Name ____ Quiz 3

1) Find the Taylor series of the function below, centered at $x = -1$

$f(x) = (2 - x)^{-3}$

Function	Expression	Evaluated at -1	Coefficient of
$f(x)$	$(2-x)^{-3}$	$\frac{1}{3^3}$	x^0
$f'(x)$	$3(2-x)^{-4}$	$\frac{3}{3^4}$	x^1
$f''(x)$	$12(2-x)^{-5}$	$\frac{12}{3^5}$	x^2
$f'''(x)$	$60(2-x)^{-6}$	$\frac{60}{3^6}$	x^3

$$f(x) = \frac{1}{3^3} + \frac{3}{3^4}x + \frac{12}{3^5}x^2 + \frac{60}{3^6}x^3 + \cdots = \sum_{k=0}^{\infty} \frac{(k+2)!}{2 \cdot 3^{k+3}} x^k$$

Well I intended this to be an easy question so you wouldn't get lost in the algebra. In particular I intended it to be evaluated at positive 1, as shown below. Difficulty correction = +2 points.

Function	Expression	Evaluated at 1	Coefficient of
$f(x)$	$(2-x)^{-3}$	1	x^0
$f'(x)$	$3(2-x)^{-4}$	3	x^1
$f''(x)$	$12(2-x)^{-5}$	12	x^2
$f'''(x)$	$60(2-x)^{-6}$	60	x^3

$$f(x) = 1 + 3x + 12x^2 + 60x^3 + \cdots = \sum_{k=0}^{\infty} \frac{(k+2)!}{2} x^k$$

2) Determine whether the series below converges or diverges. Circle which test(s) you use.

[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series] [Alternating Series]

$$\sum_{n=5}^{\infty} \frac{n + \ln(n)}{n^2}$$

$$\sum_{n=5}^{\infty} \frac{n + \ln(n)}{n^2} > \sum_{n=5}^{\infty} \frac{n}{n^2} = \sum_{n=5}^{\infty} \frac{1}{n} = \infty$$

Thus $\sum_{n=5}^{\infty} \frac{n+\ln(n)}{n^2}$ diverges.