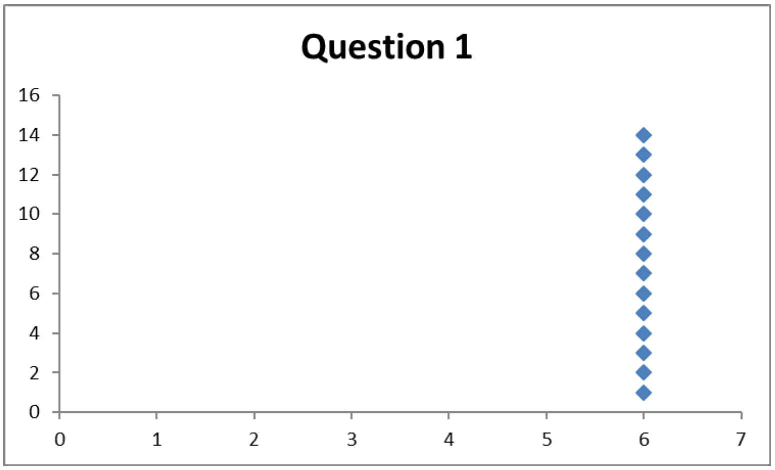
Name ____ Test 2, Spring 2019

1) Given the vectors $\vec{v} = \langle -1, 2, 4 \rangle$ and $\vec{w} = \langle 10,15,30 \rangle$ compute $4\vec{v} + \vec{w}$
(6 points)

$$4\langle -1, 2, 4 \rangle + \langle 10,15,30 \rangle = \langle -4,8,16 \rangle + \langle 10,15,30 \rangle = \langle 6,23,46 \rangle$$



2) Do these two lines intersect? Justify your answer.
(6 points)

$$r(t) = \langle t,\ 2t + 2,\ 3\rangle$$
$$s(t) = \langle 1 + 2t, 2 + 3t,\ 4 - t\rangle$$

Suppose $r(a) = s(b)$. We get three equations, for x, y and z coordinates:

$$a = 1 + 2b$$
$$2a + 2 = 2 + 3b$$
$$3 = 4 - b$$

The third equation gives:

$$b = 1$$

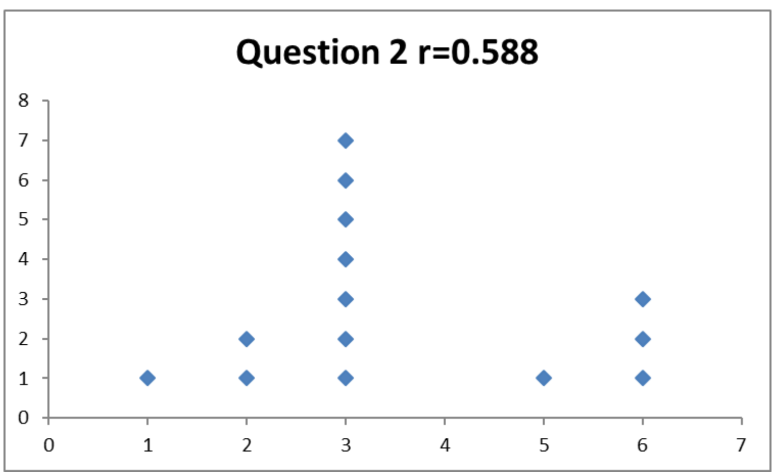
The first equation gives:

$$a = 3$$

Now look at the second equation:

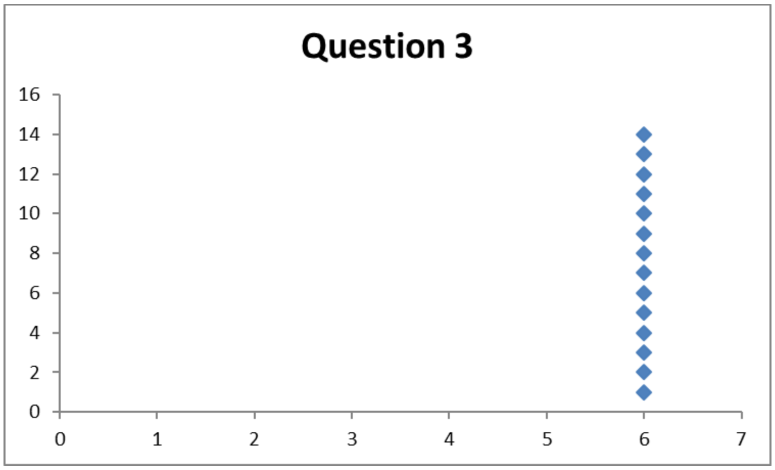
$$6 + 2 = 2 + 3$$

This is impossible.



3) Find the length of $\vec{v} = \langle -10, 4, 5 \rangle$
(6 points)

$$L = \sqrt{(-10)^2 + 4^2 + 5^2} = \sqrt{141}$$



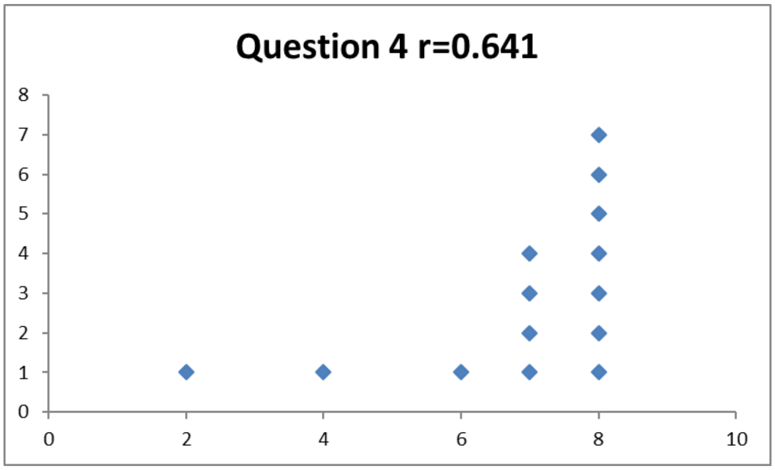
4) Find an equation of the line through $(2,3,5)$ and $(1,0,2)$
(8 points)

First find the direction:

$$\langle 2,3,5\rangle - \langle 1,0,2\rangle = \langle 1,3,3\rangle$$

Then the line is:

$$\vec{r}(t) = \langle 1,0,2\rangle + t\langle 1,3,3\rangle$$

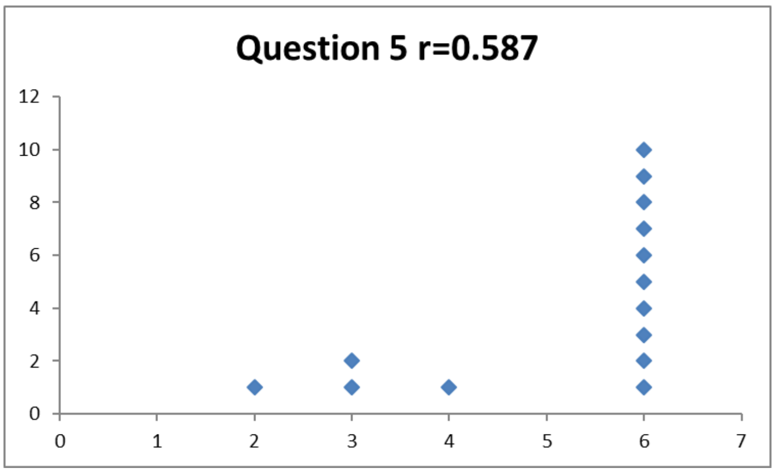


5) A certain plane is given by the two equations below. Find a vector orthogonal to the entire plane.

$$r(t) = \langle 0,0,0\rangle + t\langle 1,2,3\rangle$$
$$s(t) = \langle 0,0,0\rangle + t\langle 0,2,4\rangle$$

(6 points)

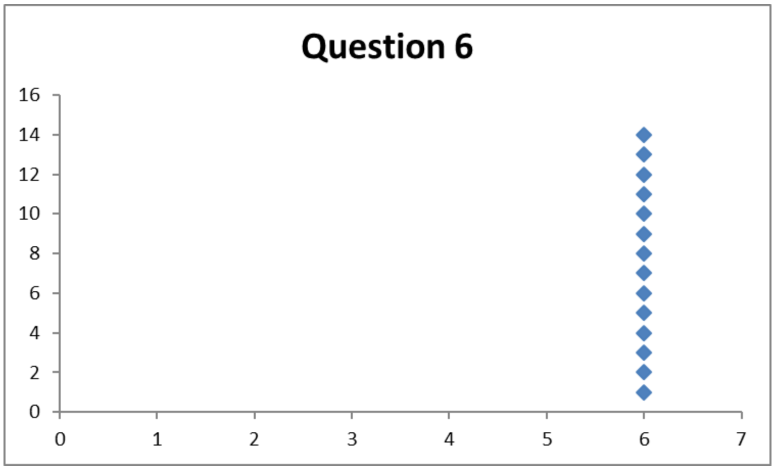
$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 0 & 2 & 4 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = \langle 2,-4,2\rangle$$



6) Find the dot product of $\langle -2,4,5\rangle$ and $\langle 3,2,1\rangle$

(6 points)

$$-6+8+5=7$$



7) Which of these formulas correctly describe the angle θ between two vectors $\vec{v}$ and $\vec{w}$? Two of these are correct, six of them are not.

(4 points)

A. $\cos^{-1}\left(\frac{\vec{v}\bullet\vec{w}}{|\vec{v}|\cdot|\vec{w}|}\right)$

B. $\sin^{-1}\left(\frac{\vec{v}\bullet\vec{w}}{|\vec{v}|\cdot|\vec{w}|}\right)$

C. $\cos^{-1}\left(\frac{\vec{v}\times\vec{w}}{|\vec{v}|\cdot|\vec{w}|}\right)$

D. $\sin^{-1}\left(\frac{\vec{v}\times\vec{w}}{|\vec{v}|\cdot|\vec{w}|}\right)$

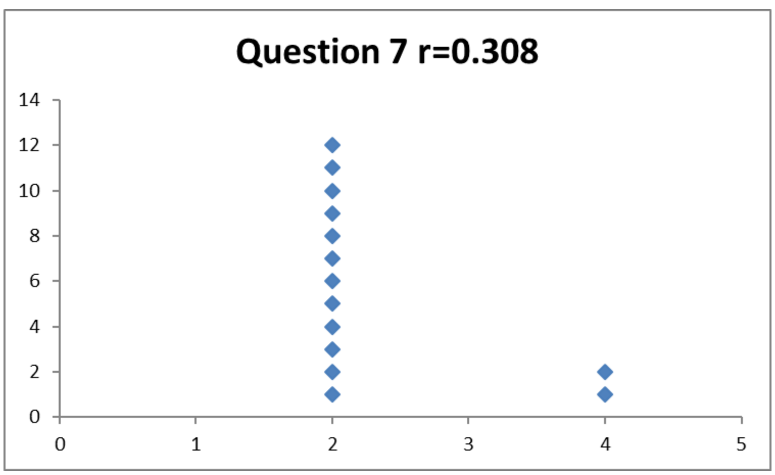
E. $\cos^{-1}\left(\frac{|\vec{v}\bullet\vec{w}|}{|\vec{v}|\cdot|\vec{w}|}\right)$

F. $\sin^{-1}\left(\frac{|\vec{v}\bullet\vec{w}|}{|\vec{v}|\cdot|\vec{w}|}\right)$

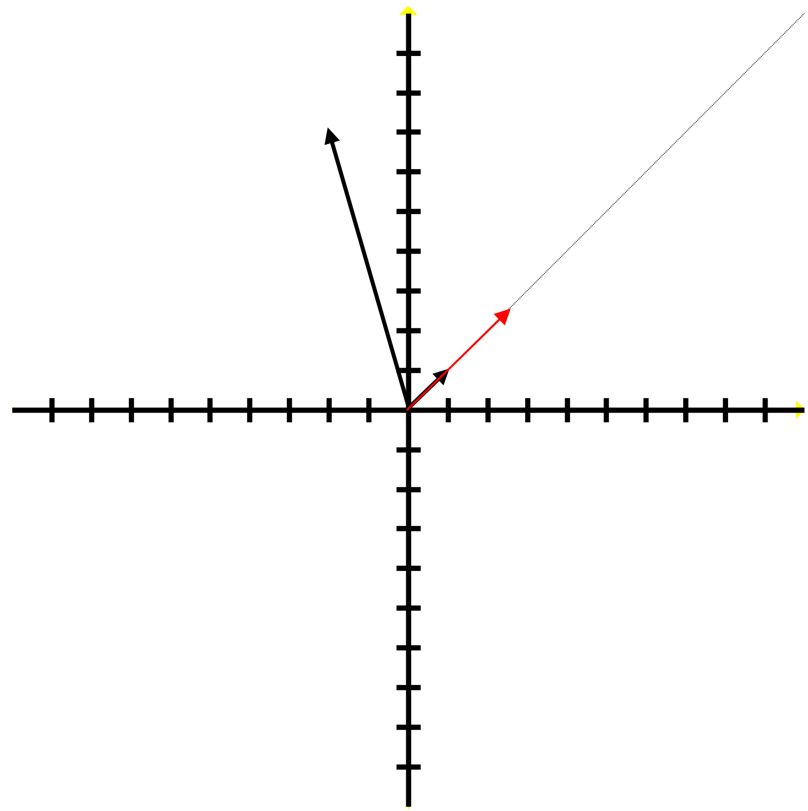
G. $\cos^{-1}\left(\frac{|\vec{v}\times\vec{w}|}{|\vec{v}|\cdot|\vec{w}|}\right)$

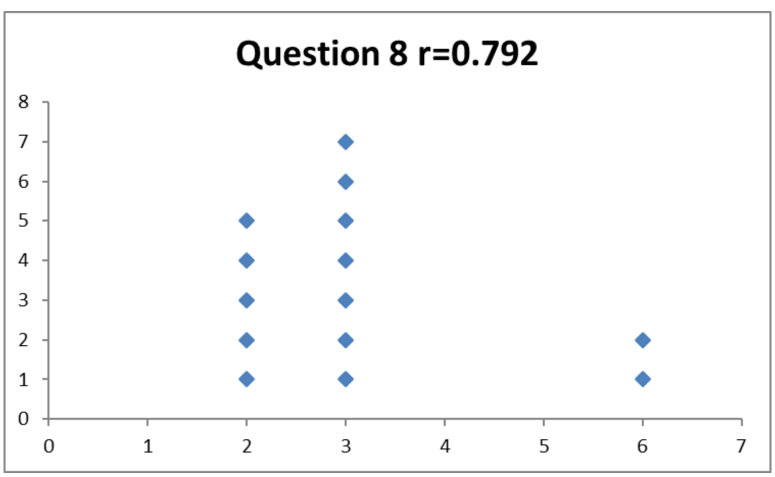
H. $\sin^{-1}\left(\frac{|\vec{v}\times\vec{w}|}{|\vec{v}|\cdot|\vec{w}|}\right)$

$$\vec{v}\bullet\vec{w} = |\vec{v}|\cdot|\vec{w}|\cdot\cos(\theta)$$
$$|\vec{v}\times\vec{w}| = |\vec{v}|\cdot|\vec{w}|\cdot\sin(\theta)$$



8) Illustrate the projection of $\langle -2,7 \rangle$ onto $\langle 1,1 \rangle$
(6 points)

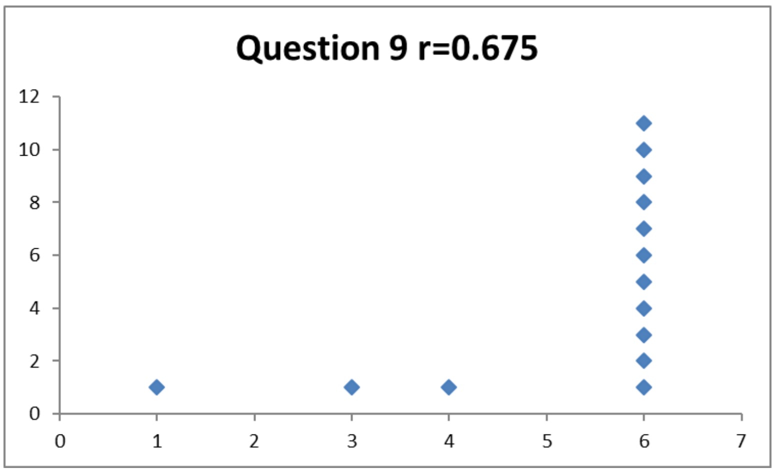




9) Find the limit below.
(6 points)

$$\lim_{t \to 0} \langle \frac{\sin(t)}{t}, \frac{t^2 + t}{t}, 4t + 2 \rangle$$

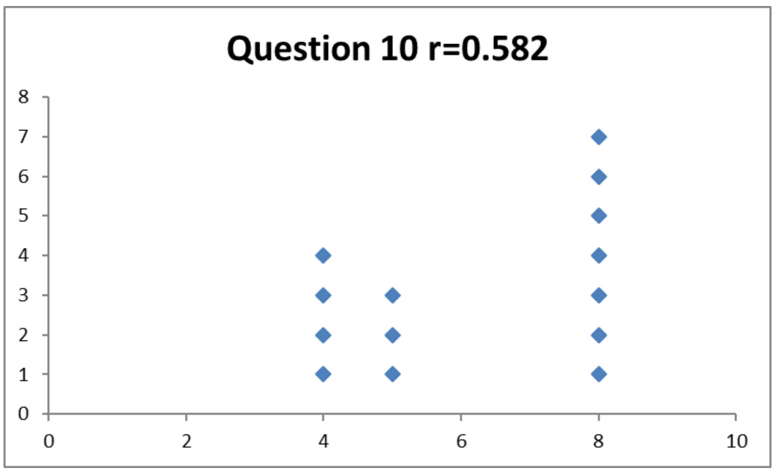
$$\lim_{t \to 0} \langle \frac{\sin(t)}{t}, \frac{t^2 + t}{t}, 4t + 2 \rangle = \lim_{t \to 0} \langle \frac{\sin(t)}{t}, t + 1, 4t + 2 \rangle = \langle 1,1,2 \rangle$$



10) Find the derivative below.
(8 points)

$$\frac{d}{dt}\langle\frac{\sin(t)}{t},\frac{t^2+t}{t},\ 4t+2\rangle$$

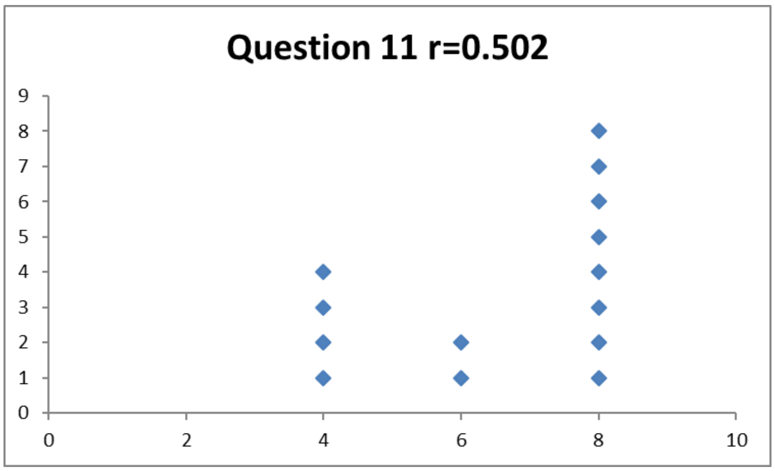
$$\frac{d}{dt}\langle\frac{\sin(t)}{t},\frac{t^2+t}{t},\ 4t+2\rangle\ =\langle\frac{\cos(t)\,t-\sin(t)}{t^2},1,4\rangle$$



11) Find the integral below.
(8 points)

$$\int \langle \frac{1}{1+2t}, \frac{t^2+t}{t}, 4t+2 \rangle \, dt$$

$$\int \langle \frac{1}{1+2t}, \frac{t^2+t}{t}, 4t+2 \rangle \, dt = \int \langle \frac{1}{1+2t}, t+1, 4t+2 \rangle \, dt = \langle \frac{1}{2} \ln|1+2t|, \frac{t^2}{2} + t, 2t^2 + 2t \rangle + \vec{C}$$



12) Find the integral below.

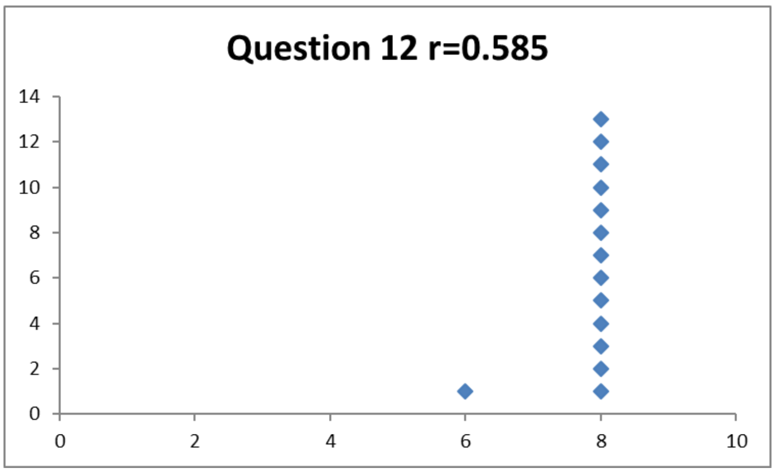
(8 points)

$$\int x\cosh(x)\,dx$$

$u = x$ $\qquad dv = \cosh(x)\,dx$

$du = dx$ $\qquad v = \sinh(x)$

$$\int x\cosh(x)\,dx = x\sinh(x) - \int \sinh(x)\,dx = x\sinh(x) - \cosh(x) + C$$



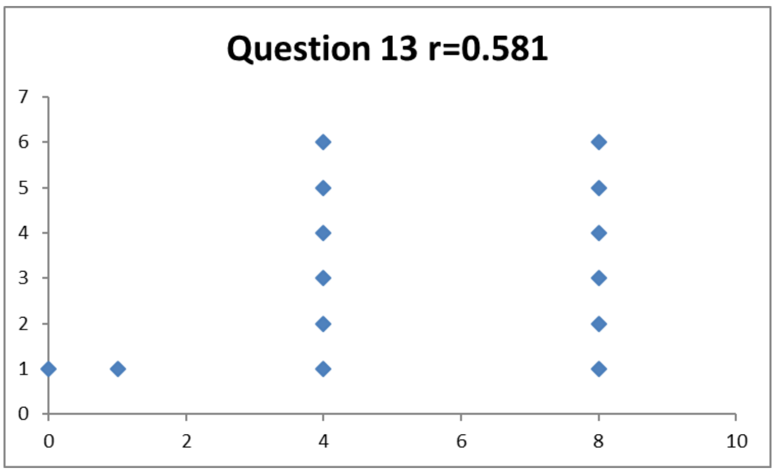
13) Find the integral below.

(8 points)

$$\int \frac{e^x}{1+e^{2x}}\,dx$$

$$\int \frac{e^x}{1+e^{2x}} = \int \frac{e^x}{1+(e^x)^2} = \int \frac{1}{1+u^2}\,du = \tan^{-1}(u) + C = \tan^{-1}(e^x) + C$$

$$u = e^x$$
$$du = e^x dx$$



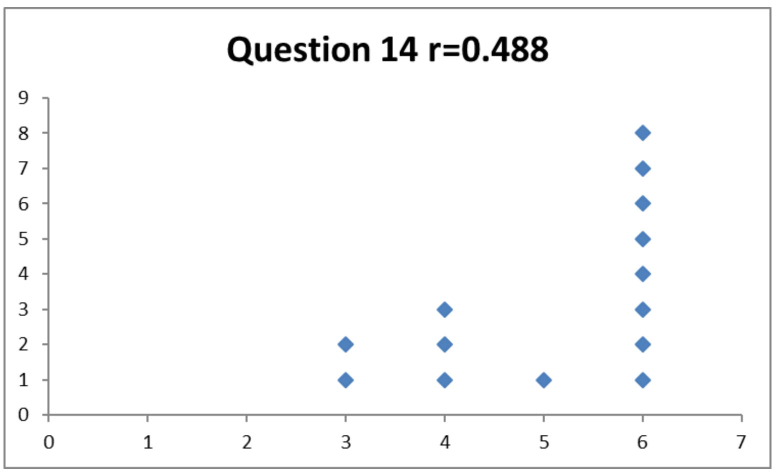
14) Determine whether the series below converges or diverges. Circle which test(s) you use. (6 points)

[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series] [Alternating Series]

$$\sum_{k=1}^{\infty} \frac{k^2+1}{2k-3}$$

$$\lim_{k\to\infty} \frac{k^2+1}{2k-3} = \infty$$

By the divergence test, $\sum_{k=1}^{\infty} \frac{k^2+1}{2k-3}$ diverges because $\lim_{k\to\infty} \frac{k^2+1}{2k-3} \neq 0$



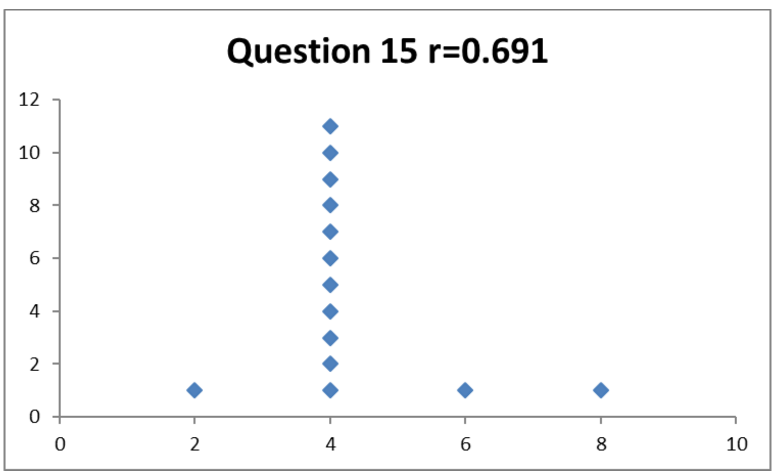
15) Determine whether the series below converges or diverges. Circle which test(s) you use. (8 points)
[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series] [Alternating Series]

$$\sum_{k=1}^{\infty} \frac{2k-3}{k^2+1}$$

$\lim_{k\to\infty} \frac{2k-3}{k^2+1} = 0$, but notice that it looks a lot like $\frac{1}{k}$. So let's a limit comparison test!

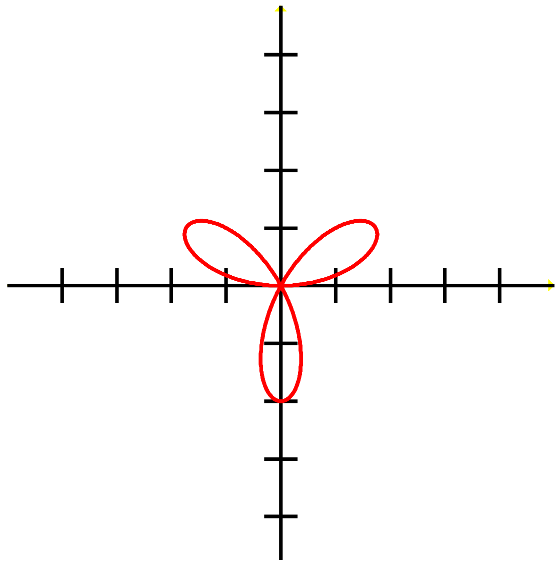
$$\lim_{k\to\infty} \frac{\frac{2k-3}{k^2+1}}{\frac{1}{k}} = \lim_{k\to\infty} \frac{2k^2-3k}{k^2+1} = 2$$

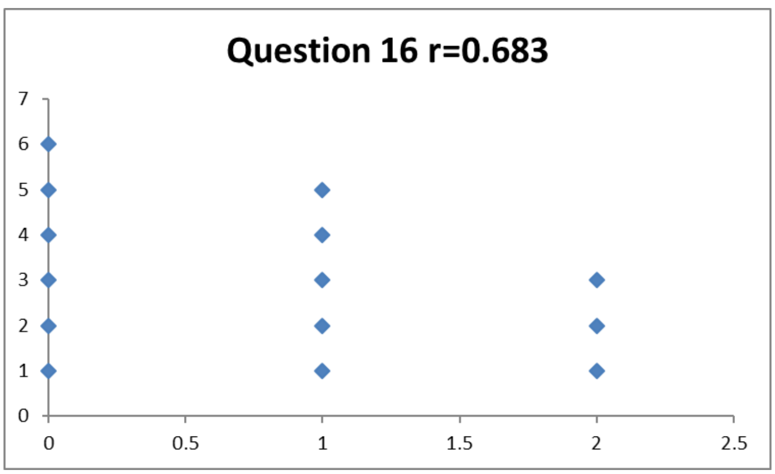
By the limit comparison test, $\sum_{k=1}^{\infty} \frac{2k-3}{k^2+1}$ and $\sum_{k=1}^{\infty} \frac{1}{k}$ both converge or both diverge. We know the latter diverges, and so the former also diverges.



16) **(2 point bonus)** Graph the curve $r = 2\sin(3\theta)$, then set up the integral to find the area enclosed.

$$A = 3\int_0^{\pi/3} \frac{1}{2}(2\sin(3\theta))^2 d\theta$$





17) **(2 point bonus)** Graph the curve $r = 3\cos(5\theta)$, then set up the integral to find the perimeter of the figure created.

$$L = 10\int_0^{\pi/10} \sqrt{(r)^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = 10\int_0^{\pi/10} \sqrt{(3\cos(5\theta))^2 + (-15\sin(5\theta)}\, d\theta$$