Name ____ Test 2, Spring 2019

1) Determine whether the series below converges or diverges. Circle which test(s) you use. (6 points)
[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series] [Alternating Series]

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$$

2) Determine whether the series below converges or diverges. Circle which test(s) you use. (6 points)
[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series] [Alternating Series]

$$\sum_{k=1}^{\infty} \frac{1}{k^2+4}$$

3) Find the 3rd order Taylor Polynomial of $f(x) = e^{2x}$ centered at $x = 1$.

(6 points)

4) Use your Taylor Polynomial from the previous problem to write down a formula that approximates the number $e^{2.2}$

(6 points)

5) How accurate is your approximation from the previous problem? Note that the remainder in a n^{th} order Taylor Polynomial is bounded by $\frac{M(x-a)^{n+1}}{(n+1)!}$.

(6 points)

6) Complete ONE of the following problems.

(a) Find the radius of convergence of the Taylor series below. (8 points maximum)

$$\sum_{k=1}^{\infty} (2x)^k$$

(b) Find the interval of convergence of the Taylor series below. (10 points – full credit)

$$\sum_{k=1}^{\infty} 2(x-5)^k$$

7) Find the series expansion for ONE of the functions below, centered at $x = 0$. Use any method.

(a) $f(x) = \cosh(x)$

(8 points maximum)

(b) $f(x) = e^{x^2}$

(10 points – full credit)

8) Find the series expansion for ONE of the functions below, centered at $x = 0$. Use any method.

(a) $f(x) = \frac{1}{(1-x)^2}$

(8 points maximum)

(b) $f(x) = \frac{1}{(2-x)^2}$

(10 points – full credit)

9) Given the parametric equations below, find a single equation that describes the same curve. (6 points)

$$x = t^2 + 2$$
$$y = 4t$$

10) Given the parametric equations below, find the slope of the tangent line at $t = 2$. (8 points)

$$x = 3 - t$$
$$y = 3t^2 + 12$$

11) Find the equation of the line tangent to the curve in the previous problem at $t = 2$. (6 points)

12) On the axis provided, graph ONE of the polar curves below.

(a) $r = 3\sin(2\theta)$ for $0 \leq \theta \leq \frac{\pi}{2}$.

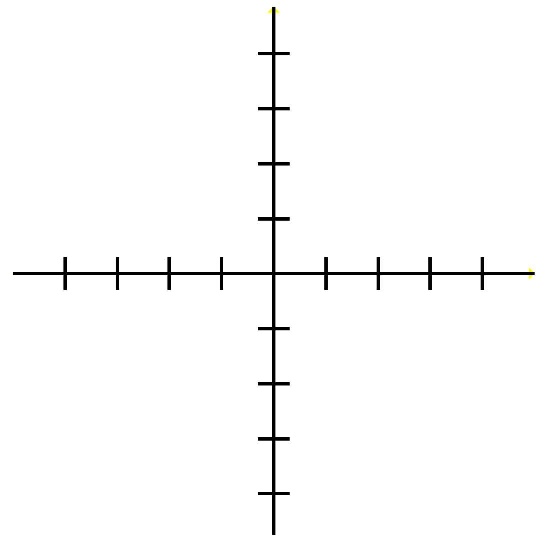
(8 points – full credit)

(b) $r = 3\cos(3\theta)$ for $0 \leq \theta \leq \frac{\pi}{6}$.

(8 points – full credit)

(c) $r = 3\cos(2\theta)$ for $0 \leq \theta \leq \pi$.

(6 points maximum)



13) Complete ONE of the following problems.

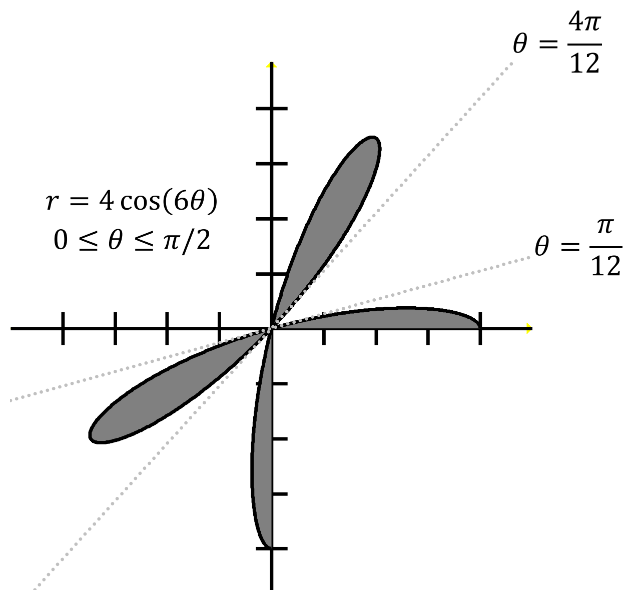
(a) Given a polar equation $r = f(\theta)$, what is the formula for $\frac{dy}{dx}$ in terms of r and θ?

(4 points maximum)

(b) Find the slope of the line tangent to $r = \cos(3\theta)$ at $\frac{\pi}{6}$.

(6 points – full credit)

14) Write down an integral formula for the area shown in the graph below. You do not need to calculate it. (6 points)



15) **2 point bonus:** Use Taylor series to find the limit below.

$$\lim_{x \to 0} \frac{e^x - 1}{x}$$

16) **10 points bonus**: Write the series below as a summation in sigma notation.

(Warning: Don't waste all you're time thinking about this problem. It's a huge bonus for a reason – it's hard and I don't expect many people to figure it out)

$$2 + 4 - 8 - 16 + 32 + 64 - 128 - 256 + 512 + 1024 - 2048 - 4096 + \cdots$$