1) Determine whether the series below converges or diverges. Circle which test(s) you use. (6 points)

\[ \sum_{k=1}^{\infty} \frac{(-1)^k}{k^3} \]

2) Determine whether the series below converges or diverges. Circle which test(s) you use. (6 points)

\[ \sum_{k=1}^{\infty} \frac{1}{k^2 + 4} \]
3) Find the 3rd order Taylor Polynomial of $f(x) = e^{2x}$ centered at $x = 1$.
(6 points)

4) Use your Taylor Polynomial from the previous problem to write down a formula that approximates the number $e^{2.2}$
(6 points)

5) How accurate is your approximation from the previous problem? Note that the remainder in a $n^{th}$ order Taylor Polynomial is bounded by $\frac{M(x-a)^{n+1}}{(n+1)!}$.
(6 points)
6) Complete ONE of the following problems.

(a) Find the radius of convergence of the Taylor series below. (8 points maximum)
\[ \sum_{k=1}^{\infty} (2x)^k \]

(b) Find the interval of convergence of the Taylor series below. (10 points – full credit)
\[ \sum_{k=1}^{\infty} 2(x - 5)^k \]
7) Find the series expansion for ONE of the functions below, centered at $x = 0$. Use any method.

(a) $f(x) = \cosh(x)$  
(8 points maximum)

(b) $f(x) = e^{x^2}$  
(10 points – full credit)
8) Find the series expansion for ONE of the functions below, centered at $x = 0$. Use any method.

(a) $f(x) = \frac{1}{(1-x)^2}$
   (8 points maximum)

(b) $f(x) = \frac{1}{(2-x)^2}$
   (10 points – full credit)
9) Given the parametric equations below, find a single equation that describes the same curve. (6 points)

\[ x = t^2 + 2 \]
\[ y = 4t \]
10) Given the parametric equations below, find the slope of the tangent line at \( t = 2 \). (8 points)

\[
\begin{align*}
x &= 3 - t \\
y &= 3t^2 + 12
\end{align*}
\]

11) Find the equation of the line tangent to the curve in the previous problem at \( t = 2 \). (6 points)
12) On the axis provided, graph ONE of the polar curves below.

(a) \( r = 3 \sin(2 \theta) \) for \( 0 \leq \theta \leq \frac{\pi}{2} \).
(8 points – full credit)

(b) \( r = 3 \cos(3 \theta) \) for \( 0 \leq \theta \leq \frac{\pi}{6} \).
(8 points – full credit)

(c) \( r = 3 \cos(2 \theta) \) for \( 0 \leq \theta \leq \pi \).
(6 points maximum)
13) Complete ONE of the following problems.

(a) Given a polar equation $r = f(\theta)$, what is the formula for $\frac{dy}{dx}$ in terms of $r$ and $\theta$?
(4 points maximum)

(b) Find the slope of the line tangent to $r = \cos(3\theta)$ at $\frac{\pi}{6}$.
(6 points – full credit)
14) Write down an integral formula for the area shown in the graph below. You do not need to calculate it. (6 points)

\[ r = 4 \cos(6\theta) \]
\[ 0 \leq \theta \leq \pi/2 \]
15) **2 point bonus:** Use Taylor series to find the limit below.

\[
\lim_{x \to 0} \frac{e^x - 1}{x}
\]

16) **10 points bonus:** Write the series below as a summation in sigma notation.

(Warning: Don’t waste all you’re time thinking about this problem. It’s a huge bonus for a reason – it’s hard and I don’t expect many people to figure it out)

\[
2 + 4 - 8 - 16 + 32 + 64 - 128 - 256 + 512 + 1024 - 2048 - 4096 + \cdots
\]