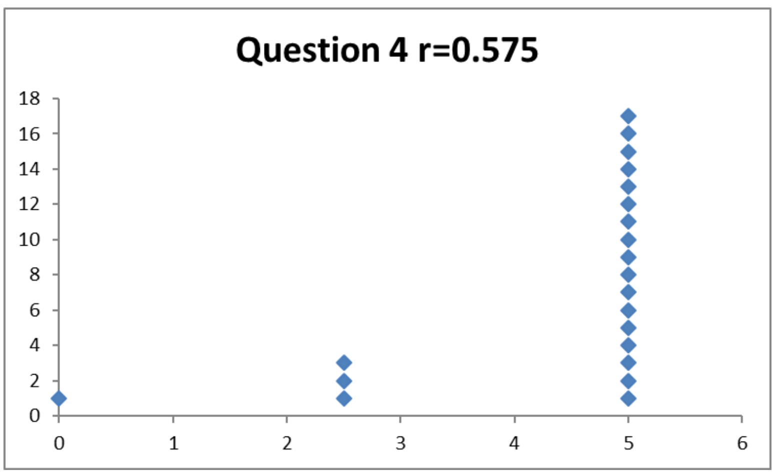
Answer: $\boxed{\frac{\ln|2x+4|}{2} + C}$

$u = 2x + 4$

$du = 2dx$

$\frac{du}{2} = dx$

$$\int \frac{1}{2x+4}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{\ln|2x+4|}{2} + C$$

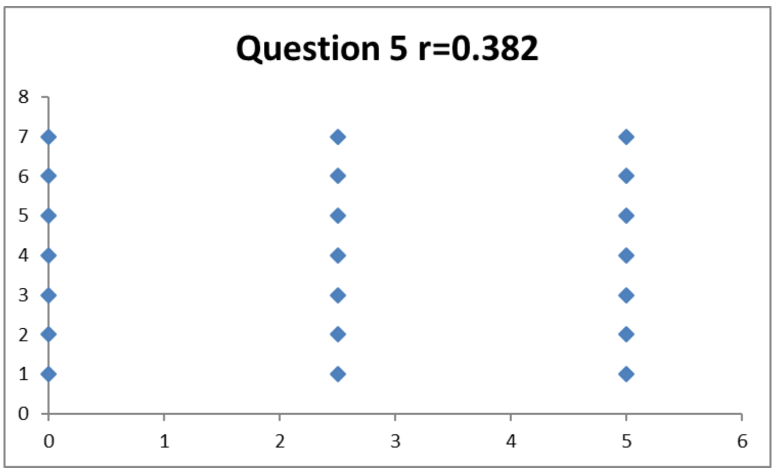


5) Evaluate. (5 points)

$$\frac{d}{dt}\int_0^t \frac{x^4+5x}{x^2-3}\,dx$$

Answer: $\boxed{\frac{t^4+5t}{t^2-3}}$

The above answer is what I intended … but the question is kind of foobar'd because of the t/x typo. Thus 0 was also given full credit, because if t and x are independent, with no t's, whatever that expressions means, it's derivative with respect to t would be 0.



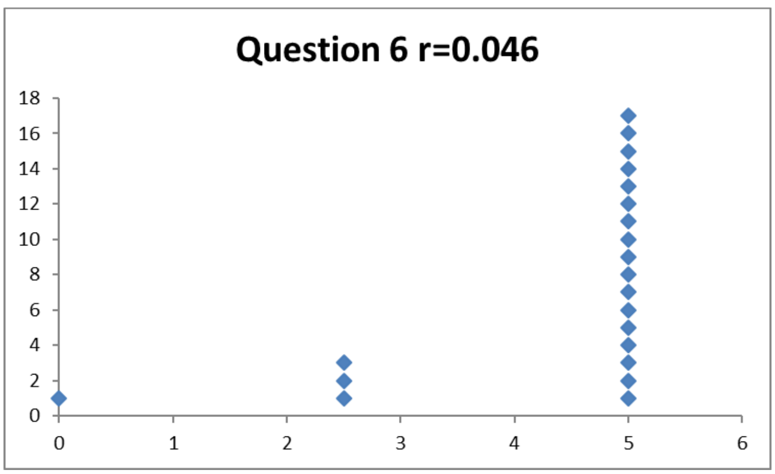
6) Evaluate. (5 points)

$$\int xe^{x^2}dx$$

Answer: $\boxed{\frac{1}{2}e^{x^2}+C}$

$u = x^2$
$du = 2xdx$
$\frac{du}{2x} = dx$

$$\int xe^{x^2}dx = \frac{1}{2}\int xe^{u}\frac{du}{x} = \frac{1}{2}\int e^{u}du = \frac{1}{2}e^{u} + C = \frac{1}{2}e^{x^2} + C$$

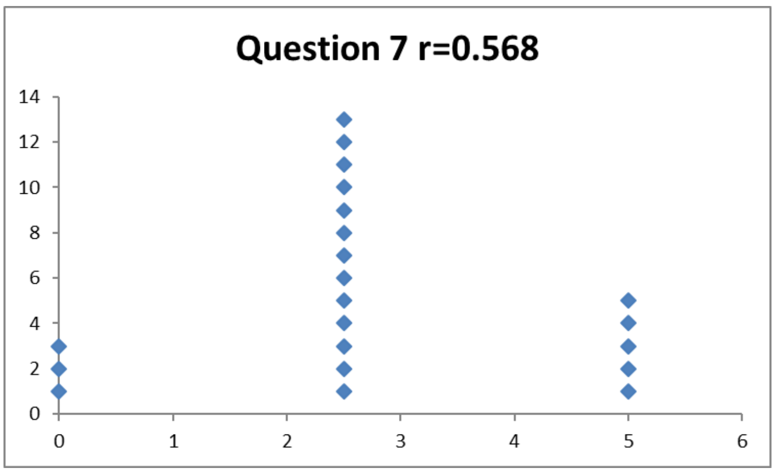


7) Evaluate. (5 points)
(Hint: $25^2 - 22 \cdot 25 - 75 = 0$)

$$\lim_{x \to 25} \frac{x^2 - 22x - 75}{\sqrt{x} - 5}$$

Answer: $\boxed{280}$

$$\lim_{x \to 25} \frac{x^2 - 22x - 75}{\sqrt{x} - 5} = \lim_{x \to 25} \frac{2x - 22}{\frac{1}{2\sqrt{x}}} = \frac{50 - 22}{\frac{1}{2 \cdot 5}} = 28 \cdot 10 = 280$$

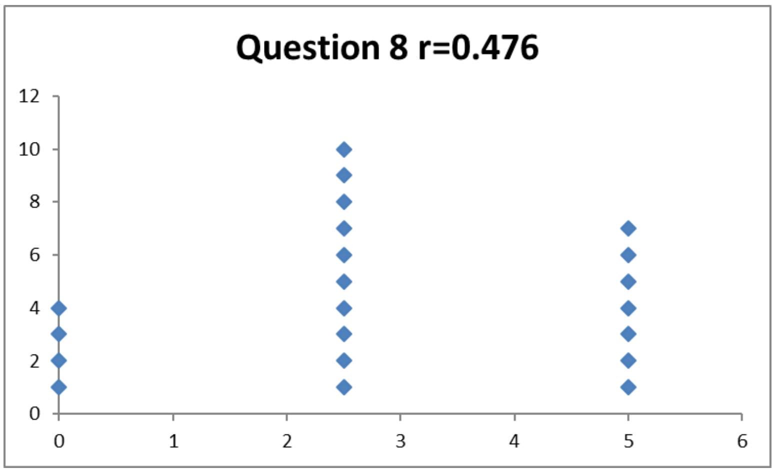


8) Set up, but do not evaluate the integral for: (5 points)

The area between $y = \sin(x)$ and $y = \cos(x)$ between $x = 0$ and $x = \frac{\pi}{4}$

Answer:

$$\boxed{\int_0^{\frac{\pi}{4}} \cos(x) - \sin(x)\, dx}$$



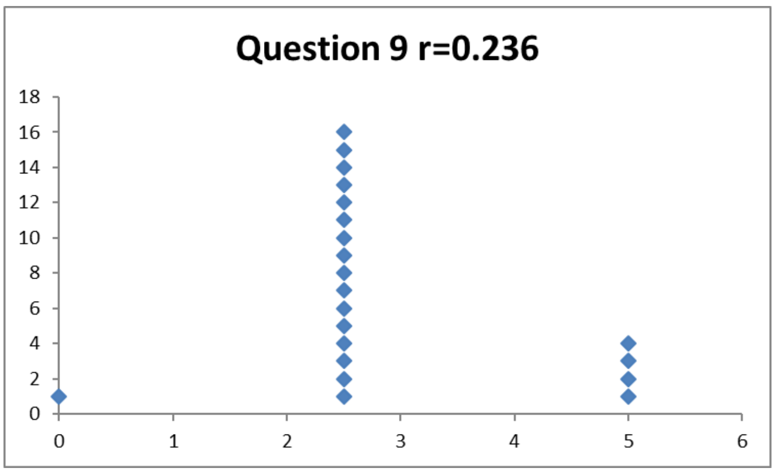
9) Set up, but do not evaluate the integral for: (5 points)

The volume of the region bounded by the curves below, rotated around the x-axis

$y = 9 - x^2$
The x-axis
The y-axis

Answer:

$$\boxed{\int_0^3 \pi(9 - x^2)^2 dx}$$



10) Set up, but do not evaluate the integral for: (5 points)

The volume of the region bounded by the curves below, rotated around the x-axis

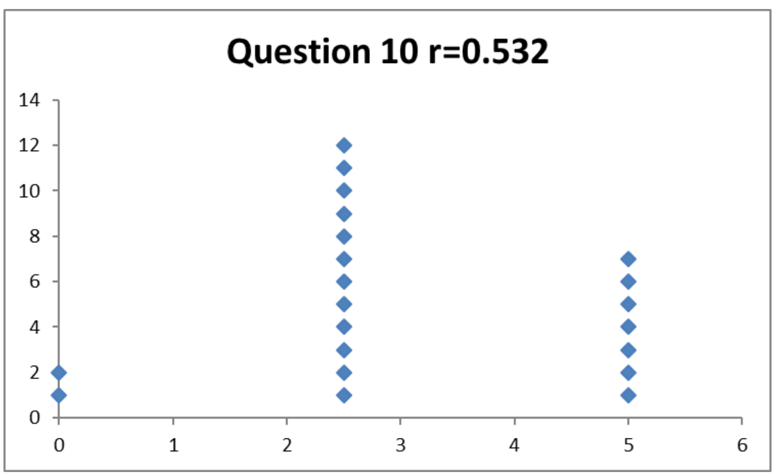
$y = x + 12$
$y = 6$
$x = 0$
$x = 8$

Answer:

$$\boxed{\int_0^8 \pi(x+12)^2 dx - \int_0^8 \pi(6)^2 dx}$$

OR

$$\int_0^8 \pi(x+12)^2 dx - \pi \cdot 6^2 \cdot 8$$



Part 2: Conceptual Understanding

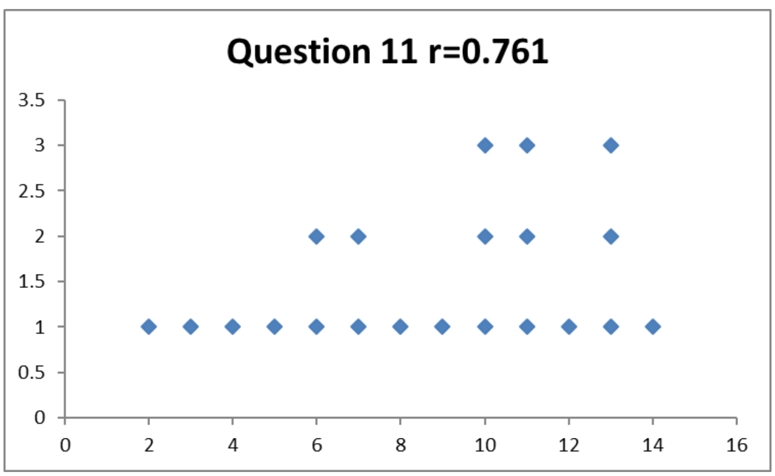
11) Find the integral below. Show your work.
(14 points)

$$\int_0^1 (x+1)(x^2+2x)^2 dx$$

$u = x^2 + 2x$
$du = (2x+2)dx$
$\frac{du}{2x+2} = dx$

When $x = 0, u = 0$
When $x = 1, u = 3$

$$\int_0^1 (x+1)(x^2+2x)^2 dx = \int_0^3 \frac{(x+1)u^2 du}{2x+2} = \int_0^3 \frac{u^2 du}{2} = \frac{1}{2}\int_0^3 u^2 du = \frac{1}{2}\cdot\frac{u^3}{3}\bigg|_0^3 = \frac{u^3}{6}\bigg|_0^3 = \frac{3^3}{6} - \frac{0^3}{6} = \frac{27}{6} = \frac{9}{2}$$



12) Given the information below, find the integrals that follow. (6 points)

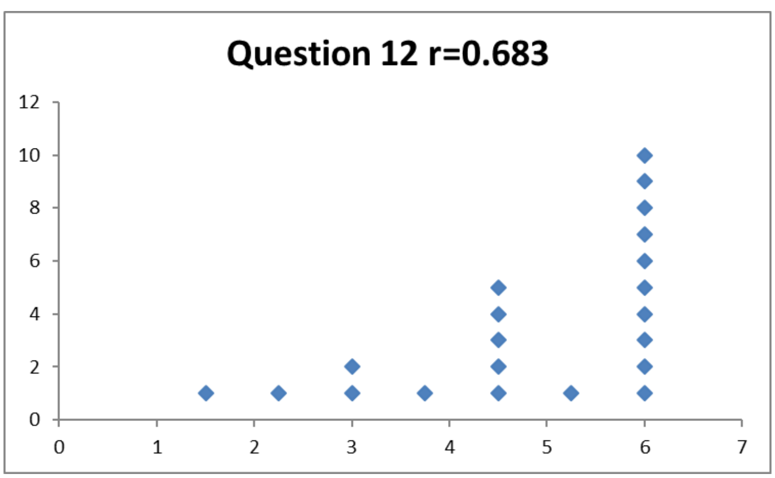
$$\int_0^5 f(x)dx = 10$$
$$\int_0^5 g(x)dx = 25$$
$$\int_0^2 g(x)dx = 1$$

$$A) \int_5^0 f(x)dx = -10$$

$$B) \int_2^2 3g(x) + 2f(x)dx = 0$$

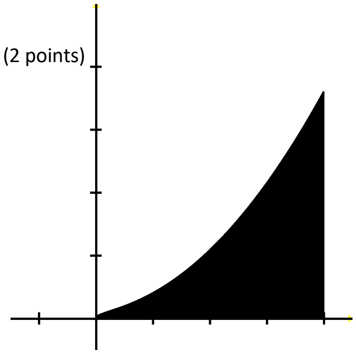
$$C) \int_2^5 g(x)dx = 24$$

$$D) \int_0^5 g(x) - 2f(x)dx = 25 - 20 = 5$$



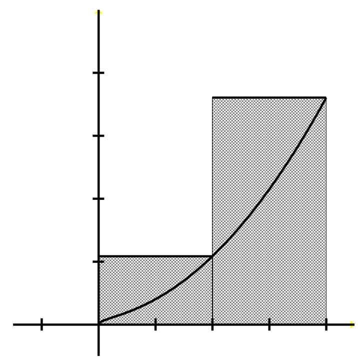
13) Answer each of the following parts.

(A) Illustrate (do not calculate) the area under the curve given below. (2 points)



(B) Illustrate (do not calculate) an approximation to the area under the curve given below. (2 points)

(There are multiple answers)



(C) Calculate the approximation you illustrated in part B. (2 points)

(There are multiple answers, but each is unique and based on your answer to #B)

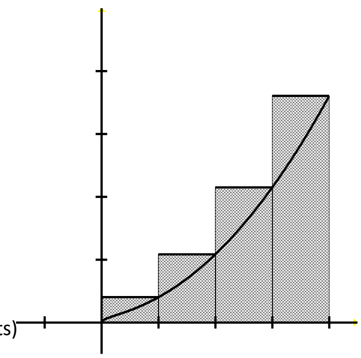
$1.1 \cdot 2 + 3.5 \cdot 2$

(D) Is your approximation in Part C an overestimate or an underestimate? (1 point)

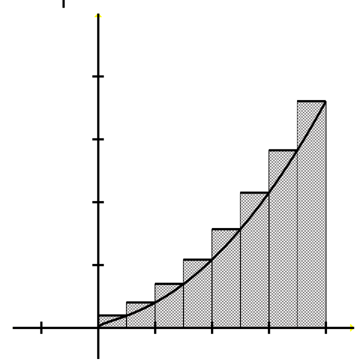
(Again based on #B)

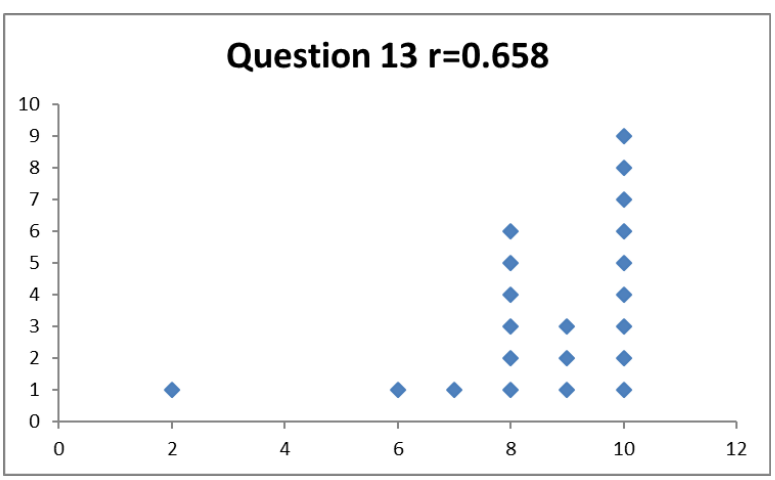
Overestimate

(E) Illustrate a better approximation than you came up with in part B. (2 points)



(F) Illustrate an even better approximation than you came up with in part E. (1 points)





Part 3: Applications

14) Let $g(s)$ be a velocity function. Consider the function below and answer the following parts. (6 points)

$$f(t) = \int_0^t g(s)ds$$

(A) Which one best describe $f(t)$?

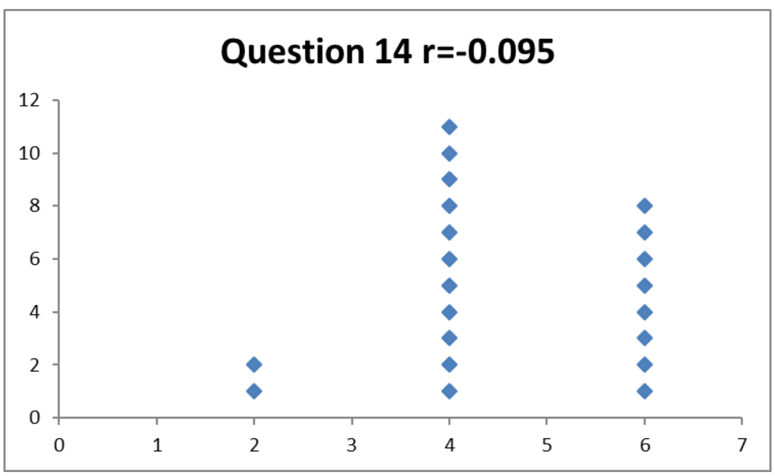
- I. Position
- II. Velocity
- III. Acceleration

(B) If $g(s)$ is positive, which one best describes $f(t)$?

- I. Positive
- II. Negative
- III. We cannot know

(C) If $g(s)$ is positive, which one best describes $f(t)$?

- I. Increasing
- II. Decreasing
- III. We cannot know



15) Consider the 3D figure you've been given. Construct it as a solid of revolution by giving equations to create the 2D region and an axis of rotation: (4 points)

It doesn't have to be perfect, as long as your region and axis give the key features.

It is the region bounded by:

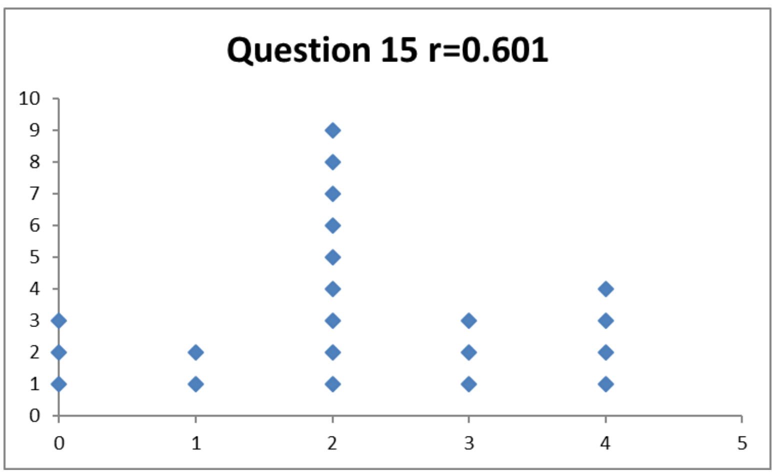
$y = x^2$_________________________,

$y = 0$___________________________, and

$x = 2$___________________________

Rotated around the axis: _$x = 3$_________________

(There are multiple answers – the key idea is to have a solid base, curved outside, and a cylindrical hole in the inside.

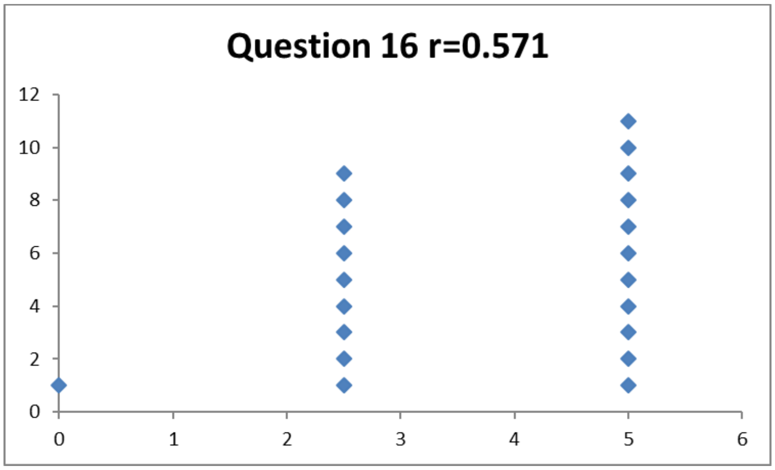


Part 4: Review

16) Find the derivative of the function below. (5 points)

$$f(x) = \sec(3x^2)$$

Answer: $\boxed{\sec(3x^2)\tan(3x^2) \cdot 6x}$



17) Find the critical values of the function below. (5 points)

$$f(x) = x^3 - 12x$$

Answer: $\boxed{x = -2, 2}$

$$f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x - 2)(x + 2) = 0$$