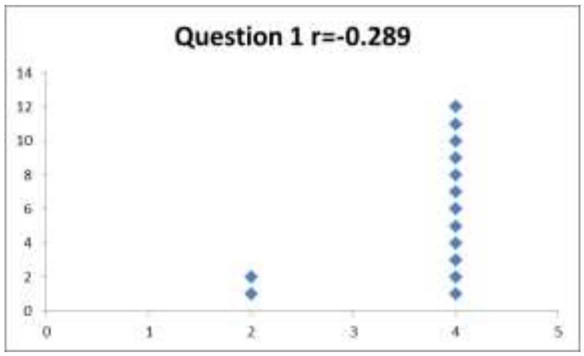
Name ____ Test 1, Spring 2019

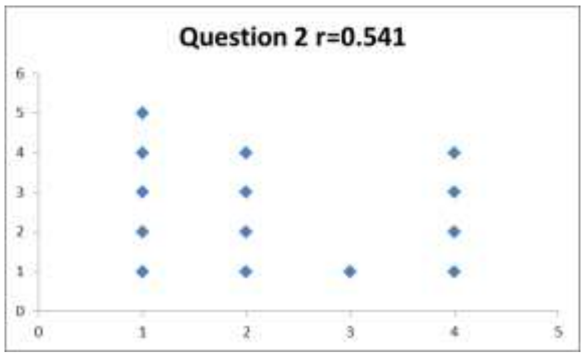
1) Write out the first 4 terms of the series below. (4 points)

$$\sum_{k=0}^{\infty} \frac{1}{10^k} = 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots$$



2) Find the summation below. You do not need to simplify your answer. (4 points)

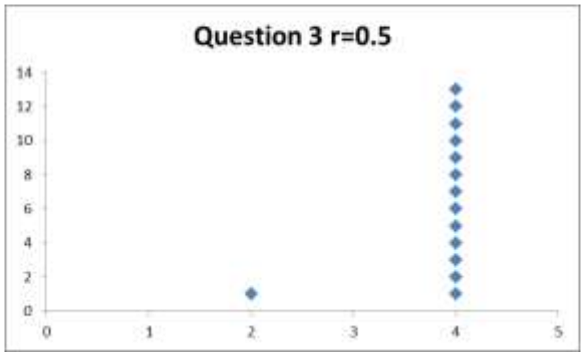
$$\sum_{k=0}^{473} \frac{1}{10^k} = \frac{1-\left(\frac{1}{10}\right)^{474}}{1-\frac{1}{10}}$$



3) Write out the first 4 terms of the sequence below. (4 points)

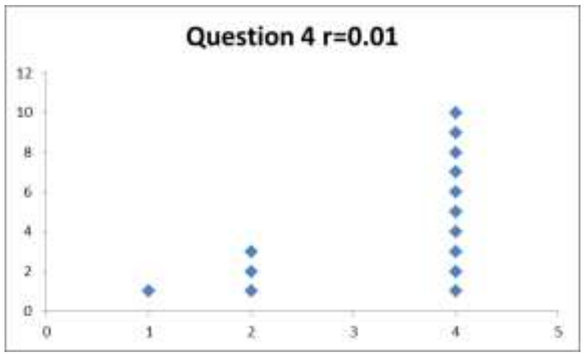
$$\left\{\frac{1}{10^n}\right\}$$

$$\frac{1}{10}, \frac{1}{100}, \frac{1}{10^3}, \frac{1}{10^4}, \dots$$



4) What does the sequence in the above question converge to? (4 points)

$$\lim_{n\to\infty} \frac{1}{10^n} = 0$$



5) Find the integral below. (8 points)

$$\int \tan^3(x)\,dx = \int (1+\sec^2(x))\tan(x)\,dx = \int \tan(x)\,dx + \int \sec^2(x)\tan(x)\,dx$$

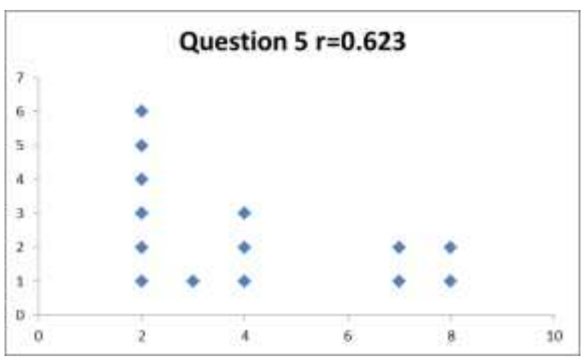
$$= -\ln|cos(x)| + C + \frac{\tan^2(x)}{2} + C$$

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}dx = -\int \frac{du}{u} = -\ln|u| + C = -\ln|cos(x)| + C$$

$$u = \cos(x)$$
$$du = -\sin(x)$$

$$\int \sec^2(x)\tan(x)\,dx = \int u du = \frac{u^2}{2} + C = \frac{\tan^2(x)}{2} + C$$

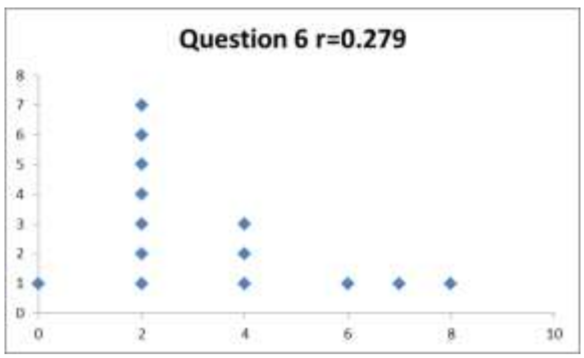
$$u = \tan(x)$$
$$du = \sec^2(x)$$



6) Find the integral below. (8 points)

$$\int \frac{1}{1-\sqrt{x}}dx = -2\int \frac{1-u}{u}du = -2\int \frac{1}{u} - 1du = -2\ln|u| - \frac{u}{2} + C = -2\ln\left|1-\sqrt{x}\right| - \frac{1-\sqrt{x}}{2} + C$$

$u = 1 - \sqrt{x}$
$du = -\frac{1}{2\sqrt{x}}dx$
$-2\sqrt{x}du = dx$
$-2(1-u)du = dx$

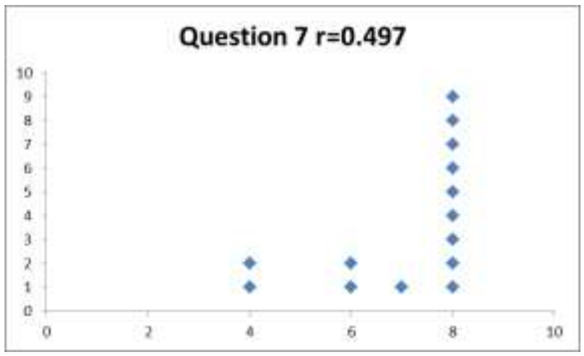


7) Find the integral below. (8 points)

$$\int x\ln(x^2)\,dx = \frac{x^2\ln(x^2)}{2} - \int \frac{2}{x}\cdot\frac{x^2}{2}\,dx = \frac{x^2\ln(x^2)}{2} - \int x\,dx = \frac{x^2\ln(x^2)}{2} - \frac{x^2}{2} + C$$

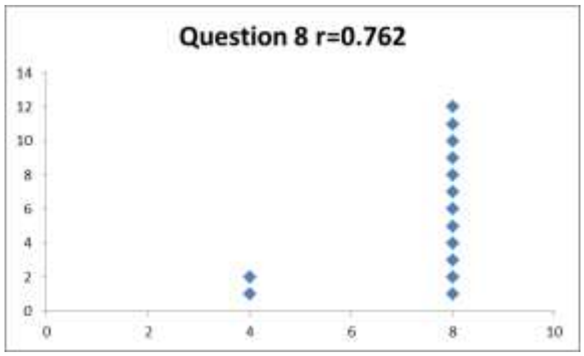
$u = \ln(x^2) \qquad dv = x$

$du = \frac{2x}{x^2} = \frac{2}{x} \qquad v = \frac{x^2}{2}$



8) Find the integral below. (8 points)

$$\int_2^{\infty} \frac{1}{\sqrt{x}} dx = \lim_{t \to \infty} \int_2^{t} \frac{1}{\sqrt{x}} dx = \lim_{t \to \infty} 2\sqrt{x}\Big|_2^t = \lim_{t \to \infty} 2\sqrt{t} - 2\sqrt{2} = \infty$$



9) Find the integral below. (8 points)

$$\int \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx$$

Set up the trig functions:

$\sin(\theta) = x$

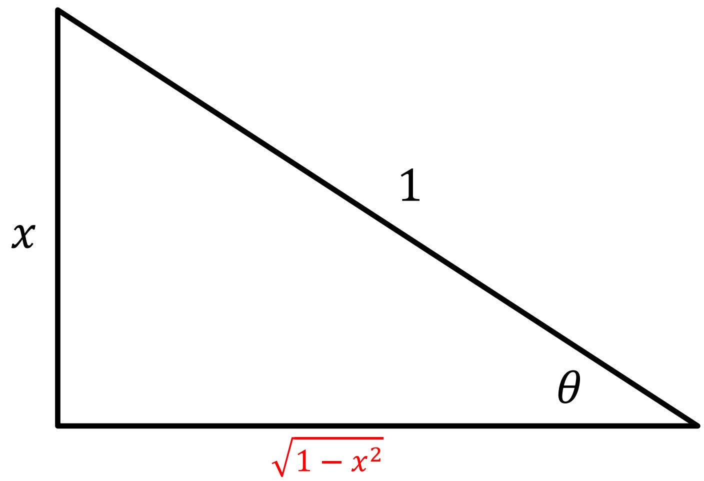
$\cos(\theta) = \sqrt{1-x^2}$

$\tan(\theta) = \frac{x}{\sqrt{1-x^2}}$

$\sec(\theta) = \frac{1}{\sqrt{1-x^2}}$

$\csc(\theta) = \frac{1}{x}$

$\cot(\theta) = \frac{\sqrt{9-x^2}}{x}$



Find the new differential:

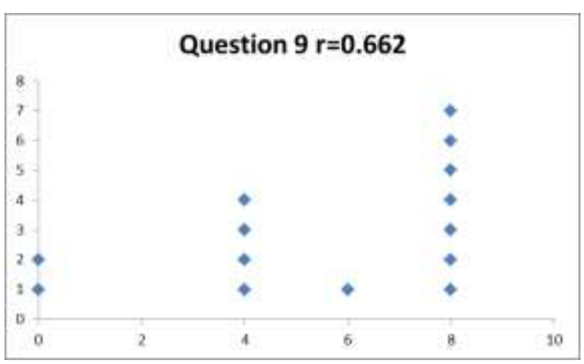
$\sin(\theta) = x$

$\cos(\theta)\,d\theta = dx$

Find the integral:

$$\int \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx = \int \frac{1}{\left(\sqrt{1-x^2}\right)^3}\,dx = \int \frac{1}{\cos^3(\theta)}\cos(\theta)\,d\theta$$

$$= \int \sec^2(\theta)\,d\theta \; = \tan(\theta) + C = \frac{x}{\sqrt{1-x^2}} + C$$



10) Find the integral below. (8 points)

$$\int \frac{x^2+20x-15}{x^3+4x^2-5x}dx = \int \frac{3}{x}dx - \int \frac{3}{x+5}dx + \int \frac{1}{x-1}dx = 3\ln|x| - 3\ln|x+5| + \ln|x-1| + C$$

Partial Fraction Decomposition:

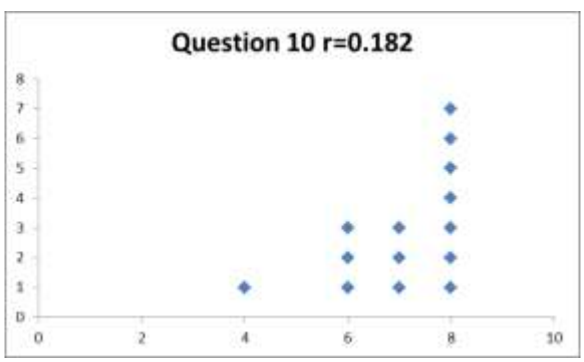
$$\frac{x^2+20x-15}{x^3+4x^2-5x} = \frac{a}{x} + \frac{b}{x+5} + \frac{c}{x-1}$$

$$x^2+20x-15 = a(x+5)(x-1) + bx(x-1) + cx(x+5)$$

$x=0 \quad \Rightarrow \quad -15=-5a \quad \Rightarrow \quad a=3$

$x=1 \quad \Rightarrow \quad 6=6c \quad \Rightarrow \quad c=1$

$x=-5 \quad \Rightarrow \quad -90=30b \quad \Rightarrow \quad b=-3$

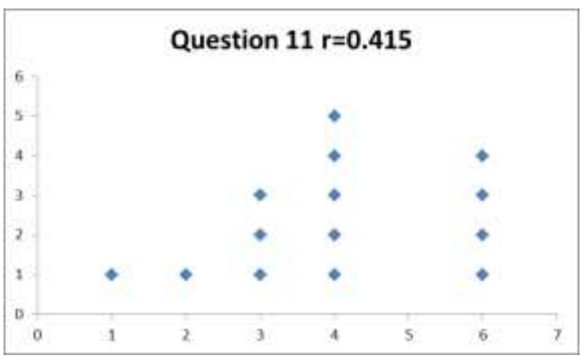


11) Hourly temperature data in Fictoria is given over a 4 hour period. Assume the data comes from a continuous temperature function $T(t)$. Using the midpoint rule, find a good approximation of the average temperature during that time, $\frac{1}{4}\int_0^4 T(t)dt$. Write out the formula you would plug in a calculator, do not simplify your answer. (6 points)

t	$T(t)$
0	15
1	17
2	18
3	21
4	22

$$\int_0^4 T(t)dt \approx 2(17 + 21)$$

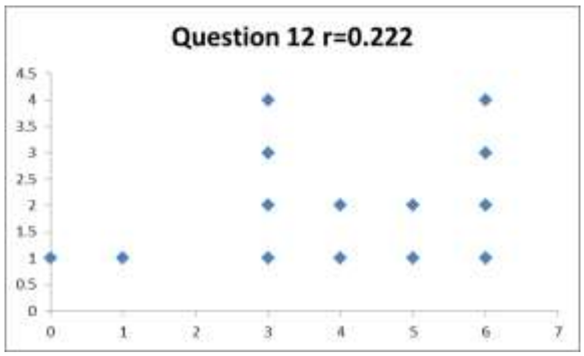
$$\frac{1}{4}\int_0^4 T(t)dt \approx \frac{17 + 21}{2}$$



12) Give the appropriate form of the partial fraction decomposition for the following function. (6 points)
DO NOT DO ANY WORK TO SOLVE IT; I ONLY WANT TO SEE THE "FORM" WITH VARIABLES a, b, c, ETC.

$$\frac{2x^2+3}{(x^2-8x+16)(x^2+3x+4)} = \frac{a}{x-4} + \frac{b}{(x-4)^2} + \frac{cx+d}{x^2+3x+4}$$

One thing to note is that $x^2 - 8x + 16 = (x-4)^2$, but $x^2 + 3x + 4$ is irreducible.



13) Determine whether the series converges or diverges. Circle which test(s) you use. (8 points)

[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series]

$$\sum_{k=5}^{\infty} \frac{\sqrt{k}}{(\ln(k)^7)}$$

Divergence Test:

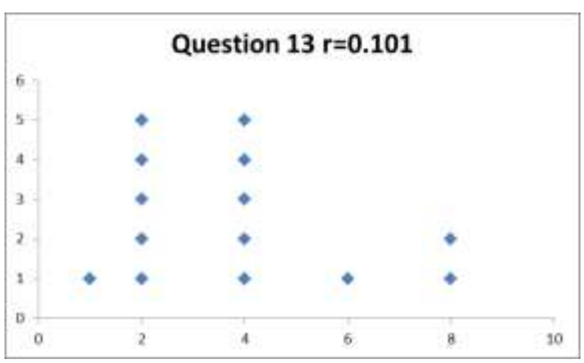
$$\lim_{k\to\infty} \frac{\sqrt{k}}{(\ln(k))^7} = \infty \neq 0, \text{ and so the series diverges}$$

Comparison Test:

$$\sum_{k=5}^{\infty} \frac{\sqrt{k}}{(\ln(k)^7)} > \sum_{k=5}^{\infty} \frac{1}{(\ln(k)^7)} = \infty, \text{ and so the series diverges}$$

The ratio test is inconclusive thus does not work

$$\lim_{k\to\infty} \frac{\frac{\sqrt{k+1}}{(\ln(k+1))^7}}{\frac{\sqrt{k}}{(\ln(k))^7}} = \lim_{k\to\infty} \frac{\sqrt{k+1}}{(\ln(k+1))^7} \cdot \frac{(\ln(k))^7}{\sqrt{k}} = 1 \nless 1$$

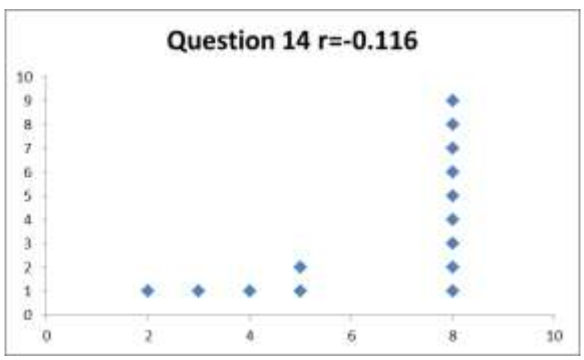


14) Determine whether the series converges or diverges. Circle which test(s) you use. (8 points)
[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series]

$$\sum_{k=1}^{\infty} \left(\frac{2k}{k+1}\right)^k$$

Root Test:

$$\lim_{k\to\infty} \sqrt[k]{\left(\frac{2k}{k+1}\right)^k} = \lim_{k\to\infty} \frac{2k}{k+1} = 2, \text{ and so the series diverges}$$



15) Determine whether the series converges or diverges. Circle which test(s) you use. (8 points)

[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series]

$$\sum_{k=1}^{\infty} \frac{2^k}{k!}$$

Ratio Test:

$$\lim_{k\to\infty} \frac{\frac{2^{k+1}}{(k+1)!}}{\frac{2^k}{k!}} = \lim_{k\to\infty} \frac{2^{k+1}}{(k+1)!} \cdot \frac{k!}{2^k} = \lim_{k\to\infty} \frac{2}{k+1} = 0\text{, and so the series converges}$$