Part 1: Computational Skills

1) Determine whether the series converges or diverges. Circle which test(s) you use. (10 points)

[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series] [Alternating Series]

$$\sum_{n=1}^{\infty} \frac{3^n + n}{2^n - 1}$$

2) Determine whether the series converges or diverges. Circle which test(s) you use. (10 points)

[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series] [Alternating Series]

$$\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^3}$$

3) Determine whether the series converges or diverges. Circle which test(s) you use. (10 points)

[Divergence Test] [Integral Test] [Comparison Test] [Limit Comparison Test] [Ratio Test] [Root Test] [Geometric Series] [p-Series] [Alternating Series]

$$\sum_{n=1}^{\infty} \frac{n}{(\ln(n))^n}$$

4) Find the radius of the convergence of the power series below.

$$\sum_{k=3}^{\infty} \frac{k^2}{5^k}(x-2)^k$$

5) Find a power series expansion of the function below, centered at $x = 1$. Write your answer using sigma notation. (10 points)

$$\frac{1}{x^2}$$

6) Find a power series expansion of the function below, centered at $x = 0$. (10 points)

$$\cos(5x)$$

Part 2: Conceptual Understanding

7) Let $f(x)$ be a function satisfying with power series expansion $\sum_{k=0}^{\infty} a_k (x-2)^k$, centered at $x = 2$ with radius of convergence 7. It is known that $f(2) = 5$. What is $\sum_{k=0}^{\infty} a_k (2-2)^k$? (10 points)

8) If a power series has radius of convergence 4 and the interval of convergence does not include either endpoint, for how many integer x-values can the series possibly converge? (10 points)
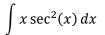
Part 3: Applications

9) Below is a Taylor series (centered at $x = 0$) that represents a function. Suppose we are going to ask a computer to evaluate this function at $x = 1$. The computer uses the first 13 terms. How accurate is the computer's approximation? It is known that f is infinitely differentiable, and each derivative is no larger than 5. (10 points)

$$\sum_{k=0}^{\infty} \frac{(k-3)x^k}{k}$$

Part 4: Review

10) Find the integral below. (5 points)

$$\int x \sec^2(x)\, dx$$

11) Find the integral below. (5 points)

$$\int \sin(x) \cos^4(x)\, dx$$

Part 5: Small Bonus

12) A restaurant offers 20 different entries. Your party has 7 people, and you all want different meals so you can share. How many meal choices are there? Use correct mathematical notation. (2 points)