

1) Find the negation of  $\forall_{x \in U} \exists_{y \in U} (p \wedge q)$

$$\neg(\forall_{x \in U} \exists_{y \in U} (p \wedge q)) \Leftrightarrow \exists_{x \in U} \forall_{y \in U} (\neg p \vee \neg q)$$

2) Prove that 17 is odd.

Recall that a number, say  $x$ , is odd iff it can be written as  $x = 2k + 1$  for some integer  $k$ .

$$17 = 2 \cdot 8 + 1$$

3) Prove that the product of any two even numbers is even.

Let  $x$  and  $y$  be arbitrary even numbers. Then there are some integers  $k_1$  and  $k_2$  such that:

$$x = 2k_1$$

$$y = 2k_2$$

Multiplying these we see that  $xy$  is even:

$$xy = 2k_1 \cdot 2k_2 = 2 \cdot (2k_1k_2)$$